ERROR ANALYSIS IN THE JOINT EVENT LOCATION/SEISMIC CALIBRATION INVERSE PROBLEM

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ABSTRACT

In previous years of this project we have formulated an approach to analyzing seismic event location uncertainty that considers the effects of both observational errors in arrival-time data and prediction errors incurred in travel-time forward modeling. The analysis takes place in the setting of the multiple-event location problem, whereby arrival-time data from a set of events are used to simultaneously solve for the locations of the events and travel-time corrections associated with the observed event-station paths. One of the events is taken as the target of the uncertainty analysis while the remaining events are treated as calibration events whose data constrain the path corrections within some level of uncertainty. Based on this joint inversion formulation, we developed a numerical scheme for computing a confidence region on the target event location that implicitly accounts for the calibration errors in the path-corrections, including the effects of uncertainty in the calibration event locations, nonlinearity of the travel-time forward problem, and non-Gaussian pick errors in observed arrival times. The confidence region is found by mapping a likelihood function, as defined for the multiple-event problem, on a hypocenter grid for the target event. The previous work focused on the basic multiple-event location problem, in which travel-time corrections comprise a simple time term for each observed station/phase combination.

This paper describes our recent efforts to adapt our location uncertainty approach to the more general, and more difficult, problem in which travel-time corrections are linked to errors in the velocity model used for travel-time prediction. The resulting multiple-event location problem identifies travel-time calibration with 3D tomography. Replacing time-term corrections with tomographic corrections in our formulation is straightforward conceptually, but not computationally. The likelihood-mapping scheme is computationally intensive and, in the tomographic case, would require performing 3D travel-time tomography hundreds or thousands of times to obtain one confidence region. To address tomographic corrections in a practical manner, we must separate the location and calibration aspects of the problem, as in customary treatments of event location uncertainty, while seeking a computational scheme that retains the generality of our joint inversion formulation as much as possible. Therefore, we have designed a two-stage approach that relies on a Gaussian description of the errors in the travel-time corrections involved with the target event, as inferred from a tomographic analysis of the calibration data (the first stage). This defines a Gaussian prior distribution on travel-time prediction errors for the second stage, the location of the target event. The computation of the target event confidence region via likelihood mapping otherwise retains the full generality of our formulation, allowing for non-Gaussian pick errors and nonlinearity of the forward problem. This paper describes the theoretical basis for our location uncertainty approach, presenting the two-stage formulation as an approximation to the complete joint location/calibration formulation. It also describes numerical algorithms under development for implementing the two-stage approach.

OBJECTIVE

This project is developing new mathematical and computational techniques for quantifying the errors in seismic event locations, including the effects of observational errors and errors in the forward model for travel-time prediction. Our approach associates the latter, or *model errors*, with the uncertainty in path travel-time corrections that have been inferred from a calibration analysis. The problems of event location and calibration (with arrival-time data from calibration events) are coupled through the travel-time corrections, and the objective of this project has been to analyze the two problems as a joint inversion problem. In so doing, one avoids making some of the simplifying, and ad hoc, assumptions that are often made when calibration results are exported to an event locator. In previous years we have applied the joint inversion framework to a simple situation (corrections as station time terms) to compute confidence regions on event locations that account for many complexities that are typically ignored or approximated: nonlinearity of the forward problem, non-Gaussian observational errors, and uncertainty in calibration-event locations.

The current year of the project has focused on the problem of applying our joint location/calibration uncertainty framework to more realistic parameterizations of travel-time corrections, in particular, corrections based on a 3D velocity model. Calibration analysis then becomes the task of 3D travel-time tomography. It was clear from the start that the joint approach, which is computationally intensive even for the time-term problem, could not be practically applied to this situation without some sort of approximation. The project goal became to discover a way to separate the calculations along traditional lines, i.e. a calibration stage providing inputs to a follow-up location stage. A key part of this goal was to retain the completeness and generality of the joint approach as much as possible. This paper describes our location uncertainty framework in general and its approximate formulation as a two-stage process, including some specific algorithms under development for implementing this prioress.

