SEISMIC SOURCE SCALING AND DISCRIMINATION IN DIVERSE TECTONIC ENVIRONMENTS

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ABSTRACT

The objectives of this study are to improve low-magnitude regional seismic discrimination by performing a thorough investigation of earthquake source scaling using diverse, high-quality datasets from varied tectonic regions. Local-to-regional high-frequency discrimination requires an estimate of how earthquakes scale with size. Walter and Taylor (2002) developed the Magnitude and Distance Amplitude Corrections (MDAC) method to empirically account for these effects through regional calibration. The accuracy of these corrections has a direct impact on our ability to identify clandestine explosions in the broad regional areas characterized by low seismicity. Unfortunately our knowledge of source scaling at small magnitudes (i.e., mb < ~4.0) is poorly resolved. It is not clear whether different studies obtain contradictory results because they analyse different earthquakes, or because they use different methods. Even in regions that are well studied, such as test sites or areas of high seismicity, we still rely on empirical scaling relations derived from studies taken from half-way around the world at inter-plate regions.

We investigate earthquake sources and scaling from different tectonic settings, comparing direct and coda wave analysis methods. We begin by developing and improving the two different methods, and then in future years we will apply them both to each set of earthquakes. Analysis of locally recorded, direct waves from events is intuitively the simplest way of obtaining accurate source parameters, as these waves have been least affected by travel through the earth. But there are only a limited number of earthquakes that are recorded locally, by sufficient stations to give good azimuthal coverage, and have very closely located smaller earthquakes that can be used as an empirical Green’s function (EGF) to remove path effects. In contrast, coda waves average radiation from all directions so single-station records should be adequate, and previous work suggests that the requirements for the EGF event are much less stringent. We can study more earthquakes using the coda-wave methods, while using direct wave methods for the best recorded subset of events so as to investigate any differences between the results of the two approaches.

Finding “perfect” EGF events for direct wave analysis is difficult, as is ascertaining the quality of a particular EGF event. We develop a multi-taper method to obtain time-domain source-time-functions by frequency division. If an earthquake and EGF event pair are able to produce a clear, time-domain source pulse then we accept the EGF event. We then model the spectral (amplitude) ratio to determine source parameters from both direct P and S waves. We use the well-recorded sequence of aftershocks of the M5 Au Sable Forks, NY, earthquake to test the method and also to obtain some of the first accurate source parameters for small earthquakes in eastern North America. We find that the stress drops are high, confirming previous work suggesting that intraplate continental earthquakes have higher stress drops than events at plate boundaries.

We simplify and improve the coda wave analysis method by calculating spectral ratios between different sized earthquakes. We first compare spectral ratio performance between local and near-regional S and coda waves in the San Francisco Bay region for moderate-sized events. The average spectral ratio standard deviations using coda are ~0.05 to 0.12, roughly a factor of 3 smaller than direct S-waves for 0.2 < f < 15.0 Hz. Also, direct wave analysis requires collocated pairs of earthquakes whereas the event-pairs (Green’s function and target events) can be separated by ~25 km for coda amplitudes without any appreciable degradation. We then apply coda spectral ratio method to the 1999 Hector Mine mainshock (Mw 7.0, Mojave Desert) and its larger aftershocks. We observe a clear departure from self-similarity, consistent with previous studies using similar regional datasets.
OBJECTIVES

The objectives of this study are to improve low-magnitude regional seismic discrimination by performing a thorough investigation of earthquake source scaling using diverse, high-quality datasets from varied tectonic regions. Local-to-regional high-frequency discrimination requires an estimate of how earthquakes scale with size. Walter and Taylor (2002) developed the MDAC method to empirically account for these effects through regional calibration. The accuracy of these corrections has a direct impact on our ability to identify clandestine explosions in the broad regional areas characterized by low seismicity. Unfortunately our knowledge at small magnitudes (i.e., \( m_b < \sim 4.0 \)) is poorly resolved, and source scaling remains a subject of on-going debate in the earthquake seismology community. Recently there have been a number of empirical studies suggesting scaling of micro-earthquakes is non-self-similar (e.g. Kanamori et al., 1993; Abercrombie, 1995; Mayeda and Walter; 1996, Mori et al., 2003; Stork and Ito, 2004; Izutani and Kanamori, 2001, yet there are an equal number of compelling studies that would suggest otherwise (e.g., McGarr, 1999, Ide and Beroza, 2001, Imanishi et al., 2004, Prieto et al., 2004. It is not clear whether different studies reach different conclusions because they use different datasets and scaling varies with location, or because they use different methods. Sonley and Abercrombie (2006) show that small variations in the commonly used methods can lead to significant differences in results. Even in regions that are well studied, such as test sites or areas of high seismicity, we still rely on empirical scaling relations derived from studies taken from half-way around the world at inter-plate regions.

