A UNIFIED APPROACH TO JOINT REGIONAL/TELESEISMIC CALIBRATION AND EVENT LOCATION WITH A 3D EARTH MODEL

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ABSTRACT

This project addresses the problem of locating seismic events from combined data sets of regional and teleseismic arrival times. While the use of both types of data theoretically yields better location accuracy than can be achieved with either type alone, experience has shown that the realization of such improvement depends on the ability to predict regional and teleseismic travel times accurately and consistently. Inconsistencies between regional and teleseismic predictions arise from discrepancies between regional and global velocity models used for travel-time calculation, as well as from disparities in the techniques used for travel-time calibration with ground-truth data; e.g. tomographic vs. empirical calibration. Additionally, improper weighting of teleseismic vs. regional data in the event location process can degrade location accuracy.

To address these problems, this project is investigating a framework for seismic event location based on the use of a single 3D Earth model for calculating travel times for all seismic phases. A number of research problems ensue from this framework. One is how tomographic calibration of the Earth model can be most accurately and efficiently performed; for example, whether the velocities in the crust and upper mantle of different geographic regions and in the deeper mantle can be determined in separate analyses or must be done in a single, grand calibration. A second issue is whether and how one can use tomographic uncertainty analysis to provide an optimal weighting of different teleseismic and regional phases used in locating an event. In addition, we are addressing the practical difficulties involved in efficiently calculating travel times with a 3D Earth model as part of an event location procedure. In particular, the project is investigating which approximate methods – such as linearization around a 1D reference model or a hybrid technique that matches 3D and tau-p calculations done in different depth ranges – might be adequate for the purpose of event location.

This paper describes our progress to date in acquiring and developing the tools needed for the project, and shows the results of some preliminary tests with joint regional/teleseismic travel-time calibration in southern Asia.

OBJECTIVE

The development of three-dimensional models of Earth structure continues to be an active field of seismology advancing along several fronts. A small sampling of these efforts include global travel-time tomography (e.g. van der Hilst et al., 1997; Antolik et al., 2003), global surface-wave tomography (e.g., Ritzwoller et al., 2002), and regional models for Asia (Murphy et al., 2005; Li et al., 2006) and the Middle East (Pasyanos et al., 2004). An important question for nuclear monitoring is whether these, or improved, 3D Earth models can produce routine event locations with significantly reduced errors compared to 1D models when data at all event-station distances are used together.

Many previous studies have demonstrated improvement in regional event locations based on 3D crust/upper mantle models, using either *a priori* models (Ryaboy et al., 2001; Pasyanos et al., 2004) or tomographic models (e.g., Murphy et al., 2005; Reiter et al., 2005). However, relatively little work has been done on joint regional/teleseismic location with 3D models. In one of the few published studies, Yang et al. (2004) found that event locations obtained by combining regional and teleseismic data were only more accurate than locations found from the separate data sets *if* they corrected for a bias between the teleseismic and regional travel-time predictions, each made from a different 3D model (CUB1.0 for regional, J362 for teleseismic). The correction they inferred is small (0.79 s), which is consistent with the fact that 3D crust/upper mantle effects on teleseismic travel times are relatively small (<2 s). But even small, systematic shifts in teleseismic travel times can induce large location errors owing to the high apparent velocity of teleseismic arrivals.

We are investigating a methodology for seismic event location based on the principle that travel times for all seismic phases should be predicted consistently from a single 3D Earth model. This unified modeling approach would be applied as the forward model used in locating events from combined sets of regional and teleseismic data. Moreover, the 3D model would be calibrated with regional and teleseismic ground-truth data in a joint tomographic analysis. A unified approach to location and location calibration is not a new concept and, in fact, has been the approach taken with 1D Earth modeling for many decades. Applying the approach to 3D Earth models, however, presents many strategic and computational challenges, which this project is attempting to address. The techniques we develop will be tested on a variety of available 3D Earth models, including the JWM model of south-central Asia (Reiter and Rodi, 2009), the WENA1.0 model (Pasyanos et al., 2004), and global mantle models such as J362D28 (Antolik et al., 2003) and MITP07 (Li et al., 2008).

