ISOLATING SOURCE INFORMATION IN *mb* AND *Ms* WITH MODEL-BASED CORRECTIONS: NEW *mb* VERSUS *Ms* DISCRIMINANT FORMULATIONS

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ABSTRACT

Path and signal processing corrections made to amplitudes give magnitudes *mb* and *Ms*, that in principle, have only source information. We propose the addition of a model-based magnitude prediction as a correction term, that is, a source correction in addition to path and signal processing corrections under the null (H0) hypothesis that a seismic event is a single-point fully contained explosion. This additional correction removes explosion source information from the *Ms* versus *mb* discriminant with the remaining source information represented as a constant. There are effects such as depth, focal mechanism, and local material properties that cannot easily be determined and mathematically included in amplitude corrections. We develop a mathematical model to capture these near source effects as random (unknown) giving an error partition of two sources: model inadequacy and station noise. This mathematical model is the basis for two new *Ms* versus *mb* discriminant formulations. Both methods are designed to utilize advances in source physics theory by using new source models to predict (correct) observed surface wave magnitudes. *The network average discriminant formulation includes source model error and magnitude correlation in the standard error (SE) of the discriminant, effectively placing a lower bound on the discriminant SE and accounting for scaling between Ms and mb. This new property of the Ms versus mb discriminant correctly reduces the SE only through network averaging of magnitudes. A second discriminant formulation derived from order statistics theory is potentially robust to station coverage for an event by basing discrimination on the maximum observed station Ms value.*

OBJECTIVES

When completed, this research will leverage advances in *Ms* signal processing into significant reformulations of the *Ms* versus *mb* discriminant to include: 1) a new correction for source in addition to established path corrections under the null hypothesis that a seismic event is an explosion, 2) a new sources-of-error model that correctly partitions total error between two components – model and background noise, and provides the correct formulation for scaling between *Ms* and *mb*, 3) and a new *Ms* versus *mb* discriminant formulation that can be robust to surface wave radiation pattern (station coverage) for earthquakes. We review progress to date on this research.

RESEARCH ACCOMPLISHED

An instrument-corrected amplitude spectrum can be thought of as a convolution between the source and the path. In the frequency domain this can be mathematically represented as

$$A(\omega, \Delta) = S(\omega)G(\Delta)P(\omega)B(\omega, \Delta)$$ (1)

where *S* is the source spectrum, *G* is geometrical spreading, *P* is the frequency-dependent site effect, and *B* is the an-elastic attenuation with function arguments epicentral distance Δ and angular frequency ω. Here we have split the path effect into three components: 1) a frequency independent geometrical spreading component, 2) a range independent and frequency dependent site effect, and 3) an an-elastic attenuation component.

The logarithm of both sides of Equation (1) gives

$$\log A(\omega, \Delta) = \log S(\omega) + \log G(\Delta) + \log P(\omega) + \log B(\omega, \Delta)$$ (2)

In general, to remove source, size and path trends in the data, we can correct the observed $\log A_O(\omega, \Delta)$ spectrum so that

$$\log A_c(\omega, \Delta) = \log A_o(\omega, \Delta) - \log A(\omega, \Delta)$$ (3)

where $\log A_C(\omega, \Delta)$ is the corrected spectrum. Equation (3) gives corrected magnitudes (residuals) *Y* that are then used to construct discriminants. Specifically, from Equation (3), the corrected magnitude *Y* is a log observed magnitude minus a model prediction of magnitude under the hypothesis that the event was generated by source $S(\omega)$ (e.g, single-point fully contained explosion). The residuals *Y* provide data (evidence) to test this hypothesis. Parseval's theorem provides the signal processing for magnitude calculations (total phase energy) in the time domain.

A baseline source model $S(\omega)$ will forecast estimates of *mb* and *Ms* for an event in question. Fully developed, the source model $S(\omega)$ is a linear superposition of two axisymmetric force systems for explosions conducted in hard rock. First, a monopole represents the explosion as a spherical source. Second, a compensated linear vector dipole (CLVD) with vertical axis in extension represents shock-induced, deep-seated tensile failure, a form of source medium damage (see Patton and Taylor (2008), and Ben-Zion and Ampuero (2009)). Both sources are assumed to be time and space coincident, and have a common Mueller and Murphy (1971) source function. The monopole source alone is used to forecast *mb*. *Ms* is forecast by half-space calculations of Rayleigh wave interference between the monopole and CLVD force systems (Patton and Taylor (2008)).