In summary, we address the following problems.

1. Do different studies obtain different results because they use different methods, or because they analyse different data sets? We will investigate whether coda and direct wave methods applied to the same datasets provide the same scaling.
2. Is scaling dependent upon the tectonic setting? We will investigate earthquakes from different tectonic settings and depth ranges, using the same coda and direct wave methods.
3. There have been few studies in intra-plate areas where seismicity is low and/or in regions where a clandestine test might occur. The MDAC method currently assumes earthquake source scaling that was derived exclusively from the western United States. Can we extrapolate or transport results from one region to others, or must we calibrate to each specific region? We will analyse earthquakes from both interplate (e.g., California) and intraplate (e.g., Eastern North America) regions to specifically address this question.

RESEARCH ACCOMPLISHED

Our approach to obtaining improved source parameters for small earthquakes focuses on the direct and coda wave methods: to improve and investigate them both, and then to apply them to diverse data sets. Locally recorded, direct waves from events have been least affected by travel through the earth, and so are thought to be the best candidate for obtaining accurate source parameters. But there are only a limited number of earthquakes that are recorded locally, by sufficient stations to give good azimuthal coverage. Even fewer of these have an equivalently well recorded, very closely located smaller earthquake that meets the stringent criteria required to be a good EGF to remove path effects. This EGF method is the preferred one for isolating the source, but concern about the quality of the EGFs is a major source of uncertainty in studies that use these methods. In contrast, coda waves average radiation from all directions so single-station records should be adequate, and previous work suggests that the requirements for the EGF event are much less stringent. Our approach is to:

1. Develop an easy to apply coda wave spectral ratio method to obtain source parameters for large groups of earthquakes
2. Identify the mainshock – EGF earthquake pairs that meet stringent criteria for selecting the EGF, and obtain source parameters from the direct waves for this subset of events.
3. Use the direct wave results to confirm, and if necessary correct, the coda wave results
4. Apply these methods to data sets from a range of tectonic environments.
5. Determine the implications of the determined source scaling results for both coda calibration and regional discrimination using MDAC and other similar means of source and path-corrected discriminants.

So far we have worked to develop and improve both the coda wave and direct wave methods we will apply consistently to all the data sets. The coda wave approach is now published (Mayeda et al., 2007).
Development of Coda Spectral Ratio Method: the San Francisco Bay Area

We chose 10 stations operated by the Berkeley Digital Seismic Network (BDSN) along with 62 events between 3.5 < $M_L < 5.2$ spanning ~1 to 220 km epicentral distance. All stations are three-component high-frequency (80 sps) channels and the events are all located to within 2 km in absolute location by the Northern California Data Center (NCDC) in conjunction with the University of California, Berkeley Seismological Laboratory (BSL).

Each waveform was corrected for instrument response, then visually inspected and clipped data were not processed. Since the aim of this part of the study was to look at amplitude ratio variability with event separation and azimuth, we selected event pairs without consideration of magnitude. In total, we found 259 possible event pairs that ranged between ~0 and 25 km distance in absolute separation. For both the coda and direct waves we chose 14 narrow frequency bands ranging between 0.2 and 25.0 Hz. For the coda, we formed averaged narrowband time-domain envelopes using the two horizontal components following the method of Mayeda et al. (2003)]. For the direct S-waves, log RMS amplitudes from the amplitude spectra were obtained for a 5 second window starting at the S-wave onset for each component. To further screen the data, we required a minimum of 5 stations recording the event pairs per frequency band, and ended up with as many as 184 pairs at 4.0-6.0 Hz and as few as 35 pairs at 15.0-20.0 Hz. Virtually all events have source mechanisms derived from moment-tensor inversion and, as expected, strikes and dips are consistent with the orientation of the regional faults in the greater San Francisco Bay region.