RESEARCH ACCOMPLISHED

The implementation of an event location and travel-time calibration methodology with 3D Earth models involves three main components:

1. Raytracing algorithms for computing travel times of arbitrary regional or teleseismic phases in a 3D model, and the sensitivity of the travel times to the model velocities.
2. An event location algorithm that accommodates arrival-time data from multiple phases, with forward modeling based on the raytracing algorithms of Item 1 applied to a unified 3D model.
3. A tomography algorithm that fits a 3D model of *P*- and *S*-wave velocity in the crust and mantle to ground-truth observations of regional and teleseismic arrival times. This algorithm is also based on the modeling techniques of Item 1.

Our project is developing these components primarily as modifications and enhancements to algorithms previously developed by MIT, Weston Geophysical, and LLNL in past or concurrent projects on location and tomography. The goal of this project is not to develop a complete and operations-ready 3D capability but rather to demonstrate the value and practicality of the unified approach. The following sections describe the techniques we are developing in more detail and our progress to date.

3D Raytracing

The choice of a raytracing algorithm involves somewhat different issues in location and tomography. An event locator requires travel-time predictions for arbitrary hypocenters as it searches for a best-fitting event location, while tomography is usually performed with event locations held fixed. In addition, tomography needs the sensitivity of each calculated travel time to the model velocity parameters, as derived from the ray trajectory connecting the source and receiver.

For travel-time calculation in a 1D model (e.g., AK135, Kennett et al., 1995) the choice of a raytracing algorithm is straightforward since analytical solutions for travel times and rays, for any seismic phase (direct, reflected, diffracted or converted) are available and easily implemented in fast algorithms. This project is developing a general 1D raytracer based on the tau-p method (Buland and Chapman, 1983) that generates travel-time tables and travel-time sensitivities to 3D velocity parameters that are compatible with 3D event location and tomography software that is being used in this project (discussed more below).

Travel-time calculation in 3D models is much more problematic. For the calculation of travel times of regional direct *P* arrivals, Reiter et al. (2005) used the finite-difference raytracing method of Podvin and Lecomte (1991), as implemented by Lomax et al. (2000), in their nonlinear inversion of *Pn* arrival times. Since the P-L algorithm is written for finely gridded, flat-Earth models in Cartesian coordinates, they developed appropriate transformations from more coarsely gridded, spherical Earth models, incorporating an earth-flattening transformation (Müller, 1971). Reiter et al. (2005) further augmented the Podvin-Lecomte (P-L) algorithm with a back-chaining algorithm to generate sensitivities of travel times with respect to velocity parameters of the Cartesian model, and the translation of these sensitivities to the spherical Earth parameterization. These same regional raytracing capabilities were used by Reiter and Rodi (2009) in the generation of model JWM.

The finite-difference method can also be applied to teleseismic paths, but the computation time and storage requirements grow with epicentral distance. Furthermore, the earth-flattening transformation, which is only exact for 1D models, becomes less accurate with distance. Another problem with using a finite-difference method as a general raytracer is that the method is designed for first-arrival *P* or *S* waves. This is especially a problem for teleseismic arrivals, which are often well identified secondary phases such as pP.

Global travel-time tomography has generally been performed with a linear approximation to the travel-time function. In this approximation, the travel time *T* through a 3D model of interest is obtained by integrating the model's slowness function along the Fermat raypath computed in a different, reference model velocity model:

$$T \approx \int d\mathbf{x}\, a(\mathbf{x}; v_{ref})\, v^{-1}(\mathbf{x}) \qquad (1)$$

where $\mathbf{x}$ is the position vector; $v(\mathbf{x})$ is the 3D velocity function of interest; and $a(\mathbf{x}; v_{ref})$ is a sensitivity distribution concentrated on the Fermat raypath evaluated in a reference velocity function, $v_{ref}(\mathbf{x})$. When $v_{ref} = v$, the travel time *T* is exact. Linearization of travel times offers great computational advantages in both computation time and storage if v_{ref} is 1D. It also greatly simplifies the computation of times for secondary arrivals.