Estimates of *mb* and *Ms* and error bounds are derived from statistical analysis of model parameters. With density and P velocity fixed for hard rock, the model has four parameters: Poisson ratio ν, depth of burial *h*, yield *W*, and a low-frequency CLVD source strength to monopole parameter *K* developed in Patton and Taylor (2008). Independent observations constrain or bracket values for each parameter. Such observations might be drawn from site geology and tectonic history (ν), previous testing history (mature versus new test sites; *h*, *K*), and seismic measurements, both regional and teleseismic, of source size (*W*). The two discriminants developed here utilize a prediction of *mb* and *Ms* derived from a reasonable suite of models covering the parameter space. Thus, the discrimination methods developed in this paper have the potential to provide monitoring capability in settings with no explosion calibration data.

Consistent with the established approach to *Ms* versus *mb* discrimination, we develop the mathematics of two new discriminant formulations – a multi-station discriminant constructed from the network average of the corrected magnitudes *Y*, and a multi-station discriminant constructed from the maximum of station *Ms* values. Both discriminants are built from random effects analysis of variance (Scheffe (1959), Searle (1971) and Searle et al. (1992)) which has been applied to other path correction theories in seismology (e.g., Chen and Tsai (2002), Tsai and Chen (2003) and Tsai et al. (2006)). We model any remaining physical structure in corrected magnitudes as a source bias plus two random effect components – model inadequacy and station noise. This approach to *Ms* versus *mb* discriminant formulation properly forms the discriminant standard error with these two variance components. Model inadequacy decreases with scientific advances in source and path correction theory and improved calibration, and consistent with signal processing theory, station noise is reduced through station averaging in the network average formulation. The conceptual motivation for the new discrimination formulations is illustrated in Figure 1.
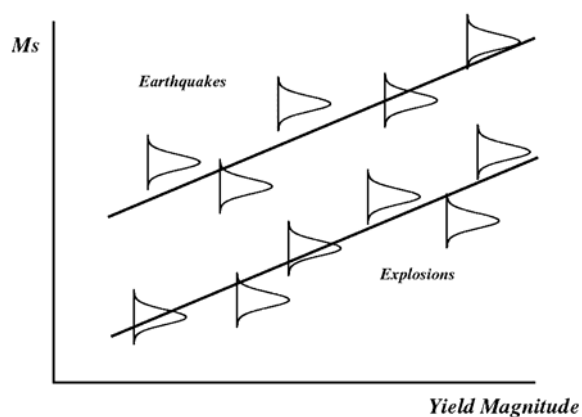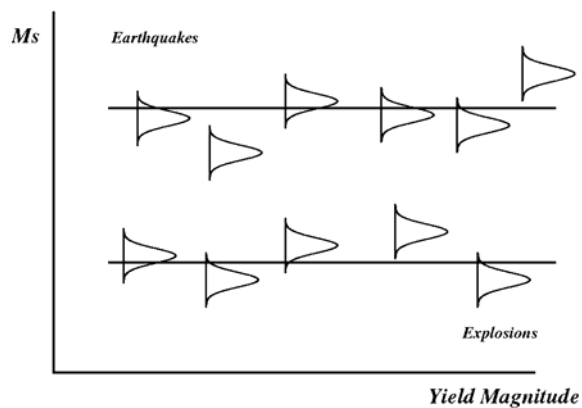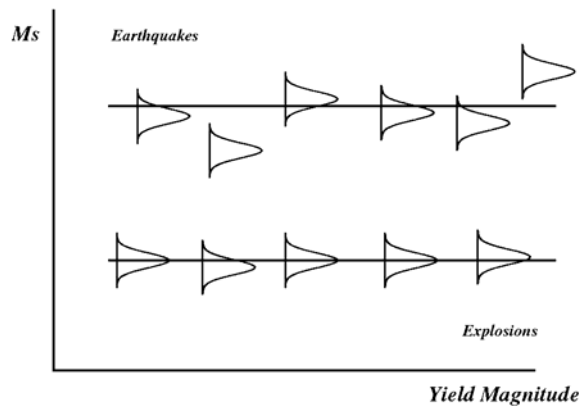
(a) Uncorrected Ms Magnitudes

(b) Ms Magnitudes Corrected for $Yield\ Magnitude$

(c) Ms Magnitudes Corrected for $Yield\ Magnitude$ and $Source$

Figure 1. Conceptual illustration of corrections applied to *Ms* magnitudes, conditional on known *Yield Magnitude*. Normal curves represent the distribution of station *Ms* values. Figure 1(a) illustrates uncorrected *Ms* magnitudes. Figure 1(b) illustrates the event populations after correction for *Yield Magnitude* giving a significant reduction in model error. Figure 1(c) illustrates the event populations after correction for *Yield Magnitude* and additionally a *Source* correction (observed minus prediction). Note the additional reduction in model error and the significant source population separation.