Based on prior work on local and regional coda, we hypothesize that amplitude ratios of the same event-pair will be much more stable for coda than for direct S-waves. We tested this hypothesis by forming narrowband amplitude ratios for both wave types and compared their standard deviations for many event pairs. In practice, direct wave empirical Green’s function studies have limited their data to co-located events with the same source mechanism (e.g., Mori et al., 2003; Izutani and Kanamori, 2001; Venkataraman et al., 2002). This, however, severely limits the useable amount of data, and if proven feasible, the coda’s stability and minimal move-out will allow inclusion of more events that are separated in distance and not necessarily of the same focal mechanism. This will allow for redundancy and lower variance estimates, a point thoroughly discussed recently by Prieto et al. (2007).

For both the coda and direct S-waves, we formed amplitude ratios for event pairs by simply subtracting the log10 amplitudes for each station that recorded the event pair. Since the site and path are the same for both events, the ratio should reflect the source differences in the frequency band. Figure 1a shows an example of direct wave amplitude ratios (top) and coda wave ratios (bottom) for an event pair that has the same depth of 7 km, with an epicentral location difference of 15.7 km. In general, for a magnitude 4 event, the measurement window for the coda envelope was ~300 seconds for 0.2-0.3 Hz, and ~100 seconds at 2-3 Hz. We note that for the direct waves, there was no difference if we used horizontal or vertical elements. Both events are roughly the same size and, as expected, the log10 average of the ratios is close to 0; however, the direct wave results are significantly more scattered.

Using all available ratios, such as the example shown in Figure 1a, we plot the amplitude ratio standard deviation as a function of event-pair offset for each frequency band (Figure 1b). We see that the coda amplitude ratios are roughly a factor of 3 smaller and do not show any appreciable increase with event separation, in sharp contrast to the direct waves. This means that the use of the coda will allow for the inclusion of many more events in spectral ratios studies, whereas in direct wave studies, only those events that are virtually co-located are used. Equally important, the coda spectral ratios are significantly less scattered and thus source parameters, such as corner frequency, will be better constrained when we fit the observed data with theoretical source models, such as the commonly used omega-square model (Aki, 1967; Brune, 1970).

We also investigated whether any of the pairs of earthquakes used in the coda wave study met the criteria for use in direct-wave analysis. Unfortunately, we found only one pair of events that were able to produce a source pulse. This is inadequate to perform any sort of rigorous comparison so we did not pursue this work further. The comparison does demonstrate how useful the coda methods should be when we have closely compared and calibrated them to some high-quality direct wave analyses. They will enable the study of many earthquakes which do not meet the criteria needed for high-quality direct wave EGF analysis.

Initial Application of the Coda Spectral Ratio Method to the $M_w$ 7.0 Hector Mine Sequence

Next, we turn our attention to local and regional recordings of the $M_w$ 7.0 Hector Mine mainshock and 6 aftershocks ranging between $M_L$ 3.7 and 5.4. In this case we consider 6 broadband stations ranging between ~60 and 700 km: GSC, PFO, MNV, CMB, TUC, and ELK.
All the events have independent regional seismic moment estimates from full waveform inversion by G. Ichinose using the methodology outlined in Ichinose et al., [2003]. For consistency, we used the regional estimates of $M_w$ to avoid any biases since they were all computed using the same method, velocity model, and station distribution. For the mainshock, we use an $M_w$ of 7.0 following the results of G. Ichinose.

Assuming a simple single corner frequency source model (Aki, 1967; Brune, 1970), the ratio of the moment-rate functions for two events (1 and 2) is given by,

$$\frac{M_1(\omega)}{M_2(\omega)} = \frac{M_{01}}{M_{02}} \left[ 1 + \left( \frac{\omega}{\omega_c} \right)^2 \right]^{\beta/2}$$

(1)

where $M_0$ is the seismic moment and $\omega_c$ is the angular corner frequency ($2\pi f_c$) and $\beta$ is the high frequency decay rate.