Accuracy Test for Linearization

Our preliminary examination of the linearization technique indicates that it is not accurate for regional phases but it may be adequate for teleseismic phases, in application to earthquake location as well as tomography. We base this tentative conclusion on numerical tests performed with the JWM model of Reiter and Rodi (2009). JWM is a 3D model of crust and upper mantle *P* and *S* velocity in southern Asia derived from the joint inversion of *Pn* travel times from the EHB database (Engdahl et al., 1998) and Rayleigh-wave group delays in the period range 10–150 seconds. A vertical slice through the model is shown in Figure 1. First, we compared travel times calculated through JWM using the Podvin-Lecomte algorithm to the travel times calculated through AK135. The times were calculated between station JYP (33.1° N, 74.85° E) and a line of fictitious surface-focus events extending to near-teleseismic distances along a great circle at azimuth N30°W. (The model slice in Figure 1 also follows this great circle.) To calculate teleseismic times in JWM with Podvin-Lecomte, JWM was extended below 410 km depth with the AK135 velocity profile. The left panel of Figure 2 shows the difference between JWM and AK135 travel times as a function of distance. At regional distances the difference is large (up to 4 s), reflecting the complex crust and upper mantle structure along this path. At teleseismic distances (>18°) the difference between the models is less than 1 s, due in part to our 1D extrapolation of JWM.

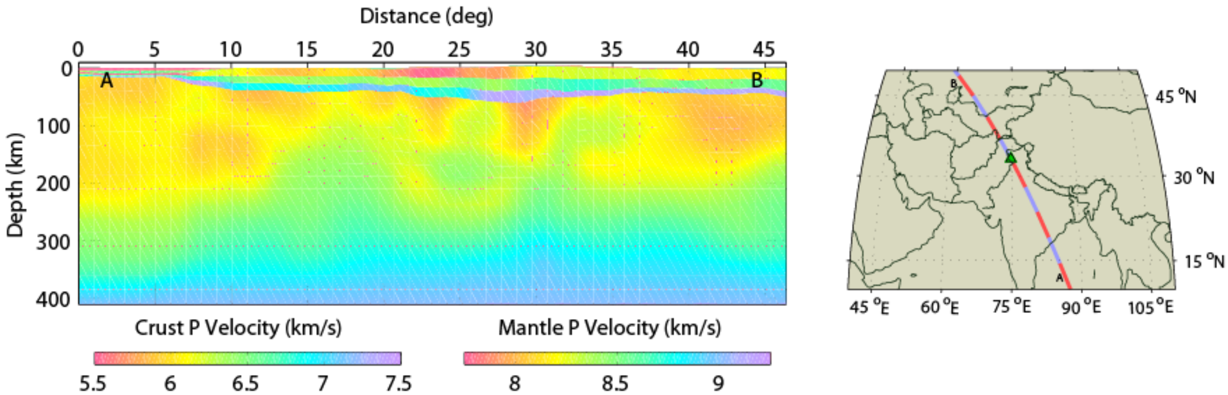

Figure 1. A vertical section through JWM, the Weston/MIT tomographic model of southern Asia (Reiter and Rodi, 2009). The section line (A-B) and station JYP (green triangle) are displayed on the accompanying map. Only the JWM *P*-wave velocity is shown. Note that a separate color scale is used for the crust and mantle.

The second part of the exercise tested whether travel times through JWM can be approximated with linearization around AK135, using Equation (1) with v taken as the JWM velocity and v_{ref} taken as the AK135 velocity. The right panel of Figure 2 shows the difference between the P-L calculated times ("Nonlin.") and the AK135 linearized approximation ("Linear"). We see that the errors in the linearized times at regional distances are as high as 25 s. Much of this difference owes to the thick crust over much of the traverse. Therefore, large pieces of the AK135 Pn raypaths are incorrectly located within the JWM crust. While it might be possible to correct for this effect, or optimize the 1D model used to obtain the reference raypaths, the fact is that integrating a 3D velocity model along reference rays from a 1D model cannot accurately model *Pn* travel times through a complex, 3D Earth model. However, we also see that the linearized travel times are much more accurate for the longest *Pn* paths (beyond 14°), corresponding to rays bottoming between 210 and 410 km in the AK135 model, which are much less affected by crustal thickness variations. Figure 2 also shows that the linearized approximation at teleseismic distances is quite accurate. While some of the accuracy may owe to the use of AK135 to extend JWM to below the upper mantle, the result still bodes well for linearization as a fast and accurate method for computing travel times at larger epicentral distances. We plan to conduct many more such tests with JWM and other 3D Earth models, and with secondary arrivals in addition to first arrivals as just shown.