RESEARCH ACCOMPLISHED

Single-Event Location

First, we consider the single-event location problem in which the hypocenter **x** and origin time t of an event are to be determined from n arrival-time data, d_i, $i = 1,\ldots n$, observed for various seismic phases at a network stations. We can write this inverse problem as

$$d_i = T_i(\mathbf{x}) + t + c_i + e_i, \tag{1}$$

where T_i is a travel-time function for the ith station/phase, determined by a reference Earth velocity model; c_i is an unknown correction to this function; and e_i is an observational (pick) error. Since each correction c_i is not known, it can be considered a travel-time prediction error, or *model* error.

To solve this problem we have adopted a maximum-likelihood framework. The likelihood function is the product of two factors: one based on an assumed probability distribution for the pick errors, and one based on a prior distribution of the travel-time corrections. The latter is needed because we are treating the c_i as explicit, unknown variables. We assume a generalized Gaussian distribution for pick errors (Billings et al., 1994). The negative logarithm of the likelihood function, ignoring constant terms, is then given by

$$\Lambda_{tar}(\mathbf{x}, t, \mathbf{c}) = \Psi_{\mathrm{tar}}(\mathbf{x}, t, \mathbf{c}), + \Phi_c(\mathbf{c}; \mathbf{x}), \tag{2}$$

where Φ_c is the prior (negative) log likelihood on **c**, which we have allowed to depend on the event hypocenter (but not its origin time), and where the *data misfit* function Ψ_{tar} is given by

$$\Psi_{\mathrm{tar}}(\mathbf{x}, t, \mathbf{c}) = \frac{1}{p}\sum_{i=1}^{n}\left|d_i - T_i(\mathbf{x}) - t - c_i\right|^p / \sigma_i^p. \tag{3}$$

The σ_i are standard errors assigned to the data; this paper will assume these are known *a priori*.

The maximum-likelihood estimate of the event location parameters are those values $(\hat{\mathbf{x}}, \hat{t})$ that, together with $\mathbf{c} = \hat{\mathbf{c}}$, minimize Λ_{tar}:

$$\Lambda_{\text{tar}}(\hat{\mathbf{x}}, \hat{t}, \hat{\mathbf{c}}) = \min_{\mathbf{x},t,\mathbf{c}} \Lambda_{\text{tar}}(\mathbf{x}, t, \mathbf{c}). \tag{4}$$

We note that in this equation, and others to follow, minimization with respect to a set of parameters is the equivalent of solving an inverse problem for those parameters.

Location confidence regions

Our approach to uncertainty analysis in single-event location derives a confidence region on a subset of the unknown problem parameters based on a "reduced" log-likelihood function that is minimized with respect to the remaining parameters. We will explain our uncertainty approach by considering confidence regions on the target event hypocenter, **x**, which considers the function

$$\Lambda_x(\mathbf{x}) = \min_{t,\mathbf{c}} \Lambda_{\text{tar}}(\mathbf{x}, t, \mathbf{c}). \tag{5}$$

A confidence region on the epicenter, for example, would involve an additional minimization over focal depth.

Stated simply, a confidence region on **x** is a region in **x**-space that encompasses the smallest values of Λ_x, which of course includes the maximum-likelihood estimate $\hat{\mathbf{x}}$. The confidence region is defined by the inequality

$$\tau(\mathbf{x}) \equiv \Lambda_x(\mathbf{x}) - \Lambda_x(\hat{\mathbf{x}}) \leq \tau_\beta \tag{6}$$

where $\tau(\mathbf{x})$ defines a test statistic and τ_β is a critical value of the test statistic selected for a given confidence level β (e.g. $\beta = 0.95$ for a 95% confidence region). Rodi (2006) discusses different ways of defining τ_β, in either a Neyman-Pearson or Bayesian framework, and for computing τ_β numerically. Amongst these choices, the preferred approach we have settled on is as follows.

We base the definition of τ_β on a Bayesian framework, which treats $\tau(\mathbf{x})$ as the exponent of a posterior distribution on **x**. Then τ_β is the solution to

$$\int_{\tau(\mathbf{x}) \leq \tau_\beta} d\mathbf{x}\, e^{-\tau(\mathbf{x})} = \beta \int_{\text{all } \mathbf{x}} d\mathbf{x}\, e^{-\tau(\mathbf{x})}. \tag{7}$$

Rodi (2006) refers to this definition as "quasi-Bayesian" because the implied posterior distribution of **x** is not derived from a posterior joint distribution on all the parameters by integrating out *t* and **c**. The minimization over *t* and **c** in equation (5) is used instead.

