

A MATHEMATICAL STATISTICS FORMULATION OF THE TELESEISMIC EXPLOSION IDENTIFICATION PROBLEM WITH MULTIPLE DISCRIMINANTS

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ABSTRACT

Seismic monitoring for underground nuclear explosions answers three questions for all global seismic activity: Where is the seismic event located? What is the event source type (event identification)? If the event is an explosion, what is the yield? The answers to these questions involve processing seismometer waveforms with propagation paths predominately in the mantle. Four discriminants commonly used to identify teleseismic events are depth from travel time, presence of long-period surface energy (mb versus Ms), depth from reflective phases, and polarity of first motion. The seismic theory for these discriminants is well established in the literature (see for example Pomeroy et al. [1982] and Blandford [1982]). However the physical basis of each has not been formally integrated into probability models to account for statistical error and provide discriminant calculations generally appropriate for multi-dimensional event identification. This paper develops a mathematical statistics formulation of these discriminants and offers a novel approach to multi-dimensional discrimination that is readily extensible to other discriminants. For each discriminant a probability model is formulated under a general null hypothesis of H₀: Explosion Characteristics. The veracity of the hypothesized model is measured with a p-value calculation (see Stuart et al. [1994] and Freedman et al. [1991]) that is filtered to be approximately normally distributed and is in the range [0, 1]. A value near zero rejects H₀, and a moderate to large value indicates consistency with H₀. The hypothesis test formulation ensures that seismic phenomenology is tied to the interpretation of the p-value. These p-values are then embedded into a multi-discriminant algorithm that is developed from regularized discrimination methods proposed by Smidt and McDonald (1976), DiPillo (1976) and Friedman (1989). Performance of the methods is demonstrated with 102 teleseismic events with magnitudes (mb) ranging from 5 to 6.5 in Anderson et al. (2007). Example p-value calculations are also given for two of these events. Preliminary studies on the statistical properties of p-values are presented here.

OBJECTIVES

Anderson et al. (2007) propose a unifying framework for seismic event identification that can be populated with a diversity of seismic discriminants. For inclusion in the framework, a discriminant's physical theory must be mathematically embedded into a probability model designed to capture significant sources of error. This is accomplished by formulating each discriminant as a statistical hypothesis test under a general null hypothesis of H_0 : Explosion Characteristics. For example, a depth null hypothesis under Explosion Characteristics might be H_0 : event depth ≤ 10 km with the logical alternative hypothesis H_A : event depth > 10 km. The veracity of the null hypothesis for each discriminant is measured with a p-value calculation, which is used as the discriminant. The p-value ranges between zero and one, with a value near zero indicating inconsistency with Explosion Characteristics and a moderate to large value indicating consistency with Explosion Characteristics. With this approach to discriminant construction, the p-value carries information about source type fully adjusted for natural and measurement variability. This places a high standard on the construction of the discriminants—seismic phenomenology and path corrections must be integrated into an appropriate probability model, and a seismic-based hypothesis test must be constructed. The p-values under this formulation can be viewed as standardized discriminants with common interpretation across geographical regions and different discriminants.

For continuous discriminants such as depth from travel time, spectral ratios, or mb versus Ms, when the null hypothesis is true (e.g., explosion), the p-value will have a uniform probability distribution, and when the null hypothesis is false (earthquake), the p-value will have a probability distribution with most of its mass near zero. Here, the concentration of probability mass at zero is determined by the degree of disagreement between the true probability model of the data and the null hypothesis model.

The hypothesis test p-values can be mildly transformed to become standardized discriminants Y that also possess predictable statistical properties. They also range between zero and one, their interpretation is completely analogous to that of p-values, and they are approximately Gaussian. Therefore, established Gaussian discrimination methods can be used to formulate a unified decision from standardized discriminants. Specifically, the equation

$$Y = \frac{2}{\pi} \arcsin(\sqrt{p - \text{value}})$$

is well established in statistical theory as a transformation to achieve Gaussian behavior in data bounded between zero and one. Figure 1 illustrates the effect of standardizing the hypothesis test p-values. Precedence for interpreting p-values as discriminants can be found in Maharaj (2000), and Dumbgen and Homke (2000).

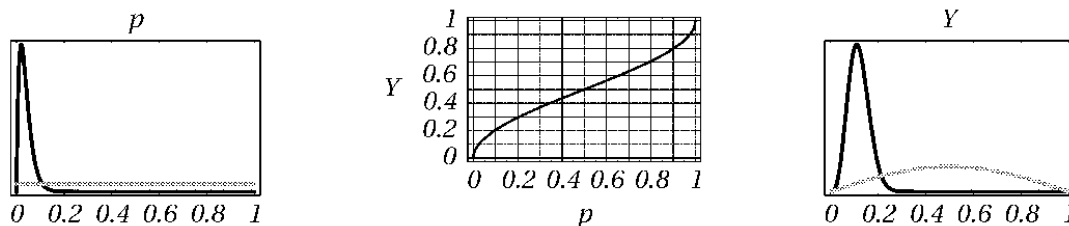


Figure 1. Transformation to induce an approximate Gaussian distribution on individual p-values (denoted p in the graphs) to derive standardized discriminants. The H_0 probability distribution is gray, and the H_A probability distribution is black.