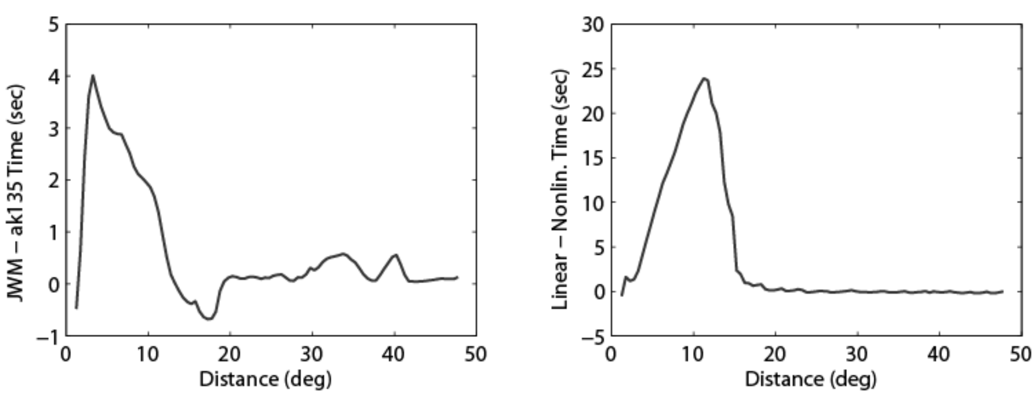

Figure 2. Experiments with regional and teleseismic travel-time calculation performed with model JWM. *Left*: Difference between calculated travel times for JWM and AK135. The JWM times were calculated with the Podvin-Lecomte (P-L) finite-difference algorithm. *Right*: Difference between calculated travel times from JWM, obtained two ways: "nonlinear" times obtained by the P-L method, and "linearized" times obtained by integrating the JWM slowness over AK135 raypaths. In both panels, times are plotted as a function of distance from JYP at an azimuth of N30°W.

Ray Bending Benchmark

Since the P-L method itself involves approximations and deals poorly with secondary arrivals, we have adapted a 3D raytracing procedure from Zhao et al. (1992) and Zhao and Lei (2004) as an alternative way to check the accuracy of travel-time calculations. This technique combines the pseudo-bending method for continuous media developed by Um and Thurber (1987) and refraction (Snell's law) at velocity discontinuities. The specific approach is outlined in Simmons et al. (these Proceedings). We find that the ray-bending approach is robust, always returning a travel time between two points in the Earth. There is no guarantee, however, that the bending method will converge to the minimum-time ray. We address this issue by testing a set of starting rays that span reasonable ray configurations. Using this approach, we have found that the ray bending method agrees with tauP calculations to within 0.01 s in spherically symmetric models and is in good agreement with finite-difference calculations in 3D models. Efficiency-wise, the ray bending is slow, taking approximately one second to compute a single travel time. This makes it impractical for routine use in an event locator run on commonly available computer platforms. However, the method is well suited as a benchmark for evaluating faster but less accurate methods.

Event Location with a 3D Model

The primary event locator we are using on this project is GMEL (Grid-search Multiple-Event Location), developed at MIT under previous projects and extended to 3D models in a previous joint project with Weston Geophysical (Reiter et al., 2001). GMEL uses travel-time predictions for either 1D reference velocity models or 3D reference models, using interpolation of pre-calculated tables in each case. A description of the numerical techniques used in GMEL is available in Rodi (2006). We are investigating the feasibility of replacing 3D table interpolation with “on-the-fly” linearized travel-time calculations for teleseismic phases, using Equation (1) with a 1D reference model. This entails incorporating into GMEL what we have built as stand-alone software for performing the linearization technique. This change to GMEL would avoid the need for pre-calculating and storing (on disk and in run-time memory) large 3D travel-time tables, each of which must contain travel times on a 3D hypocenter grid for a particular station and phase. Instead, only a single 3D Earth model, appropriate for all stations and phases, would be stored. A travel time calculation for a particular path and teleseismic phase would involve an on-demand tau-p calculation with the 1D reference model, followed by numerical integration to evaluate Equation (1). This will take more CPU time than a table interpolation but much less than the time needed for on-demand 3D raytracing. Furthermore, with much smaller memory requirements, the number of teleseismic phases GMEL can handle will increase. Whether this new version of GMEL will be sufficiently fast for routine use on a garden-variety computer remains to be seen since grid search typically tests 1000 or more hypocenter locations to find a solution.

