suppose there are a bunch of elements arranged such that \( N_1 \) are in quantum state 1, \( N_2 \) are in quantum state 2, and \( N_i \) are in quantum state \( i \).

The number of ways in which \( n \) elements can be arranged with \( N_i \) in each state \( i \) is

\[
\frac{n!}{T! N_1! N_2! \ldots N_i!}
\]

call this the probability of a macroscopic state \( P \). For example consider 3 elements, ABC. These can be arranged into several macroscopic states:

- ALL IN STATE 1
- ALL IN STATE 2
- ONE IN STATE 1 AND 2 IN STATE 2
- TWO IN STATE 2 AND 1 IN STATE 1

for a simple two state system.

Now, there are three microscopic ways to get, for example, one in state 1 and 2 in state 2.

Let STATE 1 STATE 2

\[
\begin{align*}
AB & \quad C \\
AC & \quad B \\
BC & \quad A
\end{align*}
\]

so the probability of this state is said to be 3. (The probability of any state is \( 3^3+3+1+1 = 8 \))

Now we wish to find all all possible macroscopic states the most probable. We however put several constraints on the macroscopic states.

- fixed \( n \) of elements \( n = \sum N_i \)
- fixed total energy \( E = \sum N_i e_i \)
so we look for a stationary value of $P$

i.e. $\delta P = 0$ or equivalently $\delta \ln P = 0$

$$\delta P = \delta \ln n_i - \sum_i \delta \ln n_i$$

now $\ln n_i = \ln n - \ln \delta n_i$ so

$$\delta P = 0 = \sum_i \ln n_i \delta n_i$$

now the small perturbations $\delta n_i$ are not arbitrary but

are related via the constraints. i.e.

$$\sum \delta n_i = 0$$

$$\sum \delta e_i \delta n_i = 0$$

Lagrange multipliers can then be used to solve the prob.

for arbitrary $\alpha$, $\beta$ it is true that

$$\sum [\ln n_i + \alpha + \beta e_i] \delta n_i = 0$$

suppose we choose $\alpha$, $\beta$ so that for $i = 1$ and 2

[ ] is identically zero. Then all the other $\delta n_i$'s

are truly independent and we have therefore that

each [ ] in the sum must be zero. for $i > 2$

$$\ln n_i + \alpha + \beta e_i = 0$$

$$n_i = \exp [-\alpha - \beta e_i] = e^{-\alpha} e^{-\beta e_i}$$

now we need only determine $\alpha$ and $\beta$. since

$$n = \sum n_i = \sum e^{-\alpha} e^{-\beta e_i}$$

$$e^{-\alpha} = n / \sum e^{-\beta e_i}$$

since $E = \sum n_i e_i = \sum e^{-\alpha} e^{-\beta e_i} e_i = n \Sigma e_i e^{-\beta e_i}$

$$\sum e^{-\beta e_i}$$