MICROMECHANICAL DAMAGE MECHANICS AT HIGH LOADING RATES

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ABSTRACT

Brittle fracture damage in the non-linear source region of a large underground explosion has been shown to make an important contribution to P and S wave radiation, particularly in the near field at frequencies in excess of 1 Hz (Johnson and Sammis, 2001). However, Johnson and Sammis made two simplifications in their model: 1) the stress field produced by the explosion was calculated using an “equivalent elasticity” approximation, and 2) the fracture damage was assumed to be in constant equilibrium with the local stress. Deshpande and Evans (2008) improved on both approximations by 1) calculating the stress field using the ABAQUS dynamic finite element program, and by 2) incorporating a dynamic crack growth law in the Ashby and Sammis (1990) micromechanical damage mechanics. However, the functional form they assume for crack growth is ad hoc and unphysical at very high loading rates. In this paper, we develop a crack growth law based on theoretical models and experimental observations of the nucleation and growth of cracks over a wide range of loading rates. In specific, our new law allows instantaneous acceleration and deceleration of crack growth typically found in explosive shock fronts. We are using the new law to describe dynamic crack growth in the Ashby and Sammis damage mechanics currently implemented in the VUMAT material constitutive subroutine of the ABAQUS finite element program.

OBJECTIVES

The primary objective of this research is to develop and test a physically based constitutive law for the source region of an underground nuclear explosion that is dominated by rock fracture. Our approach is based on micromechanics based damage mechanics in which the nucleation and growth of individual fractures is taken into account and homogenized to obtain a continuum constitutive description. The main advantages of this approach are 1) the size, density and orientations of natural fractures in the rock are physically accounted for in the model and 2) the damage evolution is accounted for by considering the growth of these natural fractures using well-know relations from dynamic fracture mechanics. This model thus offers a natural explanation for the ubiquitous observation of S-wave radiation from explosions. However, our modeling to-date has assumed quasi-static crack growth in which the cracks are always in equilibrium with their surrounding stress field. Experimental observations of crack growth at loading rates comparable to those achieved during nuclear explosions find that crack growth lags behind the applied loads. Our goal, this year, has been to incorporate recent developments in dynamic crack growth into our damage mechanics based constitutive law.

RESEARCH ACCOMPLISHED

Improving the Johnson and Sammis Micromechanical Source Model

Johnson and Sammis (2001) used the Ashby and Sammis (1990) micromechanical damage mechanics to model the Non-Proliferation Experiment (NPE) at the Nevada Test Site in September 1993 (Denny, 1994). They demonstrated that the evolving damage generated significant secondary seismic (P and S-wave) radiation in both near and far-fields. However, an obvious weak point in their approach was the approximate way in which the stresses were calculated, and in particular the empirical way in which the elastic stiffness was adjusted by the evolving damage. However, the most unphysical assumption was the use of the Ashby and Sammis (1990) damage mechanics at the high loading rates that exist in the non-linear source region. The problem is that basic assumption in Ashby and Sammis (1990) is that cracks are assumed to grow quasistatically because the stress intensity factor is always at its critical value. However, at high loading rates, the cracks require a higher value of Coulomb stress to nucleate and, once nucleated, they may not grow fast enough to keep up with the increasing stress. In this case, the stress intensity factor rises above its critical value. The good news is that the accumulation of damage at high loading rates can be modeled since the rate at which the cracks nucleate and grow is a known function of the difference between the instantaneous value of K_I and its critical value K_{IC} and the time rate of change of the stress intensity factor dK_I/dt. Modeling damage accumulation at very high loading rates is one of the primary objectives of the proposed work.

Extending Micromechanical Damage Mechanics to High Loading Rates.

If the history of the crack-tip motion is specified, then the surrounding mechanical fields in an elastic body can be obtained using linear elastic continuum mechanics, as long as the configuration of the body and the details of the loading are specified. However, since the motion of a crack-tip is totally controlled by the deformation state inside the surrounding material, the motion of the crack-tip should not be specified a priori. Due to the fact that the constitutive equation for the material does not include the possibility of material separation, we need a mathematical statement of a crack growth criterion to be added into the governing equations. Such a criterion must be stated as a physical postulate on material behavior at the same level as the kinematical theorems governing deformation, momentum balance principles, as well as the constitutive relation describing material response.

