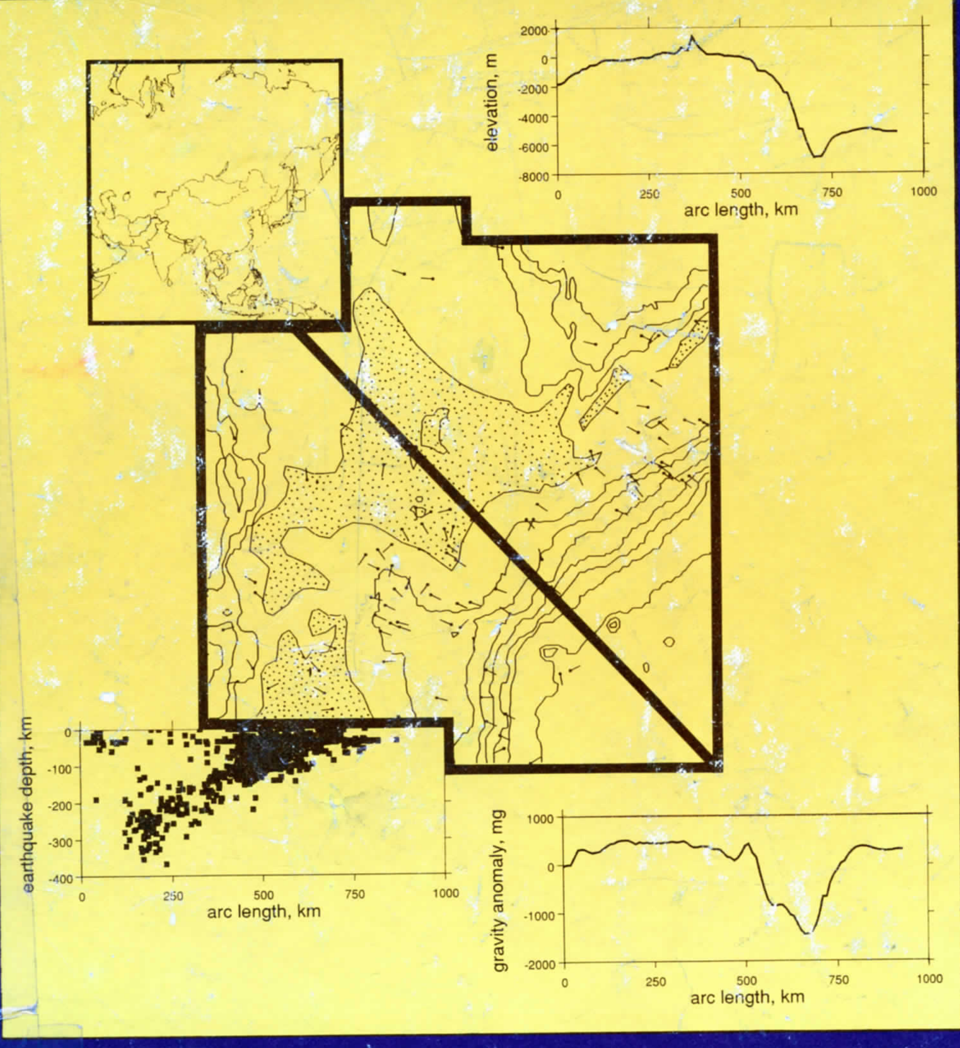
Geophysical Theory

WILLIAM MENKE AND DALLAS ABBOTT



Geophysical Theory introduces a wide range of general physical and applied mathematical techniques and shows how these techniques can be applied to answering specific questions about the earth. The authors have selected material from the disciplines which encompass *solid earth geophysics*. These include geodesy, geomagnetism and paleomagnetism, hydrology, planetology, tectonophysics, seismology, physical volcanology, and petrology. By offering a thorough and up-to-date quantitative background in theoretical geophysics, *Geophysical Theory* will be useful to both students and professionals alike.