The numerical algorithm we have developed consists of two steps. The first step, likelihood *mapping*, computes τ on a grid in **x** -space. The second step, likelihood *integration*, estimates the integral on the right-hand side of equation (7) with numerical integration of the grid values of τ. From this integral, and a histogram of the grid-sampled τ values, it is straightforward to calculate τ_β as a function of β and then identify the grid points that belong to the confidence region for any given β.

Figure 1 shows an example of our numerical confidence region algorithm applied to a Nevada Test Site (NTS) explosion at the Pahute Mesa testing area. The center panel shows the log-likelihood function mapped on an epicenter grid. The right panel shows the confidence regions, at three confidence levels, that result after performing the likelihood integration step to find τ_β. This example did not account for model errors: the travel-time corrections, c_i, were fixed to zero. Additionally, the event depth was fixed to its known value.

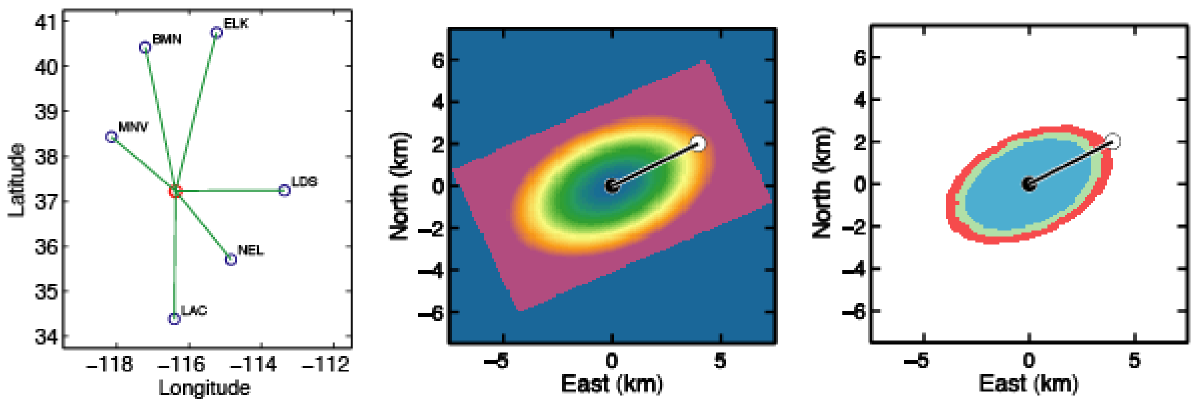

Figure 1. Likelihood function and confidence regions for a Pahute Mesa explosion, derived from 6 Pn arrivals, with the pick error distribution assumed to be Gaussian (p = 2). *Left*: The station geometry. *Center*: Log-likelihood function mapped on an epicenter grid. *Right*: Epicenter confidence regions determined from the likelihood function. Confidence regions are shown for 90, 95 and 98% confidence (blue, green and red, respectively). In the right two panels, the black circle marks the maximum-likelihood estimate for the event location, and the white circle is its GT0 location (from Walter et al., 2003).

The computational efficiency of our numerical confidence region algorithm depends on how much CPU time it takes to evaluate $\Lambda_x(\mathbf{x})$, involving the minimization in equation (5), for each grid point $\mathbf{x}$. When model errors are allowed, this is controlled largely by the nature of the prior misfit function for corrections, $\Phi_c(\mathbf{c};\mathbf{x})$, appearing in equation (2).

Prior Likelihood on Corrections and Calibration

It is common in seismic event location algorithms to treat the travel-time corrections c_i as independent Gaussian random variables (model errors) with known means and variances. Later in our development we will do the same (but including covariances). First, let us consider the formal relationship between Φ_c, the prior log-likelihood function on $\mathbf{c}$, and the problem of seismic calibration.

We assume that calibration is performed with seismic arrival-time data observed from m calibration events. Following our single-event formulation, we can write the calibration problem then as

$$d_{i,j} = T_i(\mathbf{x}_j) + t_j + c_{i,j} + e_{ij}, \tag{8}$$

where i indexes station/phase pairs and j indexes the events. Equation (8) holds for the subset of (i, j) paths for which data are available. The unknowns in this problem are the path travel-time corrections c_{ij} and, to the extent that the calibration events do not have GT0 locations, $(\mathbf{x}_j, t_j)$, $j = 1, \ldots, m$.