Discriminant Formulation with Model Inadequacy and Station Noise Error Partition

Established signal processing treats amplitudes as lognormal distributed and therefore magnitudes are normally distributed. The conceptual representation of the proposed model is

$$Y = \log(\text{corrected amplitude}) = C(\text{source}) + Event + Noise \tag{4}$$

where C(source) is a source constant, *Event* is a zero mean random effect that varies from event to event and represents model inadequacy from effects such as depth, focal mechanism, local material property and apparent stress variability, and *Noise* represents measurement and ambient noise, also with zero mean. This approach results in C(source) terms for earthquakes and explosions, with large differences in these terms indicative of good discrimination ability. In subsequent development, the hypotheses test that an event is a single point fully contained explosion will be mathematically represented in terms of these C(source) parameters. Equation (4) implies the expected value of Y is $\mathbf{E}\{Y\} = C$(source). In the development it is assumed that good signal processing practices have been applied giving high confidence in the quality of the observed amplitudes and calculated magnitudes.

For the mathematical statistics formulation of Equation (4), define the random variable Y_{ijk} to be a magnitude residual for source $i=0,1$ (explosion, earthquake), event j and station k (observed data are denoted y_{ijk}). The statistical linear model representation of Equation (4) is then

$$Y_{ijk} = \mu_i + E_j + \epsilon_{(ij)k} \quad j = 1, 2, \ldots m_i \quad k = 1, 2, \ldots n_{ij} \tag{5}$$

Analogous to Equation (4), Equation (5) reads Y_{ijk} equals a source constant μ_i plus a random event adjustment E_j (source and path model inadequacy) plus a station noise adjustment $\epsilon_{(ij)k}$. The subscript notation $(ij)k$ for $\epsilon_{(ij)k}$ specifies that observed station noise (k) is different for each source and event combination (ij). Equation (5) is a standard mixed effects (random and fixed) linear model.

The E_j are modeled as independent normal random variables with zero mean and variance τ^2. The $\epsilon_{(ij)k}$ are independent normal random variables with zero mean and variance σ^2. E_j and $\epsilon_{(ij)k}$ are independent across all subscripts. This assumption is consistent with effects local to the source being uncorrelated with station noise.

Statistical Properties of a Network Magnitude Residual

For source i and event j, denote the $1 \times n_{ij}$ vector of magnitude residuals for n_{ij} stations as $\underline{Y}'_{ij}$. Then $\underline{Y}'_{ij}$ is multivariate normal with $1 \times n_{ij}$ mean vector $\underline{\theta}'_{ij} = (\mu_i, \mu_i, \ldots, \mu_i)$ and $n_{ij} \times n_{ij}$ covariance matrix

$$\Omega_{ij} = \begin{pmatrix} \tau^2 + \sigma^2 & \tau^2 & \tau^2 & \cdots & & \tau^2 \\ \tau^2 & \tau^2 + \sigma^2 & \tau^2 & & & \tau^2 \\ \tau^2 & \tau^2 & \ddots & & & \vdots \\ \vdots & & & \tau^2 + \sigma^2 & \tau^2 \\ \tau^2 & \cdots & & \tau^2 & \tau^2 & \tau^2 + \sigma^2 \end{pmatrix} \tag{6}$$

The network magnitude residual is $Y_{ij} = \underline{1}' \, \underline{Y}_{ij} / n_{ij}$ is normal with mean μ_i and standard error $\tau^2 + \sigma^2 / n_{ij}$. Omitting the term E_j in Equation (5) implies that the magnitude residual at a station is μ_i plus station noise. This model formulation is fundamentally inconsistent with seismic observation. The standard error of Y_{ij} with E_j removed from Equation (5) is σ^2 / n_{ij} ($\tau^2 = 0$) and decreases as the number of stations n_{ij} observing an event increases. By not including the term E_j, effects such as depth, focal mechanism, local material property and apparent stress variability are not accounted for in the theoretical model representation of a magnitude, and this bias can never be diminished with a network average calculation. Equation (5) captures source and path model deficiencies as a random effect (E_j) and compensates for them as a component in the standard error of a discriminant. Also, the lower bound of a magnitude standard error, derived from Equation (5), is non-zero and therefore consistent with realistic seismic observation.

Another important property of this model is that a magnitude residual for a single event is correlated across stations. This correlation, $\tau^2 / (\tau^2 + \sigma^2)$ implies that large adjustment E_j increases correlation between stations because this

random adjustment is applied to all stations observing an event, that is, the station residuals stochastically move together. Small adjustment E_j implies the correction model is good and is conceptually equivalent to stations with incoherent noise. Small adjustment E_j also implies τ^2 is small and the standard error of Y_{ij} is fundamentally reduced through network averaging.