Thus, to properly solve this problem, one needs to understand the state of stress around a crack-tip (both stationary and propagating) under various loading conditions. These values then need to be compared with experimentally determined fracture toughness of the material, under similar conditions, to develop a crack-growth criterion. The most common form for such a criterion is the requirement that the crack must grow in

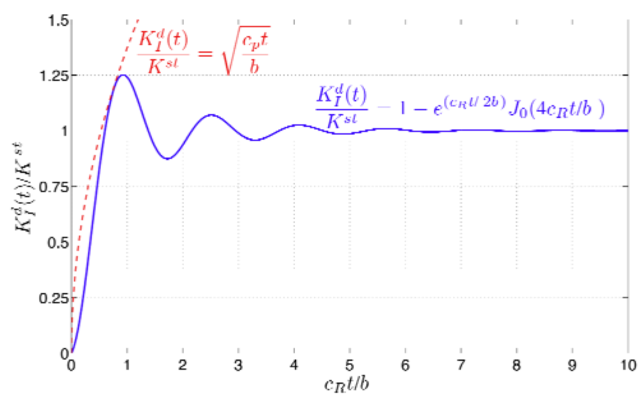

Figure 1. Temporal evolution of the dynamic stress intensity factor, scaled by its static limit.

such a way that some parameter defined as part of the crack-tip field maintains a value that is specific to the material. This value, representing the resistance of the material to the advance of the crack, is called the fracture toughness (K_{IC}) of the material, and it can be determined through experimental measurements only. In its most general form this can be represented as

$$K_I^d\,[b(t), v(t), P(t), t] = K_{IC}^d\,[v(t), \ldots] \tag{1}$$

where the left hand side of the equation represents the solution obtained from elasticity and the right hand side of the equation represents the material property determined experimentally. Here *b, v,* and *P* are the crack length, crack speed, and applied load, respectively.

Dynamic Stress Intensity Factor K_I^d *(t)*

The physical problem at hand involves transient loading of the existing fractures, in the damage mechanics framework, due to an underground explosion. Several authors have studied the problem of transient loading of a crack using analytical and numerical techniques (see Achenbach, 1970; Baker, 1962; Broberg, 1999; Brock, 1991; Freund, 1972a,b, 1973, 1974, 1990; Gross and Seelig, 2006; Kalthoff and Shockey, 1977; Kim, 1979; Kostrov, 1974, 1984; Liu and Rosakis, 1994; Liu et al., 1998; Owen et al., 1998; Ravera and Sih, 1970; Shockey et al., 1983; Sih, 1968; Sih et al., 1972; Zehnder and Rosakis, 1990, among others). The stress intensity factor for transient loading of cracks, referred to as the dynamic stress intensity factor, K_I^d , has two important general characteristics:

1) For a stationary finite crack under transient loading conditions, K_I^d evolves with time following the application of loads. As illustrated in Fig. 1, it rises sharply with time, overshoots the equivalent static value K_{st} by a considerable amount, and then oscillates around the static value with decreasing amplitude. This oscillation is due to the Rayleigh waves traveling back and forth along the surface of the crack with decreasing intensity. This generalized behavior can be summarized by the relationship

$$K_I^d(t) = K_{st}\,\Lambda\,(c_R\,t/b) \tag{2}$$

where Λ is a function of a dimensionless, crack-length-related time $c_R t/b$, describing the oscillation of K_I^d(t) around K_{st}. Here c_R is the Rayleigh wave speed of the material and b is the half-length of the crack. After an early steep increase, Λ reaches a maximum value, and then oscillates around the value 1 with diminishing amplitude, becoming equal to the average value of 1 for longer times. The increase in Λ, before the arrival of reflections, is known analytically (Kostrov, 1974) and is given by

$$K_I^d(t) = K_{st}\,(c_p\,t/b)^{1/2} \tag{3}$$

where c_p is the dilatational or the P-wave speed of the medium. This initial increase and subsequent oscillations can be approximated as

$$\Lambda\,(c_R\,t/b) = 1 - exp(c_R\,t/2b)\,J_0(4c_R\,t/b) \tag{4}$$

where $J_0(x)$ is the zero'th order Bessel Function. The above equations are plotted in Fig. 1.