The set of station/phase combinations indexed by i may be the same as those for the single-event location problem of equation (1), but we do not require this to be true. Henceforth, the n station/phases under discussion may be considered a master set, with equations (1) and (8) holding for appropriate subsets for which observations are available.

To link the target-event location problem, (1), and the calibration problem, (8), we need to link the path travel-time corrections c_i and c_{ij} through some underlying parameterization. In previous years, the project has focused on the "basic" multiple-event location problem (e.g., Pavlis and Booker, 1983), which equates the corrections to a set of time terms common to the events but different between station/phase combinations. Letting a_i be the time term for the ith station/phase, we then have

$$c_i = a_i \tag{9}$$
$$c_{ij} = a_i . \tag{10}$$

In the current year, we have considered the more complex situation of *tomographic* travel-time corrections. Under the linear approximation to the velocity dependence of travel times, these corrections can be expressed as

$$c_{i,j} = \int d\mathbf{x}' k_i(\mathbf{x}';\mathbf{x}_j)\delta u(\mathbf{x}'), \tag{11}$$

where $k_i(\mathbf{x}';\mathbf{x}_j)$ is a travel-time sensitivity kernel and $\delta u(\mathbf{x}')$ is the slowness difference between the real Earth and the reference Earth model used for evaluating the travel-time function T_i. In this case, the slowness perturbation $\delta u(\mathbf{x}')$ serves as the underlying parameterization of travel-time corrections and c_i in the single-event problem is defined by integrating this perturbation with an appropriate sensitivity kernel for the target event, $k_i(\mathbf{x}',\mathbf{x})$.

We can generalize these specific parameterizations and others as

$$c_i = \mathbf{a}_i(\mathbf{x})^T \mathbf{q} \tag{12}$$
$$c_{ij} = \mathbf{a}_i(\mathbf{x}_j)^T \mathbf{q}, \tag{13}$$

where the vector **q** contains an underlying, discrete set of parameters for generating travel-time corrections. It will be convenient to write equation (12) alternatively as

$$\mathbf{c} = \mathbf{A}(\mathbf{x})\mathbf{q}, \tag{14}$$

where the vectors $\mathbf{a}_i(\mathbf{x})$ (transposed) occupy the rows of the *sensitivity* matrix $\mathbf{A}(\mathbf{x})$.

Being consistent with our maximum-likelihood formation for single-event location, we define a (minus) log-likelihood function for the calibration problem as

$$\begin{aligned}\Lambda_{\text{cal}}(\mathbf{x}_1,t_1,\ldots,\mathbf{x}_m,t_m,\mathbf{q}) = {}& \Psi_{\text{cal}}(\mathbf{x}_1,t_1,\ldots,\mathbf{x}_m,t_m,\mathbf{q}) \\ & + \Phi_{\text{gt}}(\mathbf{x}_1,t_1,\ldots,\mathbf{x}_m,t_m) + \Phi_q(\mathbf{q}).\end{aligned} \tag{15}$$

Ψ_{cal} is the misfit function for the calibration data:

$$\Psi_{\text{cal}}(\mathbf{x}_1,t_1\ldots,\mathbf{x}_m,t_m,\mathbf{q}) = \frac{1}{p}\sum_{ij}\left| d_{ij} - T_i(\mathbf{x}_j) - t_j - \mathbf{a}_i(\mathbf{x}_j)^T \mathbf{q}\right|^p . \tag{16}$$

Φ_{gt} and Φ_q express prior information on the calibration event locations and calibration parameters, respectively. We can now state a formal relationship between calibration and a prior likelihood function for model errors in the single-event location problem:

$$\Phi_{\text{cal}}(\mathbf{c};\mathbf{x}) = \min_{\mathbf{q}:\mathbf{A}(\mathbf{x})\mathbf{q}=\mathbf{c}} \; \min_{\mathbf{x}_2,t_1\ldots,\mathbf{x}_m,t_m} \Lambda_{\text{cal}}(\mathbf{x}_1,t_1\ldots,\mathbf{x}_m,t_m,\mathbf{q}). \tag{17}$$

The two minimizations embody an inversion of the calibration data for the calibration parameters **q** in conjunction with a relocation of the non-GT0 calibration events. The minimization constrains **q** to match the given correction vector: $\mathbf{A}(\mathbf{x})\mathbf{q} = \mathbf{c}$. Equation (17) defines an implicit transformation of the posterior distribution on the calibration parameters **q**, as implied by an inversion of calibration data, into a prior distribution on $\mathbf{A}(\mathbf{x})\mathbf{q}$.