The corrected magnitudes mb and Ms can be correlated if they are both corrected for size in the source spectrum S with, for example, Mw. If Ms is corrected for size with mb, then these magnitudes will be uncorrelated. From Equation (5) the random mechanism causing this correlation is modeled with the E_j terms between magnitudes because these errors perturb the signal equally at all stations observing an event, and also the measured amplitudes. *This correlation is the mathematical mechanism that captures any scaling between magnitudes.* Extending Equation (5) to two corrected magnitudes $Y1_{ijk}$ ($= Ms_{ijk}$) and $Y2_{ijk}$ ($= mb_{ijk}$) with the $E1_j$ and $E2_j$ terms between magnitudes correlated (ρ) provides the statistical model to calculate the standard error of the multi-station discriminant given in the section that follows.

Network Average Discriminant Formulation

The Ms versus mb discriminant is constructed from a bivariate normal model of network magnitude residuals Ms and mb, with bias constants $\mu_{Ms}i$ and $\mu_{mb}i$, $i=0,1$ (explosion, earthquake). For an explosion, the network magnitude residual Ms is normal with mean μ_{Ms_0} and standard error $\tau^2_{Ms}+\sigma^2_{Ms}/n_{Ms}$ and mb is normal with mean μ_{mb_0} and standard error $\tau^2_{mb}+\sigma^2_{mb}/n_{mb}$. With the inclusion of correlation between model errors $E_{Ms}j$ and $E_{mb}j$, the standard error of the Ms versus mb discriminant is then

$$SE_{\bar{M}_s - \bar{m}_b} = \sqrt{\tau^2_{mb} + \frac{\sigma^2_{mb}}{n_{mb}} + \tau^2_{Ms} + \frac{\sigma^2_{Ms}}{n_{Ms}} - 2\,\rho\,\tau_{mb}\,\tau_{Ms}}, \tag{7}$$

for both earthquakes and explosions. Equality of the standard error for both source types is an important model property because unlike discrimination analysis with unequal source variability, it ensures that an earthquake with an unusually strong earthquake-identifying discriminant value will not be identified as an explosion. This can occur with discrimination methods that model source variance as unequal – explosion calibration data can exhibit variability that is significantly larger than earthquake calibration data.

In an operational setting, physical correction theory will never be able to adjust amplitudes for all local systematic effects. As discussed previously, these local systematic effects are modeled as random, moving out to all stations (and therefore amplitudes), hence the correlation between a network of stations. As physical (source and path) corrections improve, the model inadequacy terms E_j will be small, giving small values of τ, and the covariance between amplitudes will be small. In the limit, this conceptually gives station discriminant scatter plots for explosions and earthquakes (populations) that are small shotgun patterns of data.

Centering relative to the explosion population means, the standardized discriminant is

$$Z_{\bar{M}_s - \bar{m}_b} = \frac{(\bar{mb} - \mu_{mb_0}) - (\bar{M}s - \mu_{Ms_0})}{\sqrt{\tau^2_{mb} + \sigma^2_{mb}/n_{mb} + \tau^2_{Ms} + \sigma^2_{Ms}/n_{Ms} - 2\,\rho\,\tau_{mb}\,\tau_{Ms}}}, \tag{8}$$

which is centered at zero for explosions and has a non-zero center for earthquakes. The advantage to centering relative to explosions is consistency with the monitoring position to assume all events are explosions and then prove otherwise with seismic signatures. From Equation (8), values of Z_{Ms-mb} greater than a decision threshold fail to reject the hypothesis that an event is a single-point fully contained explosion.

Statistical Properties of the Maximum of Magnitude Residuals

The energy radiation pattern of surface waves from a single-point fully contained explosion can be reasonably assumed to be azimuthally isotropic. With $\tau^2=\tau^2_{Ms}$ and $\sigma^2=\sigma^2_{Ms}$, Equation (6) gives the Ms covariance matrix for an event observed by n_{ij} stations. Make the assumption that corrections are of sufficient high quality to give $\tau^2_{Ms} \cong 0$. With this assumption, the observed station Ms are independent (uncorrelated) for an event, and the Ms population mean is zero for explosions and greater than zero for earthquakes. For source i, event j, $Max\{Ms_{ijk}\} < m \Leftrightarrow \{Ms_{ijk} < m; k = 1, n_{ij}\}$. Under the null (*H0*) hypothesis that a seismic event is a single-point fully contained explosion, surface

wave energy is azimuthally isotropic and observed station *Ms* have the same probability distribution and so the *H0* likelihood is

$$P(Ms_1 < m, \ldots, Ms_{n_{ij}} < m \mid H0) =$$
$$P(Ms_1 < m \mid H0) \cdots P(Ms_{n_{ij}} < m \mid H0) = P(Ms_k < m \mid H0)^{n_{ij}}. \quad (9)$$