Joint Regional/Teleseismic Tomography

The stand-alone 1D raytracer we have developed converts rays to travel-time sensitivities (partial derivatives) with respect to the same 3D velocity model parameterization we use in our regional tomography algorithm. The parameterization is described by Reiter and Rodi (2009). Moreover, the sensitivities are output in the same format as the regional travel-time sensitivities we generate with our extended Podvin-Lecomte algorithm. This provides an initial capability to perform joint regional/teleseismic travel-time calibration, a major goal of the project. The term “initial” alludes to several important enhancements that await our future efforts, as discussed later.

To test our initial joint calibration capability, we experimented with the updating of JWM through linearized tomography of combined data sets of regional and teleseismic *P*-wave travel times. The regional data set was identical to that used by Reiter and Rodi (2009) to construct JWM. Two teleseismic data sets were considered, both extracted from the EHB database, and both including arrivals only in the epicentral distance range 35° – 65°. In the first teleseismic data set (termed TelE), the stations were restricted to be located in or near the JWM study region, i.e. in the box 0°– 60° N, 30°– 120° E. The event locations were unrestricted (except for the distance criterion) and fell primarily outside this box. The second data set (TelS) restricted the events to be within the study box and accepted all stations obeying the distance criterion. The regional and both teleseismic data sets included only events which have EHB focal depths less than 200 km and which are deemed well-located in accordance with criteria described by Reiter and Rodi (2009). To reduce the size and redundancy of the data sets, individual events were lumped into summary events in the same manner as done for JWM. Each summary event is a node on a 0.5° geographic grid and variable depth grid. For a given station, the observed travel-time residual (relative to AK135) for a summary event is obtained by averaging the residuals for the individual events near the summary event. In round numbers, teleseismic data set TelE contained 322,000 arrivals for 14,000 summary events and 800 stations, while TelS contained 278,000 arrivals for 3,700 summary events and 2500 stations. The regional data set contained 104,000 arrivals from 3700 summary events and 600 stations. More than 99% of the regional arrivals are the *Pn* phase, the rest being *Pg* or *Pb*. We point out that these data-set sizes approach the capacity of our current tomography software, which is why we considered teleseismic events and stations separately, and why we capped epicentral distances at 65°. Normally, linearized tomography is performed with a common reference model (v_{ref} in Equation (1)) for all data. Our tomography experiments used a different reference model for regional and teleseismic data: JWM for regional and AK135 for teleseismic. This is consistent with the necessity of doing 3D raytracing for regional travel-time calculation, and our proposed approximate modeling of teleseismic travel times with 1D reference rays.

Figure 3 shows a vertical slice through each of two joint regional/teleseismic tomography models, corresponding to the two versions of the teleseismic data set that were combined with the common regional data set. The model obtained with TelE as the teleseismic data set is labeled JWM_TelE, and the other is labeled JWM_TelS. We see that the models are quite similar to each other, and are not starkly different from JWM, for which the same slice was shown in Figure 1. The largest differences from JWM occur at depths below 200 km, where the *P* velocity is significantly slower. This is more easily seen in Figure 4, which displays the velocity deviation of the two regional/teleseismic inversion models relative to JWM.