Joint Location/Calibration Analysis

A confidence region on the target event hypocenter $\mathbf{x}$ requires mapping the function $\Lambda_x(\mathbf{x})$ in hypocenter space, which in turn requires the evaluation of $\Phi_c(\mathbf{c};\mathbf{x})$ for each grid point $\mathbf{x}$ (see equations (2) and (5)). If one could evaluate equation (17) in some closed form, it would be possible to separate the computations involved in calibration and target event location. In general, however, this is not possible without invoking some approximation.

The main goal of this project was to investigate an approach that bypassed the computation of Φ_c by combining the location and calibration problems into a joint inverse problem. In the development here, this means substituting equation (17) into (2) so that (5) becomes (with some rearranging)

$$\Lambda_x(\mathbf{x}) = \min_{\mathbf{q}}\left\{\min_t \Psi_{\text{tar}}\left(\mathbf{x}, t, \mathbf{A}(\mathbf{x})\mathbf{q}\right) + \min_{\mathbf{x}_1, t_1 \ldots, \mathbf{x}_m, t_m} \Lambda_{\text{cal}}\left(\mathbf{x}_1, t_1 \ldots, \mathbf{x}_m, t_m, \mathbf{q}\right)\right\} \tag{18}$$

The computational burden resulting from doing this is apparent. To evaluate $\Lambda_x(\mathbf{x})$ for each grid-point $\mathbf{x}$ requires performing a minimization over all the remaining parameters of the joint inverse problem, including calibration parameters ($\mathbf{q}$) and calibration event locations ($\mathbf{x}_1,t_1,\ldots,\mathbf{x}_m,t_m$). Therefore, a large inverse problem is solved repeatedly in order to calculate Λ_x on a grid.

These calculations are feasible for the basic multiple-event location problem, and we have implemented a joint inversion uncertainty analysis for this problem as part of a location program (GMEL) developed under this and previous projects. Figure 2 shows confidence regions for the same Pahute Mesa event as in Figure 1, derived using 32 other explosions at Pahute Mesa and Rainier Mesa as calibration events. Only one of the calibration events, a relatively well-recorded event at Rainier Mesa (16 Pn arrivals), was assigned a finite ground-truth level. The three panels show the resulting confidence regions, which now take model errors into account, under three assumptions about the GT level of that calibration event: GT0, GT2 or GT5 (at 90% confidence). We see that the confidence regions are larger than when model errors are assumed to be zero (Figure 1), and grow as the uncertainty in the GT calibration event location is increased. Figure 3 repeats the GT0 case with various non-Gaussian pick error distribution: p = 1.5, 1.25 and 1 (left, center, right, respectively).

The confidence regions shown in Figures 2 and 3 took between 5 and 30 CPU minutes each to compute, the ones for smaller p taking the longest. In at least one case (right panel in Figure 3) the likelihood function was not mapped sufficiently well to compute accurate values of τ_β, meaning even more computation was needed. To apply the joint inversion approach with tomographic corrections, thus, would be prohibitive since each point on a likelihood grid would require performing a full 3D tomography in conjunction with calibration event relocation.

The remainder of this paper outlines a two-stage approach to event location uncertainty which, following conventional practice, isolates the bulk of the calculations in a calibration stage, whose results can be used efficiently for calculating confidence regions for target events as they arise. The two-stage approach depends on approximating the distribution of model errors, Φ_c, as Gaussian.

Gaussian Approximation

To simplify notation in this and the next section, we will not show the dependence of Φ_c on $\mathbf{x}$. This dependence derives from the dependence of $\mathbf{A}$, the sensitivity matrix for the target event, on $\mathbf{x}$.