In the framework proposed in Anderson et al. (2007), standardized discriminants Y are mathematically combined for source identification. This is accomplished with a *typicality index* calculation (see McLachlan [1992]) that measures the degree of agreement a suite of observed discriminants have with the earthquake and explosion populations.

With the typicality index calculation, an event can be declared

- consistent with historical explosions,
- not consistent with historical earthquakes,
- consistent with explosions and earthquakes (indeterminate), or
- not consistent with either earthquakes or explosions (unidentified).

These declarations are technically defensible.

In the framework, a second source identification calculation is made that assumes only two possible decisions – earthquake or explosion (indeterminate and unidentified are not possible). Bayesian statistical methods can be used to calculate the $P(\text{earthquake} | \text{event data})$ and $P(\text{explosion} | \text{event data})$. Note again that these probabilities sum to one. One possible rule is to simply take the higher of the two probabilities as the source identification. The objective of this research was to determine some of the statistical properties Bayesian source identification calculations with simulated teleseismic discriminants X versus the corresponding standardized discriminants Y .

RESEARCH ACCOMPLISHED

A simulation was performed that emulates the first-order properties of teleseismic discriminants for earthquakes and explosions. The simulation represents the potential source populations for two discriminants e.g., mb versus M_s (denoted X_1), and a teleseismic spectral ratio discriminant (denoted X_2) (see Taylor and Marshall [1991]). A subset of the simulation is reported here. The explosion model used to simulate X_1 and X_2 had the centroid (0,0) and a

covariance matrix $\begin{pmatrix} 1 & 0.6 \\ 0.6 & 1 \end{pmatrix}$. The earthquake model had the centroid (-2,-2) with a suite of covariance

matrices $\begin{pmatrix} 1 & 0.2 \\ 0.2 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 0.4 \\ 0.4 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 0.6 \\ 0.6 & 1 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0.8 \\ 0.8 & 1 \end{pmatrix}$.

First, using one of the earthquake/explosion model combinations above (the true explosion/earthquake models), 30 explosions and 300 earthquakes were simulated emulating the acquisition of teleseismic discriminant calibration data (X_1 and X_2). These calibration data were used to calculate the centroids (calibrated centroids) and a pooled covariance (calibrated covariance) for the explosion and earthquake models for data X_1 and X_2 . The simulated calibration data were then converted to p-values from a Z-score. The Z-scores for the X_1 data are gotten by subtracting the calibrated explosion mean from the X_1 data and dividing by the standard deviation of X_1 from the calibrated covariance matrix. The p-value calculation for each data point X_1 is then the left tail probability of the standard Gaussian distribution (see Figure 2). Calculations are analogous for the X_2 data. With the p-value calculations, the explosion data for both X_1 and X_2 will have a histogram that is uniformly distributed and the earthquake data will have a distribution that is tightly packed near zero (see the left graphic in Figure 1). The calibration p-values are then transformed to standardized discriminants Y_1 and Y_2 . These data are then used to calculate the centroids (calibrated centroids) and a pooled covariance (calibrated covariance) for the explosion and earthquake models for data Y_1 and Y_2 . On completion of the calibration step, we have the models for source event identification with either teleseismic discriminants X_1 and X_2 or standardized discriminants Y_1 and Y_2 .

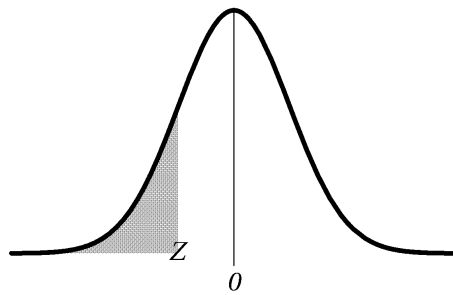
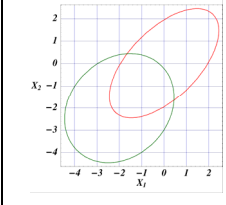
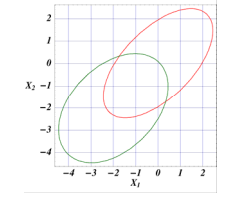
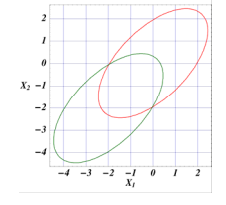
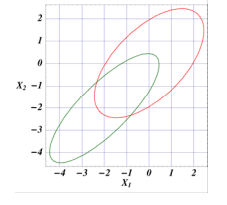


Figure 2. Graphical representation of p-value calculations from Z-scores for X_1 and X_2 .