Order Statistics Discriminant Formulation

The energy radiation pattern of surface waves from an earthquake is not azimuthally isotropic. Some stations could be on an energy lobe and some could lie on an energy node. In an event identification setting the energy radiation pattern of an event will not be known. This means that a network average of station *Ms* values could be strongly influenced by nodal stations. For example, an earthquake with four nodal stations and one station on an energy lobe could look like an explosion with *Ms* versus *mb* network average discrimination.

The right hand side of Equation (9) provides the likelihood of the random variable *Max{Ms_{ijk}}* under the null hypothesis of single-point fully contained explosion. A conservative alternative hypothesis (*Ha*) likelihood is provided by

$$P(Ms_k < m \mid Ha) \times P(Ms_k < m \mid H0)^{n_{ij}-1}, \quad (10)$$

that is, an *Ms* earthquake model at one station and an *Ms* explosion model for the other stations. The Neyman-Pearson likelihood ratio test reduces to; Reject *H0* if

$$ce^{\psi m} + \varphi(m)(n_{ij} - 1) > k, \quad (11)$$

where *c*, ψ and $\varphi(m)$ are a function of the null and alternative hypothesis parameter values. The function $\varphi(m) \in$ (0,1) and monotonic increasing, and so

$$ce^{\psi m} + \varphi(m)(n_{ij} - 1) > ce^{\psi m}. \quad (12)$$

From Equation (12), for a given value of *k* there are values of *m=Max{Ms_{ijk}}* that will fail to reject *H0* with a conservative hypothesis test based on right side of Equation (12), yet reject if the test is based on the Neyman-Pearson test (left side of Equation (12). This is graphically shown in Figure 2. The conservative hypothesis test is of the form reject *H0* if *Max{Ms_{ijk}} > k*, and is conservative in the sense of "miss no explosions". *Max{Ms_{ijk}} as a discriminant offers a cost efficient approach to discrimination in settings where path and source corrections are mature.*
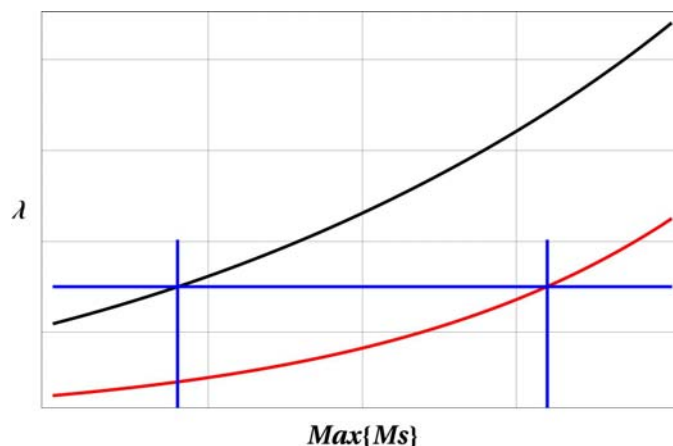
Figure 2. Graphic of the Neyman-Pearson (black) and conservative (red) likelihood ratio λ. The blue horizontal decision line projects a decision region for the discrimination *Max*(Ms). Values of *Max*(Ms) interior to this decision region reject H0 with Neyman-Pearson test and fail to reject H0 with the conservative test.

CONCLUSIONS AND RECOMMENDATIONS

The mathematical statistics of two new approaches to *Ms* versus *mb* discrimination are developed in this paper. Both methods are designed to utilize advances in source physics theory by using new source models to predict (correct) observed surface wave magnitudes. The network average discriminant formulation additionally includes source model error in the standard error (SE) of a discriminant effectively placing a lower bound on the discriminant SE. This new property of the *Ms* versus *mb* discriminant correctly reduces the SE only through network averaging of magnitudes. The order statistics discriminant formulation is potentially robust to station coverage for an event by basing discrimination on the maximum observed station *Ms* value. Validation analysis for these new formulations is in progress using global events reported in reported in Bonner et al. (2003), Bonner et al. (2006), Marshall and Bashman (1972). The Bonner et al. (2003) and Bonner et al. (2006) papers research and develop new signal processing theories for *Ms* that are design to Rayleigh wave energy from small magnitude events (stations close to source).

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