Let $\mathbf{c} = \mathbf{c}_0$ minimize $\Phi_c(\mathbf{c})$. Since the gradient of Φ_c is zero at $\mathbf{c}_0$, a quadratic approximation is given by

$$\Phi_c(\mathbf{c}) \approx \Phi_c(\mathbf{c}_0) + \frac{1}{2}(\mathbf{c}-\mathbf{c}_0)^T \mathbf{V}_c^{-1}(\mathbf{c}-\mathbf{c}_0) \tag{19}$$

where $\mathbf{V}_c$ is a symmetric matrix. This approximation is equivalent to assuming that $\mathbf{c}$ has a Gaussian prior probability distribution with moments

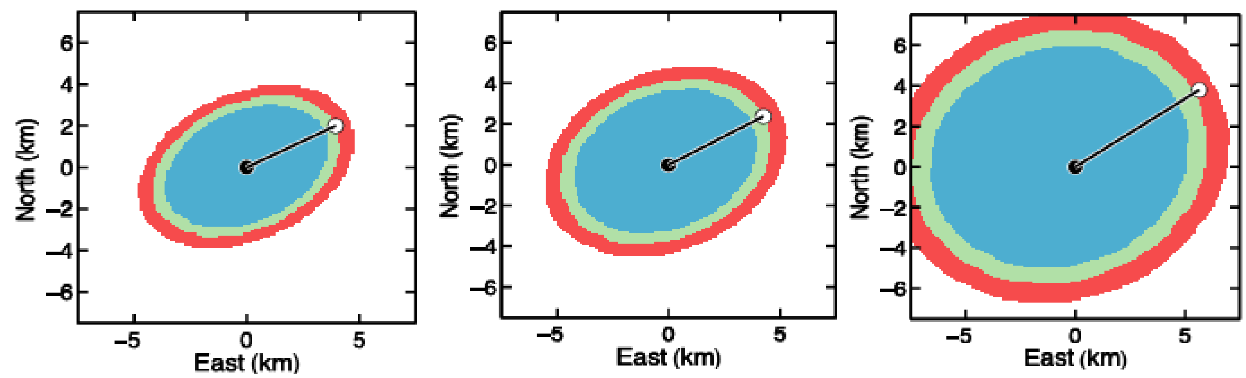

Figure 2. Confidence regions for the same Pahute Mesa event as in Figure 1, but accounting for uncertainty in travel-time corrections (model errors). The corrections were constrained by 32 calibration events (other NTS explosions) with one of them assigned a finite GT level: GT0 (left), GT2 (center), or GT5 (right). The pick error distribution was assumed to be Gaussian (*p* = 2).

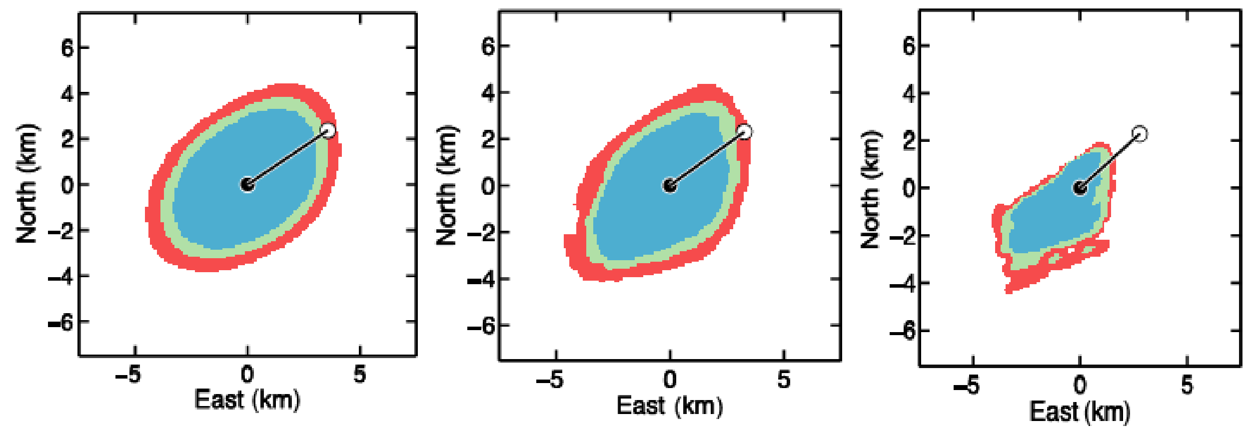

Figure 3: Confidence regions for the same Pahute Mesa event, accounting for model errors as constrained by 32 calibration events (see Figure 2). The GT calibration event is assumed to be GT0 and the pick error distribution is non-Gaussian: *p* = 1.5 (left), *p* = 1.25 (center), and *p* = 1 (right).

$$\mathrm{E}[\mathbf{c}] = \mathbf{c}_0 \tag{20}$$

$$\mathrm{Var}[\mathbf{c}] = \mathbf{V}_c. \tag{21}$$

The quadratic approximation makes it possible to separate the calculations of the joint location/calibration problem into two stages. The next two sections show how the mean and variance of **c** can be derived from the calibration analysis (stage one) and used in locating the target event (stage two).