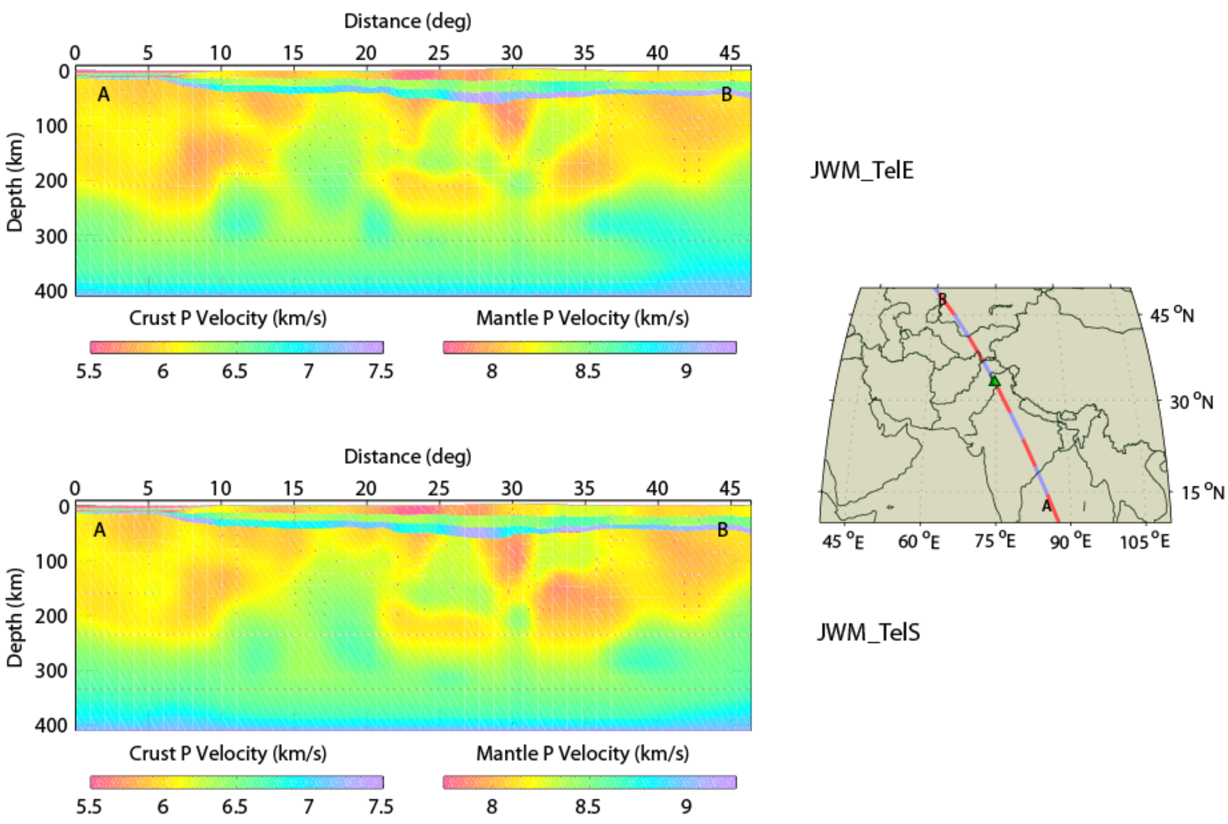

Figure 3. A vertical section through two 3D *P* velocity models obtained by joint inversion of regional and teleseismic *P*-wave travel times. The model shown at the top (JWM_TelE) used teleseismic data from local stations and teleseismic events, while the model shown at the bottom (JWM_TelS) used teleseismic data from local events and teleseismic stations. The regional data were the same in both cases and are from stations and events inside the model region.