At the low frequency limit the source ratio shown in equation 1 is proportional to the ratio of the seismic moments $M_{01}/M_{02}$, whereas at the high frequency limit, equation 1 is asymptotic to $\left[ M_{01}/M_{02} \right]^{1+\beta/3}$ under self-similarity. If we follow the usual Brune [1970] omega-square model and set $\beta=2$, the exponent of the high-frequency ratio becomes 1/3. However, it has been proposed by Kanamori and Rivera (2004) that the scaling between moment and corner frequency could take on the form,

$$M_0 \sim \omega_c^{3+\epsilon}$$

(2)
where $e$ represents the deviation from self-similarity and is usually thought to be a small positive number. For example, Walter et al. (2006) and Mayeda et al. (2005) found $e$ to be close to 0.5 for the Hector Mine mainshock and its aftershocks using independent spectral methods. For the current study we use the source spectrum portion of the MDAC methodology of Walter and Taylor (2001), which allows for the variation of the corner frequency that does not have to be self-similar. For example,

$$
\omega_c = \left( k \sigma_a / M_0 \right)^{1/3}
$$

and

$$
\sigma_a = \sigma'_a \left( M_0 / M'_0 \right)^\psi
$$

where $\sigma_a$ is the apparent stress [Wyss, 1970], $\sigma'_a$ and $M'_0$ are the apparent stress and seismic moment of the reference event, and $\psi$ is a scaling parameter. For constant apparent stress, $\psi = 0$ and $e = 0$, however, Mayeda and Walter [1996] found $\psi = 0.25$ for moderate to large earthquakes in the western United States. By using the corner frequency defined in (3) into equation 1, we can apply a grid search to find the parameters that best fit the spectral ratio data.

Using all 6 stations, we formed the average spectral ratio between the mainshock and each of the aftershocks, then grid-searched using equation 1 and 3 assuming that the reference moment corresponded to an $M_w$ 5.0 event and the reference apparent stress was varied between 0.5 and 10 bars. As observed for San Francisco Bay Area events, the coda spectral ratios for Hector Mine events were very stable, with average standard deviations less than 0.1 for all frequencies. Figure 2 shows all 6 ratios along with the high and low frequency asymptotes from equation 1 as solid black horizontal lines and dashed lines represent the case when $p = 1.5$. The average for each band is shown by red triangles and red lines represent $\pm 1 \sigma$. Blue lines represent the best fitting MDAC spectral ratios that fit all 6 ratios simultaneously, whereas the green lines represent the case when each ratio was fit individually. We find that the best fitting model corresponds to a reference event of $M_w$ 5.0 with an apparent stress of 0.1 MPa and a scaling parameter ($\psi$), of 0.25, in good agreement with previous studies using independent methods.

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Figure 2. Spectral ratios for the Hector Mine mainshock relative to 6 aftershocks. In each figure, we show the low and high frequency asymptotes from equation 1 as solid black horizontal lines and dashed lines represent the case when $p = 1.5$. The average for each band is shown by red triangles and red lines represent $\pm 1 \sigma$. Blue lines represent the best fitting MDAC spectral ratios that fit all 6 ratios simultaneously, whereas the green lines represent the case when each ratio was fit individually. We find that the best fitting model corresponds to a reference event of $M_w$ 5.0 with an apparent stress of 0.1 MPa and a scaling parameter ($\psi$), of 0.25, in good agreement with previous studies using independent methods.
If the high-frequency decay value \( p \) were close to 1.5 it would be possible to nearly match the spectral ratios shown as dashed lines in Figure 2. However, given that such shallow falloff is not consistent with most earthquake observations (e.g., Hough, 2001) and independent methods (e.g., Mayeda et al., 2005; Walter et al., 2006), our preferred interpretation is that the apparent stresses are systematically lower for the aftershocks than the mainshock. If all events have Brune-style spectra with an \( f^{-2} \) fall-off at high frequencies, this implies the corner frequency scaling is steeper than \( f^{-3} \) for self-similar, constant apparent stress scaling. Finally, we compare our estimate of the corner frequency of the mainshock with the teleseismic estimate of 0.059 Hz from Venkataraman et al. (2002). Our estimate of 0.069 Hz from simultaneous fitting of the ratios is in good agreement, supporting the validity our approach.