Stage 1: Calibration

The calibration problem involves minimizing Λ_{cal} in equation (15) with respect to both the correction parameter vector **q** and the calibration event locations ($\mathbf{x}_j, t_j$). We assume here that an algorithm is available to solve this problem, e.g. GMEL for basic multiple-event location.

Letting $\hat{\mathbf{q}}$ be the maximum-likelihood estimate of **q** obtained from solving the calibration problem, we can take the mean of **c** to be

$$c_0 = \mathbf{A}\hat{\mathbf{q}}. \tag{22}$$

To obtain the variance of **c** we can use a perturbation method described by Rodi and Myers (2007) for computing covariance matrices (their "implicit" method). Applied to the task at hand, the method solves a sequence of perturbed calibration problems, one for each row of **A**, in which the calibration log-likelihood function is augmented with an additional term. For the *i*th row, the augmented function is

$$\Lambda'_{\mathrm{cal}}(\ldots,\mathbf{q}) = \Lambda_{\mathrm{cal}}(\ldots,\mathbf{q}) + \frac{1}{2\sigma^2}(\mathbf{c}_0 + \rho\mathbf{n}_i - \mathbf{A}\mathbf{q})^T(\mathbf{c}_0 + \rho\mathbf{n}_i - \mathbf{A}\mathbf{q}), \tag{23}$$

where $\mathbf{n}_i$ is the *i*th column of the identity matrix. The residual vectors yielded by the perturbed solutions $(\mathbf{c}_0 + \rho\mathbf{n}_i - \mathbf{A}\mathbf{q})$ can be used to construct $\mathbf{V}_c$. As presented by Rodi and Myers (2007), this is an exact method applicable when Λ_{cal} is a quadratic function of **q**. Here, Λ_{cal} might not be quadratic owing to trade-offs between **q** and the calibration event locations or, more blatantly, the assumption of non-Gaussian pick errors ($p \neq 2$). For our purposes then, it is a method for fitting a quadratic function to $\Phi_c(\mathbf{c})$. The choice of the constants ρ and σ might then be crucial; ρ should anticipate the standard deviation of c_i and σ should be somewhat smaller than ρ.

The quantities $\mathbf{c}_0$ and $\mathbf{V}_c$ will depend on the target event hypocenter, **x**, exactly when **A** does. If this dependence cannot be neglected, then our two-stage approach would require that that $\mathbf{c}_0$ and $\mathbf{V}_c$ be calculated on an appropriate grid in **x**-space suitable for interpolation to an arbitrary **x**.

Stage 2: Location of a Target Event

We now restore the dependence of $\mathbf{c}_0$ and $\mathbf{V}_c$ on **x**.

Using the Gaussian (quadratic) approximation to Φ_c, the maximum-likelihood estimates of the target event location and travel-time correction vector, $(\hat{\mathbf{x}}, \hat{t}, \hat{\mathbf{c}})$, will minimize

$$\Lambda_{\mathrm{tar}}(\mathbf{x},t,\mathbf{c}) = \Psi_{\mathrm{tar}}(\mathbf{x},t,\mathbf{c}) + \frac{1}{2}(\mathbf{c} - \mathbf{c}_0(\mathbf{x}))^T \mathbf{V}_c^{-1}(\mathbf{x})(\mathbf{c} - \mathbf{c}_0(\mathbf{x})), \tag{24}$$

where Ψ_{tar} is given by equation (3).

First, we point out that when the data errors are Gaussian ($p = 2$) one can minimize Λ_{tar} analytically with respect to **c** for fixed **x** and *t*. Substituting the solution for **c** into equation (24) yields a quadratic data misfit function with the data d_i, corrected by $(c_0)_i$ and with the diagonal covariance matrix for pick errors, diag$\{\sigma_1^2,\ldots,\sigma_n^2\}$, augmented by the full matrix $\mathbf{V}_c$. Some location programs (e.g., LocOO, Ballard, 2002) can accommodate such misfit functions. However, this analytic elimination of **c** cannot be done for non-Gaussian pick errors.