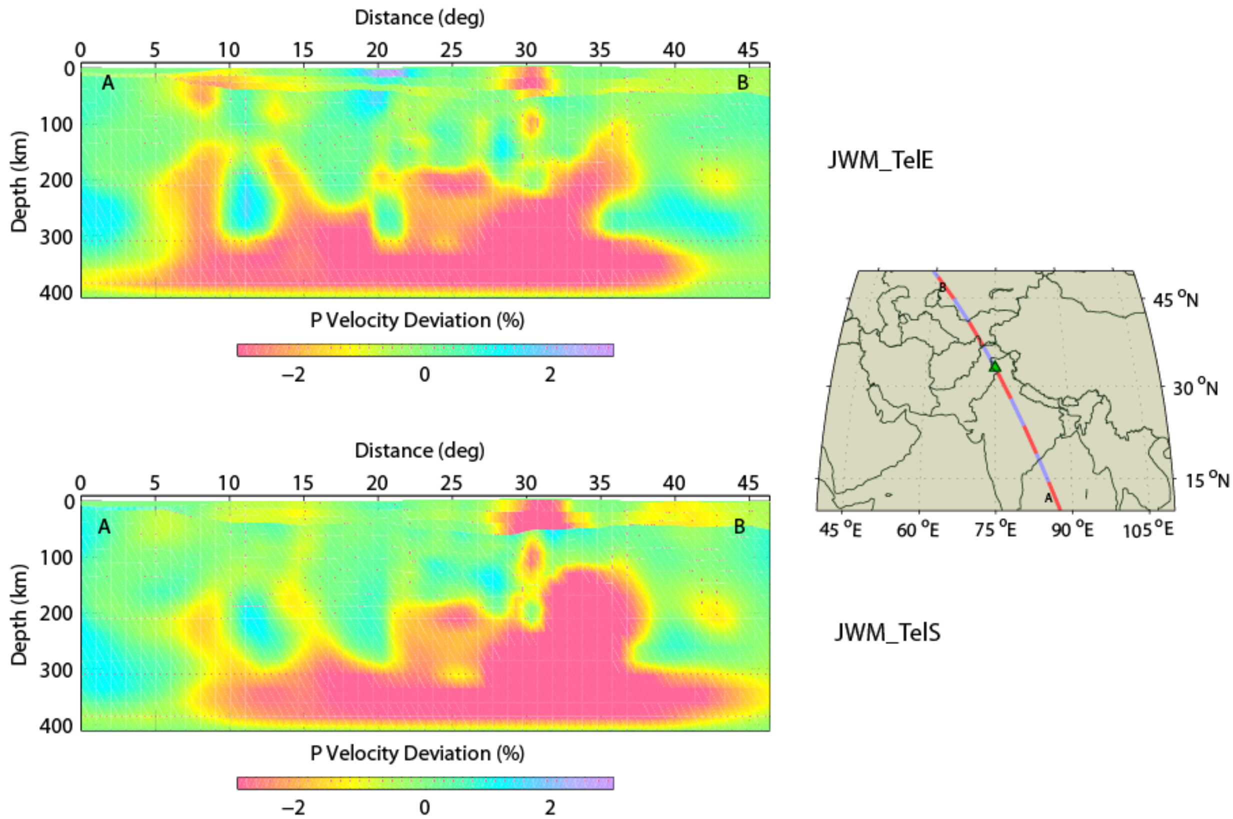

Figure 4. The inversion models shown in Figure 3 are displayed as velocity deviations from model JWM.

The reason for the lower mantle velocities in the joint regional/teleseismic models is clear if we compare their travel-time residuals to those of JWM. The comparison is quite similar for the two joint models, so we show it only for JWM_TelE. Figure 5 displays separate histograms of regional (left) and teleseismic (right) residuals, for each of the models JWM (blue lines) and JWM_TelE (red lines). We see that JWM yields regional residuals with a mean near zero but teleseismic residuals with a mean of about 1.5 s. JWM_TelE maintains the unbiased fit to the regional data and yields zero-mean teleseismic residuals as well. Apparently, the joint regional tomography removed the teleseismic travel-time bias of JWM by lowering the *P* velocity below 200 km depth, in a way that preserved the fit to the *Pn* data. Figure 6 displays the residual statistics for both models as a function of epicentral distance.

CONCLUSIONS AND RECOMMENDATIONS

In our first year we have begun acquiring and developing the tools needed to perform event location with regional and teleseismic data using a unified 3D Earth model, calibrated with both types of data, as the basis for travel-time prediction. We also began testing the effectiveness of joint regional/teleseismic tomography as a method for travel-time calibration. These initial tests involved the incorporation of teleseismic constraints to update a model of southern Asia, JWM, which had been constructed from regional *P*-wave times and surface-wave dispersion data. The tests yielded promising results in that the joint tomography preserved the fit to the regional travel-time data while removing JWM's teleseismic bias. However, these initial tests ignored several important issues that our future work will address. Among them are the effects of Earth structure below 410 km and outside our study region, which we ignored by restricting the model parameterization to that used to develop JWM. Further, we did not address the effects of event mislocations and the possible need to relocate events as part of the calibration analysis. We have also not properly validated the approximate teleseismic modeling used to obtain these results (linearization with respect to AK135 rays). Finally, our tests were limited by the unfinished status of our tomography algorithms, which are aimed toward nonlinear inversion of regional data (iteratively updating of *Pn* rays) and the incorporation of secondary teleseismic phases, among other features. Our second year's effort will pursue these improvements in concert with the enhancement of our 3D event location capabilities, and continued testing and evaluation of 3D raytracing algorithms.