In the next step, 5000 explosions and 5000 earthquakes are simulated using the true earthquake/explosion models. These data emulate new events with associated teleseismic discriminants (X_1 and X_2). The standardized discriminants Y_1 and Y_2 are calculated from these data using the calibrated explosion means and the standard deviations from the calibrated covariance matrix – the calculations are exactly as those made with the calibration data step. It is these 5000 simulated explosions and 5000 simulated earthquakes that are used to compare the properties of X_1 and X_2 versus Y_1 and Y_2 in the Bayesian source identification calculation. For this simulation study, the larger of $P(\text{earthquake} | \text{event data})$ and $P(\text{explosion} | \text{event data})$ is taken as the source identification with both

the teleseismic discriminant data X_1 and X_2 , and the standardized discriminants. With these simulated event data the probability of correctly identify an explosion (PD [=] probability of detection) and the probability of a false-alarm (FA [=] false-alarm probability) can be calculated, that is, the number of times an explosion is correctly identified divided by 5000 and the number of times an explosion is identified as an earthquake divided by 5000. *To compare the properties of Bayesian source identification with the two discriminants, we use the ratio FA/PD. In other words, this ratio is the false-alarms per detection, and the smaller this ratio the better.* The ratios FA/PD are reported in Table 1 for Bayesian source identification with simulated teleseismic discriminants and standardized discriminants. The true models used in the simulation are graphically represented in the top row. In all cases, using the standardized discriminants gives better false-alarm performance relative to explosion identification probability. This is consistent with the results observed in the full simulation.

Table 1. False-alarms per detection for simulated teleseismic discriminants and standardized discriminants. True models used to simulate teleseismic discriminants X_1 and X_2 are presented graphically in the first row. Ellipses for the models are 95% probability regions.

				
Simulated teleseismic discriminants	0.093	0.186	0.164	0.147
Standardized discriminants	0.040	0.121	0.090	0.073

CONCLUSIONS AND RECOMMENDATIONS

Conclusions are preliminary – theorems are not proven with simulations. However, we can conclude, based on simulations of statistical population behavior typical of some teleseismic discriminants, that standardized discriminants give improved operational performance over teleseismic discriminants, as measured by false-alarms per detection (FA/PD). Using p-values as discriminants has the advantage of unifying physical and statistical corrections into a single measurement. Therefore, in principle, p-values represent pure information about a seismic event source type. This is a compelling reason for using p-values as discriminants on its merits. The preliminary performance properties of p-values further supports p-values as seismic discriminants. Further research includes comprehensive simulations and potentially the development of mathematical arguments (theorems) that generalize this property of standardized discriminants.

REFERENCES

- Anderson, D. N., D. K. Fagan, M. A. Tinker, G. D. Kraft and K. D. Hutchenson (2007). A Mathematical Statistics Formulation of the Teleseismic Explosion Identification Problem with Multiple Discriminants. to appear in *Bull. Seism. Soc. Am.*
- Blandford, R. (1982). Seismic event discrimination. *Bull. Seism. Soc. Am.* 72: 69–87. DiPillo, P. (1976). The application of bias to discriminant analysis. *Communications in Statistics (Theory and Methods)* 5: 843–854.
- Dumbgen, L. and L. Homke (2000). P-values for discriminant analysis. Preprint series A-00-14, Schriftenreihe der Institute fuer Informatik/Mathematik, University of Lubeck, Germany.
- Freedman, R., R. Pisani, R. Purves, and A. Adhikari (1991). *Statistics*, Second Edition, W.W. Norton & Company Inc., New York.

- Friedman, J. (1989). Regularized discriminant analysis. *Journal of the American Statistical Association*, 84:165–175.
- Maharaj, E. (2000). Clusters of time series. *Journal of Classification*, 17:297–314.
- McLachlan, G.J. (1992). *Discriminant Analysis and Statistical Pattern Recognition*. John Wiley & Sons, New York.
- Pomeroy, P.W., W.J. Best, and T.V. McEvelly (1982). Test ban treaty verification with regional data—A review. *Bull. Seism. Soc. Am.* 72: S89–S129.
- Smidt, R. and L. McDonald (1976). Ridge discriminant analysis. Technical Report 108, University of Wyoming, Department of Statistics, Laramie, Wyoming.
- Stuart, A., K. Ord and S. Arnold (1994). *Kendall's Advanced Theory of Statistics: Volume 2A, Classical Inference and the Linear Model*, 6th Edition Arnold Publishers, London.
- Taylor, S. and P. Marshall (1991). Spectral discrimination between Soviet explosions and earthquakes using short-period array data. *Geophys. J. Int.* 106:265–273.