**Development of direct wave EGF Method, and application to eastern North America:**

Analysis of earthquakes in stable, intraplate, low seismicity regions is important to characterize these regions, but it is hard because of the sparsity of earthquakes and stations, and hence useful data. Relatively little is known about earthquake sources in intraplate regions, and often relationships based on minimal data, or simply extrapolated from interplate regions are assumed due to lack of local information. For example, Somerville et al. (2002) use the recordings of only 3 moderate-sized earthquakes to propose source scaling relationships for earthquakes in the Northeastern USA and other stable continental regions. Their results imply that earthquakes in the Northeastern USA have relatively high stress drops. This result was confirmed by Shi et al. (1998) who analysed almost 50 small earthquakes from the region. Their study included 8 empirical Green’s function pairs (Shi et al., 1996), but mostly individual earthquakes. They also found a decrease in stress drop with seismic moment, at small magnitudes (< M~3). This may indicate that earthquake source scaling is different in this intraplate region, or else may represent a limit to the resolution as was found earlier in the San Andreas Fault plate boundary region (e.g., Abercrombie and Leary, 1993). Shi et al. (1998) were mostly limited by data availability to using regional recordings at single stations, and so to frequencies less than about 25 Hz.

In the last decade there has been a significant increase in the number of broadband seismic stations deployed in eastern North America, leading to growing number of well-recorded earthquakes (M2-5). On April 20, 2002 an earthquake of magnitude Ml 5.3 occurred in the northeastern Adirondack Mountains, Seeber et al. (2002). Following the mainshock, Lamont-Doherty Earth Observatory (LDEO) deployed digital portable seismographs to monitor aftershocks. The portable network spanned an area about 24 by 20 km with an average inter-station spacing of 4 – 6 km, smaller than the average source depth. Between April 22 and November, 2002, 74 small aftershocks were detected and located in the epicentral area of the mainshock. These data (200 samples/s) represent the best recorded earthquakes in the region to date, and so provide an unprecedented opportunity to investigate source parameters in this intraplate setting.

We calculate the source parameters using two methods commonly applied to direct waves recorded at local stations: spectral modeling of the individual three-component P and S waves (e.g., Abercrombie, 1995), and the EGF method (e.g., Mori and Frankel, 1990, Abercrombie and Rice, 2005). Both methods require high frequency content data to work well. Many studies consider the EGF method superior as it corrects for all path and site effects by using a smaller, collocated earthquake as an EGF. The EGF method cannot be used to calculate the seismic moment of the earthquakes, but this is the most reliable information that can be obtained from the individual spectral analysis. Using cross-correlation we identify two clusters of earthquakes with very similar waveforms, that include the largest individual earthquakes. They also found a decrease in stress drop with seismic moment, at small magnitudes (< M~3). This may indicate that earthquake source scaling is different in this intraplate region, or else may represent a limit to the resolution as was found earlier in the San Andreas Fault plate boundary region (e.g., Abercrombie and Leary, 1993). Shi et al. (1998) were mostly limited by data availability to using regional recordings at single stations, and so to frequencies less than about 25 Hz.

It is not clear in many studies how close the EGF events are to a perfect Green’s function. They are typically too small to obtain focal mechanisms, and the location uncertainties are larger than the preferred separation between events. We select only events that are located within the uncertainties of the large event, and have a high degree of waveform similarity, determined by cross-correlation. As a further test, we transform all the spectral ratios back to the time domain, and we only use pairs where we are able to resolve a clear source pulse. If a source time function is observable in this way, then it demonstrates that the phase components of the spectra are also very similar. This test is similar to the investigation performed by Mori and Frankel (1990) to determine how closely located earthquakes pairs must be for the EGF method to work well. To calculate the source time functions, we worked with Dr. German Prieto to develop a multitaper approach to calculate the source time function. Multitaper methods have long been preferred to calculate source spectra and spectral ratios as they better represent the frequency content of the waveforms than do cosine and other tapers (e.g., Park et al., 1987). Unfortunately, existing multitaper codes only worked with amplitude spectra and so could not be used to perform the complex deconvolution and retrieve the
source time functions. Dr. Prieto adapted his code to enable it to perform the full complex deconvolution so that we could use the same frequency analysis to obtain both the best spectra, and the source time functions. The method is similar to that developed by Park and Levin (2000) to obtain receiver functions.