2) A second key result was obtained by Freund (1973) who showed that for an unbounded body subjected to time independent loading, the dynamic stress intensity factor at a running crack-tip can be expressed as a universal function of instantaneous crack-tip speed, *v(t)*, multiplied by the equilibrium stress intensity factor for the given applied loading and the instantaneous amount of crack growth i.e.

$$K_I^d\,[v(t)] = k[v(t)]\,K_I\,[b(t)]\;;\;\; k(v) \approx (1-v/c_R)/(1-v/c_p)^{1/2} \tag{5}$$

Experimental Observations of K_{IC} at High Loading Rates

As a material parameter, K_{IC} can only be obtained through experimental measurements and is found to vary with loading rate. Under impact loading conditions, high loading rates occur at a pre-existing crack-tip. A parameter $K_{I,\,t}^d$ characterizing the loading rate is defined as

$$K_{I,\,t}^d = K_{IC}\,/\,t_c \tag{6}$$

where K_{IC} is the mode-I critical stress intensity factor at the instant of crack initiation (fracture toughness) and t_c denotes the time from the beginning of loading to the instant at which fracture initiation occurs. Usually, the crack-tip loading rates range from $K_{I,t} \sim 1\ MPa\sqrt{m}s^{-1}$ for quasi- static loading to as high as $K_{I,t} \sim 10^8$ MPa$\sqrt{m}$s^{-1} for impact loading. Due to material inertia and strain rate a material may exhibit totally different behaviors from those under quasistatic loading conditions (Rosakis et al., 1984; Rosakis and Zehnder, 1985). This material property, that depends on the loading rate, is thus called the Dynamic Initiation Toughness, $K_{IC}^{(D)}$, and its quasistatic limit is K_{IC}^{SS}. This can be represented as $K_{IC}^{(D)} = \alpha\, K_{IC}^{SS}$ where $\alpha(\geq 1)$ takes into account the increased initiation toughness due to loading rate and crack-tip bluntness due to plasticity around it (Fig. 2).

Once the crack-tip starts propagating the material resistance ahead of the crack-tip has been shown to depend on the crack-tip speed (Rosakis et al., 1984; Zehnder and Rosakis, 1990). This crack-speed dependent resistance is called Dynamic Propagation Toughness, $K_{IC}^{d}(v)$. Liu et al. (1998) explain this in terms of inertial effects associated with the propagating crack-tip. A pre- existing crack in a medium extends in the following way. The stress concentration ahead of the crack-tip loads pre-existing flaws that ultimately fail and coalesce with the crack-tip thus extending it further. If this crack-tip is now moving then the inertial effects will result in less enhanced crack-tip stresses, thus delaying the loading of these flaws, making the material appear more resistant to a growing crack. These experimental observations have been summarized in Fig. 2. Liu and Rosakis (1994) represented this as

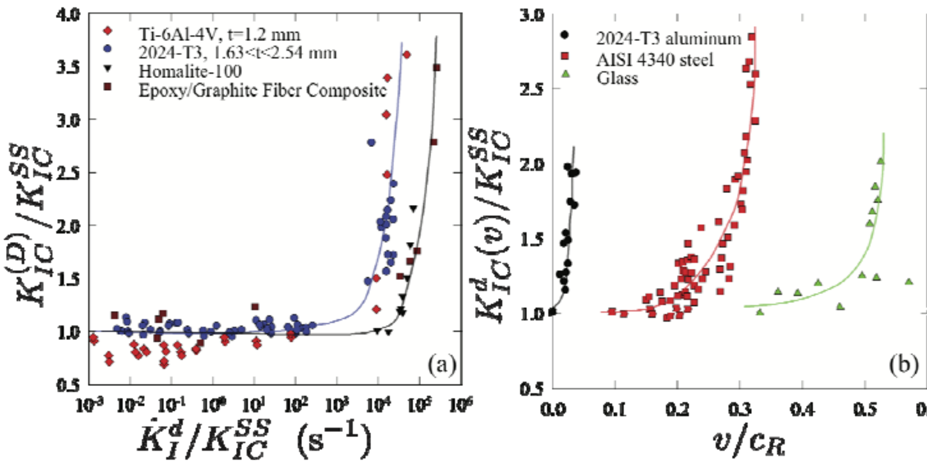

Figure 2. (a) Normalized dynamic $K_{IC}^{(D)}$ for fracture initiation as a function of loading rate for several materials. (b) Normalized K_{IC}^{d} for fracture propagation as a function of crack-tip velocity for various materials.