The event location scheme we are developing treats **c** as an explicit unknown, and performs minimization in a hierarchy. We are currently implementing the scheme in GMEL under the assumption that **A** does not depend on **x** (such as in basic multiple-event location). We describe the algorithm for this situation first.

The inner loop of the algorithm minimizes Ψ_{tar} (the data misfit) with respect to (**x**,*t*), using grid search, with **c** fixed to some current value. Denote the grid-search solution as ($\mathbf{x}^*(\mathbf{c})$, $t^*(\mathbf{c})$). We can then define an objective function for **c** as

$$\Lambda_c(\mathbf{c}) = \Psi_{\mathrm{tar}}(\mathbf{x}^*(\mathbf{c}), t^*(\mathbf{c}), \mathbf{c}) + \frac{1}{2}(\mathbf{c} - \mathbf{c}_0)^T V_c^{-1}(\mathbf{c} - \mathbf{c}_0). \tag{25}$$

The outer loop of the location algorithm minimizes $\Lambda_c(\mathbf{c})$ using a nonlinear conjugate gradients (NLCG) technique. Each step of the NLCG loop updates **c** and performs a grid search to update $\mathbf{x}^*(\mathbf{c})$ and $t^*(\mathbf{c})$. When the NLCG iteration converges, we have $\mathbf{x}^*(\mathbf{c}) = \hat{\mathbf{x}}$ and $t^*(\mathbf{c}) = \hat{t}$.

When $\mathbf{c}_0$ and $\mathbf{V}_c$ *do* depend on $\mathbf{x}$, such as for tomographic corrections, the scheme could be modified to perform the grid-search on the complete function Λ_{tar} in (24) rather than just its first term. However, this might incur a high computational cost. If the dependence of $\mathbf{c}_0$ and $\mathbf{V}_c$ on $\mathbf{x}$ is not too strong, it might suffice to update $\mathbf{c}_0$ and $\mathbf{V}_c$ only once per NLCG iteration. Rodi and Myers (2007) show evidence that $\mathbf{V}_c$, for tomographic corrections, *does* depend strongly on $\mathbf{x}$, however.

CONCLUSIONS AND RECOMMENDATIONS

This project has developed a rigorous theoretical framework for uncertainty analysis in seismic event location, linking travel-time prediction errors (model errors) to uncertainty in travel-time calibration. This link, and the fact that calibration events themselves have location errors, recognizes event location and seismic calibration as coupled inverse problems of similar structure. Our framework thus formulates the task of characterizing the location uncertainty in an event of interest as the focus of an uncertainty analysis in the larger joint location/calibration inverse problem.

In previous years we implemented this framework for a relatively simple calibration scenario (basic multiple-event location with time-term corrections) to demonstrate the validity and generality of the approach. A pessimistic, but tentative, conclusion of Rodi (2006) was that a joint location/calibration approach might not be computationally feasible for more complex scenarios as when calibration is done by 3D travel-time tomography. Aside from computational issues, we add the logistical reality that event location and, certainly, seismic calibration are not fully automated processes that can be embedded in a computer algorithm of the type we developed. These tasks are performed by different groups of people, often with different sources of data, and on different time frames.

Our recent work has attempted to reformulate our uncertainty method as a two-stage process, calibration followed by the location of new events, in a way that preserves the rigor and generality of the joint inversion approach. This task requires that calibration uncertainty be captured and exported to an event locator. We developed one such formulation that makes the new assumption that calibration uncertainty can be characterized by a Gaussian distribution on model errors, allowing for covariances between the errors. The formulation retains the accommodation of forward problem nonlinearity, non-Gaussian errors in arrival-time observations, and uncertainty in calibration event locations. Further, we have designed new algorithms to implement our two-stage formulation that place the bulk of the computations in the calibration stage and, in principle, are applicable to a wide variety of calibration scenarios including 3D tomographic calibration. We are in the process of implementing the new algorithms for basic multiple-event location in order to corroborate the computational advantages and test the Gaussian approximation for model errors in this problem.

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