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Cover: Geophysical data for northern Japan (see base map, inset), with 1000 meter contours of topography and bathymetry. The Pacific plate subducts beneath Eurasia, causing the topographically low Japan trench (lower right). Directions of maximum compressional stress, based on the observed moment tensors of earthquakes (small arrows), is approximately perpendicular to the direction of convergence. Topography, gravity and earthquake depth (inset) are profiled along the great circle diagonally crossing the map.
These figures were prepared from the Lamont-Doherty Geological Observatory's Global Geophysical Database, and includes data from Lamont-Doherty, the US Geological Survey, the National Oceanic and Atmospheric Administration and Harvard University

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Preface

Geophysics is in the curious position of being allied with two historically distinct branches of science: geology, from which it draws its subject matter; and physics, from which it draws it analytical technique. On the one hand, geophysics is a geological science. It seeks a quantitative explanation of the structure of the earth and the processes that have shaped it over geological time. Mountain building, subsidence, volcanism, sedimentation, and faulting and folding have all come under the scrutiny of geophysicists. On the other hand, geophysics is a branch of applied physics. It description of processes occuring in the earth are in terms of forces and fields, conservation laws and partial differential equations. Looking back to the nineteenth century, the dividing line between physics and geophysics blurs. W.E. Weber, for example, devoted substantial effort to the geophysical subject of terrestrial magnetism, though he is usually regarded as a physicist.

The teaching of geophysics is complicated by this duality. A student must learn an enormous body of information about the earth (much of it in a terminology peculiar to geologists), and must develop an intuition about the way the earth works. But the student must also master the quantitative tools of physical analysis. Needless to say, neither of these objectives comes easy.

This second goal, toward which this book is directed, has two parts. A student needs to learn a broad body of general physical and applied mathematical tehniques that are so ubiquitous that they transcend any particular subject area. Scalar and vector fields, for instance, appear almost everywhere is the physical sciences. Wave propagation is encountered in electromagnetism, acoustics, quantum mechanics, and seismology. Spherical harmonics are used to describe both the gravitational potential of the earth and the wave function of the hydrogen atom. This book introduces these techniques through the device of geophysical problems, yet their introduction is comparable, say, to that in many upper-class undergraduate physics textbooks. On the other hand, a student also needs to learn how these techniques are applied to answering specific questions about the earth, such as why it is elliptical or how it is known that it possesses a core. This book explores, in considerable detail, how this is done.

From the point of view of mathematics, this book is written at a level that might correspond to an upper-class undergraduate text from the point of view of a physics student, and a graduate text from the point of view of a geology student. A thorough knowledge of the calculus, linear algebra, and ordinary differential equations is presumed, as is some familiarity with complex numbers and partial differential equations. Nevertheless, part of chapter 2 is spent in a brief survey of undergraduate applied mathematics, with the purpose not so much of teaching the material as to reviewing it and establishing the notation the book will follow. We subsequentally build upon this material, beginning with vector and tensor analysis. This is such an important tool in geophysics, where almost all important physical quantities are vectors (gravity, displacement, magnetic field, heat flow, and so forth) and some are tensors (stress, strain), that we devote to it part of chapter 2 and all of chapter 3. Other mathematical tools, including partial differential equations, Fourier and Laplace transforms, orthogonal function expansions, eigenvalues and eigenvectors, spherical harmonics, and perturbation methods, are introduced as they naturally arise in the solution of a geophysical problem. We do not, however, use complex analysis, group theory, or the variational calculus, in order to limit the mathematical level of the text. We consciously emphasize modern notation, such as the vector differential operators, the Einstein notation, and Green's functions, both because this notation promotes clarity (thus revealing the underlying physics) and because a knowledge of it is necessary to be able to read the more theoretical side of the geophysical literature.

While geophysics is a very broad subject, and cannot completely be covered by any one work, we have selected material from a broad range of disciplines in what is often termed *solid earth geophysics*. Chapters 4 and 5 discuss the earth as whole—its orbital motions, its shape and its gravity field.

Chapter 6, on the flow of heat, develops some of the potential field methods introduced in the study of gravity, while solving problems that were of great importance in the discovery of plate tectonics. Chapters 7–11 are broadly concerned with the deformaiton of the earth. Chapter 7 introduces the concepts of stress and strain. Chapters 8 and 9 analyze vibrations in fluids and solids, respectively. Chapters 10 and 11 discuss permanent deformation is the lithosphere (where the stress is elastic) and the mantle (where flow occurs), respectively. Chapter 12 is devoted to electromagnetism.

Finally, a few words about what this book is not. It is neither a book on geophysical measurements nor on geophysical data analysis, though both these subjects are of great importance in geophysics. Students are encouraged to seek other works on these subjects, since they are both needed in a solid geophysical education. And it is not a book on the composition of the earth. One subject, which is rightfully a part of geophysics, and which we have asiduously avoided in this text, is thermodynamics and equations of state of geological materials. Well, any book must draw the line on its subject matter somewhere.

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