We correct the velocity seismograms for the instrument response, and then selected time windows of 0.75 seconds that start 0.1 seconds before the P or S wave onset. We use the multitaper method (Park et al., 1987; Prieto et al., 2007) to calculate the Fourier displacement amplitude spectra. We use 7 weighted tapers and a time-bandwidth-product of 4. The algorithm also removes a noise line centered at 60 Hz, if present. Figure 3 shows an example of the seismograms, spectra, and EGF analysis for 3 events in one cluster.

We first calculate source parameters using the individual spectra, to determine the seismic moments, and also to compare the results of this standard method with that of the preferred EGF method. Before modeling the individual amplitude spectra, we re-sample them on a logarithmic scale, so that the fits were not biased to the high frequencies. We use samples at intervals of 10^0.02, and the amplitude value was the average of all the points in a neighborhood of ±10^0.01. We fit the displacement amplitude spectra with an omega square source model (Boatwright, 1980; Abercrombie 1995),

\[ \Omega(f) = \frac{\Omega_0 e^{-\pi|Qf|}}{1 + (f/f_c)^\gamma} \]  
where \( \Omega(f) \) is the displacement amplitude spectrum, \( f \) is the frequency, \( \Omega_0 \) is the long period level of the spectrum, \( f_c \) is the corner frequency, \( t \) is the travel time of the seismic waves (P or S), \( Q \) is the attenuation quality factor and \( \gamma \) and \( n \) are constants (\( \gamma = n = 2 \)) that control the shape of the spectrum curvature around the corner frequency. We use the Nelder-Mead simplex method (multidimensional unconstrained nonlinear minimization method) to fit the spectra with eq. (4) using \( \Omega_0, f_c \) and \( Q \) as variables. The travel time was calculated dividing the epicentral distance (S minus P) by the P or S wave velocity (6.500 km/s or 3.7528 km/s). The displacement spectra are fitted at bandwidths with signal to noise ratio above 3.

We calculate the seismic moment for each component \( i \) at each station, \( M_{0i} \), using Brune’s (1970) relationship,

\[ M_{0i} = \frac{4\pi \rho c^3 R \Omega_{0i}}{\phi U_{\phi0}} \]  
where \( \rho \) is the density (\( \rho = 2900 \text{ kg/m}^3 \)), \( c \) is the wave of interest velocity (\( P, \alpha = 6500 \text{ m/s}; \text{or } S, \beta = 3752.8 \text{ m/s} \)), \( R \) is the hypocentral distance (determined from S minus P arrival times), \( \phi \) is the free-surface parameter (\( \phi = 2 \)), and \( U_{\phi0} \) is the mean radiation pattern (\( U_{\phi0} = 0.52 \) for P waves and \( U_{\phi0} = 0.63 \) for S waves). We take the mean of all available components to calculate a final value for each station.

We assume the circular fault model of Madariaga (1976) to calculate source radius (\( r \)) and stress drop (\( \Delta\sigma \)) from \( M_{0i} \) and \( f_{ci} \),

\[ r_i = \frac{k \beta}{f_{ci}} \]  
where \( \beta \) is the S wave velocity and \( k \) is a constant (\( k = 0.32 \) for P waves and \( k = 0.21 \) for S waves), and then take the mean of all available components. Following Eshelby (1957) the stress drop on a circular crack is

\[ \Delta\sigma = \frac{7}{16} \frac{M_0}{r^3} \]  

We then apply the EGF method to the earthquakes in the clusters we identified. We use the same amplitude spectra to calculate spectral ratios between the different events, and we also use the extension of the multitaper method to the complex spectra to deconvolve the spectral ratios back to the time domain and so obtain estimates of the source time functions. We only continued the analysis if we obtained a clear source pulse, justifying our choice of EGF event. We resample the spectral ratios on a logarithmic scale (in the same way as the individual spectra). We model the spectra ratios using

\[ \Omega_s(f) = \Omega_0 \left[ 1 + \left( \frac{f}{f_{c_2}} \right)^n \right] \] 

\[ \Omega_s(f) = \frac{\Omega_0 \left[ 1 + \left( \frac{f}{f_{c_1}} \right)^n \right]^{\gamma}}{1 + \left( \frac{f}{f_{c_1}} \right)^n} \]  