$$K_{IC}^{d}(v) = f(v)\, K_{IC}^{SS}\ ;\ f(v) = [1 + (1/M)\tan(\pi v/2v_m)] / (1-v/c_p)^{1/2} \qquad (7)$$

where M and v_m are two material constants fit to the data.

Crack-tip Equation of Motion

Based on the experimental observations discussed above we are now in a position to write an expression that describes both the initiation and growth of fractures at high loading rates.

Initiation Criterion: The crack will initiate motion when $K_I^{AS}(K_{I,t}^{AS}) = K_{IC}^{(D)}$ where K_I^{AS} is the stress intensity factor from Ashby and Sammis (1990).

Growth Criterion: During the process of crack growth, if the small scale yielding condition prevails, the fracture criterion stipulates

$$K_I^{d}[t] = K_{IC}^{d} \qquad (8)$$

where the left-hand side is the dynamic stress intensity factor (in principle entirely determined through an analysis of a boundary/initial value problem) and the right-hand side represents a material quantity called the dynamic fracture toughness that can only be determined through experiments. The above equation is an evolution equation for crack growth, i.e., a crack-tip equation of motion, since it represents a nonlinear, first order differential equation for the crack length $a(t)$. For the specific problem at hand, the dynamic stress intensity factor for the propagating crack is given by

$$K_I^d\,[t] = k[v]\,K_I^{AS}\,[D(t)/D_o]\,\Lambda(c_R\,t/b)\;;\;\; k(v) = (1-v/c_R)/(1-v/c_p)^{1/2} \tag{9}$$

where $K_I^{AS}\,[D(t)/D_o]$ is the stress intensity factor from Ashby and Sammis (1990) and D/D_o depends only on the crack length. As for the dynamic fracture toughness, one usually assumes that it is only dependent on the crack-tip velocity and on material characteristics. We can thus express

$$K_{IC}^d(v) = f(v)\,K_{IC}^{SS}\;;\;\; f(v) = [1 + (1/M)\tan(\pi v/2v_m)]\,/\,(1-v/c_p)^{1/2} \tag{10}$$

Combining the above equations, one now gets a non-linear equation for crack-tip motion as,

$$k(v)\,K_I^{AS}\,[D(t)/D_o]\,\Lambda(c_R\,t/b) = f(v)\,K_{IC}^{SS} \tag{11}$$

This expression will be incorporated into the damage module in ABAQUS to replace the ad-hoc growth law currently used by Deshpande and Evans (2008).

CONCLUSIONS AND RECOMMENDATIONS

An accurate calculation of the temporal evolution and spatial extent of rock fracture in the source region of an underground nuclear explosion requires incorporation of dynamic fracture mechanics into the micro-mechanics based constitutive laws currently used to describe the fracture damage dominated regime. Experimental observations of crack nucleation and growth, at a wide range of loading rates, have shown that the material properties pertaining to fracture are sensitive to loading rates, particularly in high loading rate regimes like in underground nuclear explosions. These material properties combined with well known theoretical and numerical observations about dynamic crack growth provide an equation of motion for the crack-tip (crack growth law) that will be implemented in our fracture damage based rheology.

We plan to use this new formulation to recalculate the Johnson and Sammis (2001) simulation of the 1993 Non-Proliferation Experiment, and to simulate recent chemical explosion in a granite quarry conducted by New England Research and Weston Geophysical.

REFERENCES

Achenbach, J. (1970). Extension of a crack by a shear wave, *Z. Angew. Math. Phys.* 21(6): 887–900.

Ashby, M. F., and C. G. Sammis (1990). The damage mechanics of brittle solids in compression, *Pure and Appl. Geophys.* 133: 489–521.

Baker, B. (1962). Dynamic stresses created by a moving crack, *J. Appl. Mech.* 29(3): 449–458.

Broberg, K. B. (1999). *Cracks and Fracture*. Academic Press.