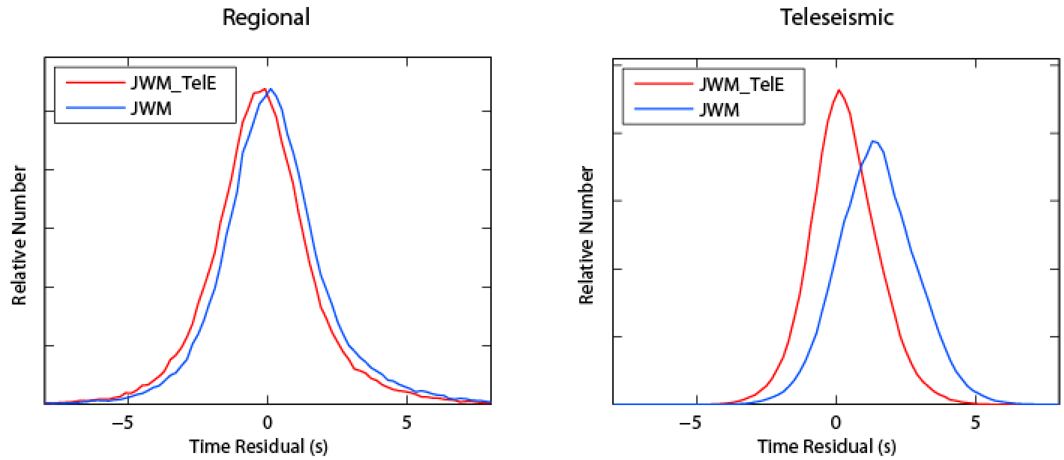

Figure 5. Distributions of regional (left) and teleseismic (right) travel-time residuals for model JWM (blue lines) and model JWM_TelE (red lines). For model JWM, the RMS residual is 2.0 s for both the regional and teleseismic data. For JWM_TelE, the RMS is 2.0 s for regional data and 1.3 s for teleseismic data.

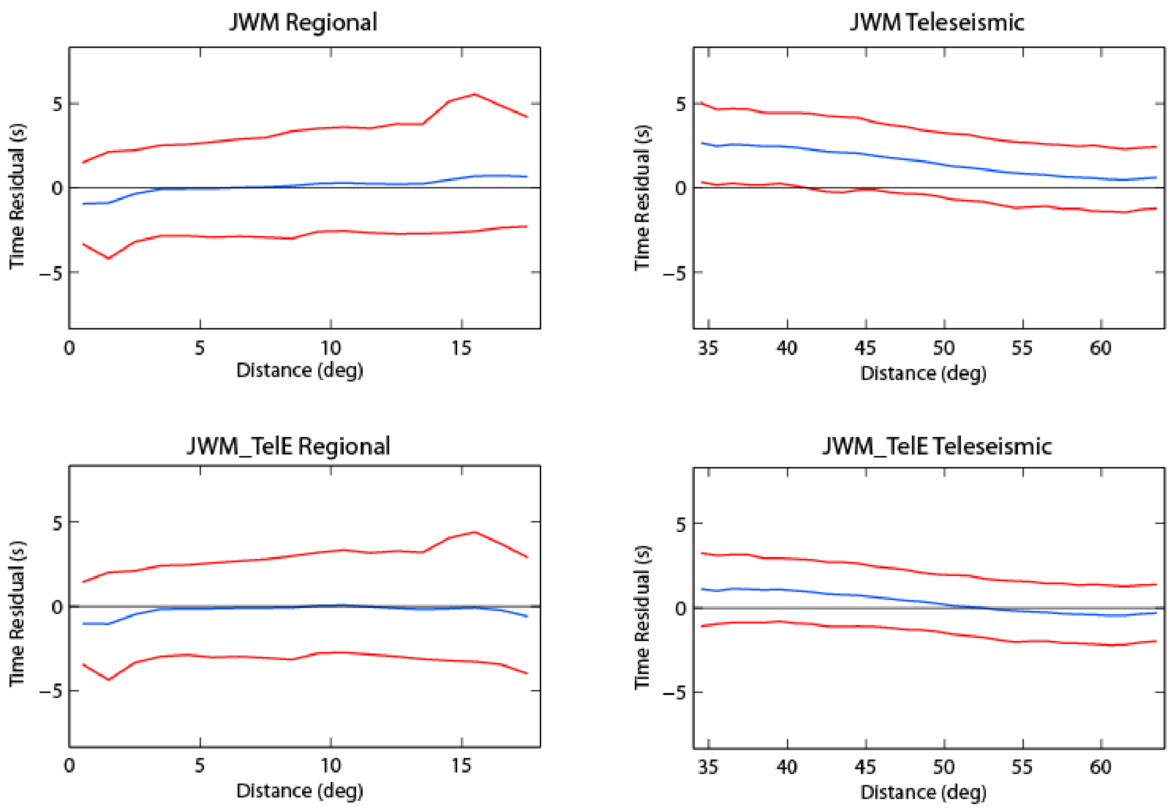

Figure 6. Travel-time residual statistics for models JWM and JWM_TelE, plotted as a function of 1° epicentral distance bins. The blue lines trace the mean residual in each bin; red lines show the 0.05 and 0.95 points (central 90%) of the residual distribution in each bin.

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