where \( \Omega_s(f) \) is the displacement amplitude spectrum, \( f \) is the frequency, \( \Omega_0 \) is the long period level of the spectrum, \( f_{c_1} \) and \( f_{c_2} \) are the corner frequencies, \( n \) is the frequency exponent, \( \gamma \) is the frequency exponent, \( t \) is the travel time of the seismic waves (P or S), \( Q \) is the attenuation quality factor, and \( \gamma \) and \( n \) are constants (\( \gamma = n = 2 \)) that control the shape of the spectrum curvature around the corner frequency.
where $\Omega_r(f)$ is the displacement amplitude spectral ratio, $\Omega_{0r}$ is the relative long period level of the spectrum, $f_{c1}$ is the corner frequency of the large earthquake, $f_{c2}$ is the corner frequency of the small earthquake. We use the same method as for the individual spectra, with variables $\Omega_{0r}$, $f_{c1}$ and $f_{c2}$, and fit the spectral ratios in the bandwidth where the signal to noise ratio of both earthquakes is above 3. We then calculate source parameters from the spectral fitting as for the individual spectra.

We obtain preliminary stress drops in the range of 10-100 MPa for ten aftershocks of the Au Sable Forks earthquake with moments in the range $10^{10}$ to $10^{15}$ Nm. These values are consistent with the previous studies which find relatively high stress drops for intraplate regions such as eastern North America. They are consistent with the results of Shi et al. (1998) for their EGF events, but we do not see a breakdown in constant stress drop scaling at lower magnitudes, suggesting that this may have been an artifact of the limited bandwidth available to Shi et al.

**CONCLUSIONS AND RECOMMENDATIONS**

In summary, we have developed and performed initial testing of our preferred analysis methods for the direct and coda waves. The next step is to use these to obtain source parameters so as to compare the methods.

The initial analysis of the aftershocks of the Au Sable Forks earthquake confirms that earthquakes in eastern North America have relatively high stress drops (10-100 MPa). They also suggest that the decrease in stress drop with decreasing moment reported by Shi et al. (1998) may be an artifact of the limitations of their data. The next stage of this analysis is to calculate the radiated seismic energy from the spectra, and then to obtain source parameters using locally recorded direct waves for further aftershocks, and other recent earthquakes in the eastern North America region that have good EGF events that meet our stringent criteria.
The averaging nature of coda waves has been shown to provide significantly lower amplitude variance than any traditional direct phase method. It is thus ideal in regions with sparse stations and events so that most events are only well-recorded by a single station. The new spectral-ratio method developed in this study makes the coda method much easier and simpler to apply, greatly extended the number of earthquakes that can be studied. We demonstrate that coda envelope amplitude ratios are an effective tool at isolating the average source effect, with average scatter that is roughly 3 times less than direct phase ratios. Moreover, the minimal move-out for coda allows event pairs to be selected that have significant lateral separation and/or focal mechanism differences, in sharp contrast to direct phase EGF and spectral ratios studies which require co-located events, and ideally, identical focal mechanisms. For the Hector Mine mainshock and selected aftershocks, there are basic conclusions that can be drawn: 1) Inferred mainshock corner frequency is similar to teleseismic estimate; 2) The observed source ratios at low frequency are in good agreement with independent moment estimates; 3) At high frequency, none of the spectral ratios is asymptotic to $\left[\frac{M_0}{M_0}\right]^0$, similar to Izutani’s [2005] finding for the mid Niigata sequence;

4) The best fitting scaling parameter, $y$, is 0.25 and $e = 1$, consistent with previous studies on the same dataset using different methodologies; 5) The scaling of seismic moment and corner frequency does not follow an $f^{-3}$ dependence and seems consistent with an $f^{-4}$ dependence.

The next stage is to compare the results of the coda wave method with those from the direct-wave analysis using EGF pairs of events that are similar enough to produce source time functions from the direct wave deconvolution. We will perform direct wave analyses on the subset of Hector Mine aftershocks that have high quality EGFs, and apply the coda wave analysis to earthquakes from eastern North America including those used for the direct wave analysis. In this way we will determine whether the results of the coda-wave ratio method really do represent average source parameters, or whether any corrections or systematic biases need to be considered.

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