Brock, L. M. (1991). Exact transient results for penny-shaped crack growth under combined loading, *Int. J. Solids. Struct.* 28(4): 517–531.

Denny, M. D. (1994). *Introduction and highlights,in Proceedings of the Symposium on the Non-Proliferation Experiment (NPE): Results and Implications for Test Ban Treaties*, M. D. Denny (editor), CONF-9404100, Department of Energy.

Deshpande, V. S., and A. G. Evans (2008). Inelastic deformation and energy dissipation in ceramics: A mechanism-based constitutive model, *J. Mech. Phys. Solids* 56(10): 3077–3100.

Freund, L. B. (1972a). Crack propagation in an elastic solid subjected to general loading-i. constant rate of extension, *J. Mech. Phys. Solids* 20(3): 129–140.

Freund, L. B. (1972b). Crack propagation in an elastic solid subjected to general loading-ii. non-uniform rate of extension, *J. Mech. Phys. Solids* 20(3): 141–152.

Freund, L. B. (1973). Crack propagation in an elastic solid subjected to general loading-iii. stress wave loading, *J. Mech. Phys. Solids* 21: 47–61.

Freund, L. B. (1974). Crack propagation in an elastic solid subjected to general loading.-iv.obliquely incident stress pulse, *J. Mech. Phys. Solids* 22: 137–146.

Freund, L. B. (1990). *Dynamic Fracture Mechanics*. Cambridge University Press, Cambridge.

Gross, D., and T. Seelig (2006). *Fracture mechanics: With an introduction to micromechanics*. Springer Verlag.

Johnson, L.R. and C.G. Sammis (2001). Effects of rock damage on seismic waves generated by explosions, *Pure Appl. Geophys.* 158(11): 1869-1908.

Kalthoff, J. F., and D. A. Shockey (1977). Instability of cracks under impulse loads, *J. Appl. Phys.* 48: 986–993.

Kim, K. S. (1979). Dynamic propagation of a finite crack, *Int. J. Solids Struct.* 15: 685–699.

Kostrov, B. V. (1974). Propagation of a crack with variable velocity, *Prikl. Mat. Mekh.* 38: 551–560.

Kostrov, B. V. (1984). On the crack propagation with variable velocity, *Int. J. Fract.* 26(4): 370–379.

Liu, C., and A. J. Rosakis (1994). *Investigation of transient effects for dynamically initiating and growing cracks under stress wave loading conditions, in Dynamic Fracture Mechanics*. edited by M. H. Aliabadi, chap. 4, Computational Mechanics Publication, Southhampton, UK; Boston, USA.

Liu, C., W. G. Knauss, and A. J. Rosakis (1998). Loading rates and the dynamic initiation tough- ness in brittle solids, *Int. J. Fract.* 90(1): 103–118.

Owen, D. M., S. Zhuang, A. J. Rosakis, and G. Ravichandran (1998). Experimental determination of dynamic crack initiation and propagation fracture toughness in thin aluminum sheets, *Int. J. Fract.* 90(1): 153–174.

Ravera, R. J., and G. Sih (1970). Transient analysis of stress waves around cracks under antiplane strain, *J. Acoust. Soc. Am.* 47: 875–881.

Rosakis, A., and A. Zehnder (1985). On the dynamic fracture of structural metals, *Int. J. Fract.* 27(3): 169–186.

Rosakis, A. J., J. Duffy, and L. B. Freund (1984). The determination of dynamic fracture toughness of AISI 4340 steel by the shadow spot method, *J. Mech. Phys. Solids* 32(6): 443–460.

Shockey, D. A., J. F. Kalthoff, and D. C. Erlich (1983). Evaluation of dynamic crack instability criteria, *Int. J. Fract.* 22(3): 217–229.

Sih, G. C. (1968). Some elastodynamic problems of cracks, *Int. J. Fract.* 4(1): 51–68.

Sih, G. C., G. T. Embley, and R. S. Ravera (1972). Impact response of a finite crack in plane extension, *Int. J. Solids Struct.* 8(12): 977–993.

Zehnder, A., and A. J. Rosakis (1990). Dynamic fracture initiation and propagation in 4340 steel under impact loading, *Int. J. Fract.* 43(4): 271–285.