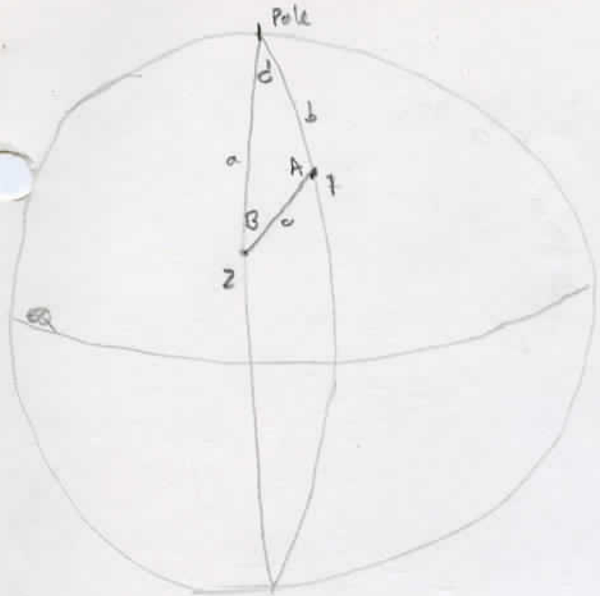
DISTANCES ON A SPHERE.



Spherical Triangle ∡ABC

$a = 90 - \lambda_2$

$b = 90 - \lambda_1$

$c = r =$ range

$d = L_1 - L_2 = \Delta L$

by law of cosines

$\cos c = \cos a \cos b + \sin a \sin b \cos d$

now $\sin 90 - x = \cos x$

$\cos 90 - x = \sin x$

so

$$\cos r = \sin \lambda_1 \sin \lambda_2 + \cos \lambda_1 \cos \lambda_2 \cos \Delta L$$

let $\frac{\lambda_1 + \lambda_2}{2} = \bar{\lambda}$ Then

$$\cos r = \tfrac{1}{2} \cos \Delta\lambda - \tfrac{1}{2} \cos 2\bar{\lambda} + \tfrac{1}{2} \cos \Delta\lambda \cos \Delta L + \tfrac{1}{2} \cos 2\lambda \cos \Delta L$$

$$\cos r = \tfrac{1}{2} \cos \Delta\lambda - \cos^2 \bar{\lambda} + \tfrac{1}{2} + \tfrac{1}{2} \cos \Delta\lambda \cos \Delta L + \cos^2 \bar{\lambda} \cos \Delta L - \tfrac{1}{2} \cos \Delta L$$

$$\left(2 \cos r = \cos \Delta\lambda - 2\cos^2 \bar{\lambda} + 1 + \cos \Delta\lambda \cos \Delta L + 2\cos^2 \bar{\lambda} \cos \Delta L - \cos \Delta L\right)$$

if $\Delta\lambda$, ΔL very small

$$2 - r^2 = 1 - \frac{\Delta\lambda^2}{2} - 2\cos^2 \lambda + 1 + 1 - \frac{\Delta\lambda^2}{2} - \frac{\Delta L^2}{2} + 2\cos^2 \bar{\lambda} - 2\cos^2 \bar{\lambda} \frac{\Delta L^2}{2} - 1 + \frac{\Delta L^2}{2}$$

$$2 - r^2 = 2 - (\Delta\lambda)^2 \cos^2 \bar{\lambda}$$

$$r^2 = (\Delta\lambda)^2 + (\Delta L)^2 \cos^2 \bar{\lambda}$$ which is the cartesian approximation.

to improve upon the cartesian approximation, we keep terms of order $(\Delta\lambda)^4$, etc. in the cosines.

$$2 - r^2 + \frac{r^4}{12} = 1 - \frac{\Delta\lambda^2}{2} + \frac{\Delta\lambda^4}{24} - 2\cos^2\bar\lambda + 1 + 1 - \frac{\Delta\lambda^2}{2} - \frac{\Delta L^2}{2} + \frac{\Delta L^4}{24} + \frac{\Delta\lambda^4}{24} + \frac{\Delta\lambda^2 \Delta L^2}{4}$$

$$+ 2\cos^2\bar\lambda - 2\cos^2\bar\lambda \frac{(\Delta L)^2}{2} + 2\cos^2\bar\lambda \frac{(\Delta L)^4}{24} - 1 + \frac{(\Delta L)^2}{2} - \frac{(\Delta L)^4}{24}$$

$$-r^2 + \frac{r^4}{12} = -(\Delta\lambda)^2 - (\Delta L)^2\cos^2\bar\lambda + \frac{(\Delta\lambda)^4}{12} + \frac{(\Delta L)^4}{12}\cos^2\bar\lambda + \frac{(\Delta\lambda)^2(\Delta L)^2}{4}$$

now let $q = (\Delta\lambda)^2 + (\Delta L)^2\cos^2\bar\lambda - \frac{(\Delta\lambda)^4}{12} - \frac{(\Delta L)^4}{12}\cos^2\bar\lambda - \frac{(\Delta\lambda)^2(\Delta L)^2}{4}$

we have $\frac{r^4}{12} - r^2 + q = 0$

$$r^2 = \frac{+1 - \sqrt{1 - \frac{1}{3}q}}{\frac{1}{6}}$$

now since $\sqrt{1+x} \approx 1 - \frac{x}{2}$ $\qquad r^2 \approx \frac{1 - 1 + \frac{1}{6}q}{\frac{1}{6}} = q$

which is also the cartesian answer. so

let $q = (\Delta\lambda)^2 + (\Delta L)^2\cos^2\bar\lambda + \left[-\frac{(\Delta\lambda)^4}{12} - \frac{(\Delta L)^4}{12}\cos^2\bar\lambda - \frac{(\Delta\lambda)^2(\Delta L)^2}{4}\right]$

$\phi = r^2 = 6 - 6\sqrt{1 - q/3}$

$$\cancel{r^2 = (\Delta\lambda^2) + (\Delta L)^2\cos^2\bar\lambda + \underbrace{A(\Delta L)^4 + B(\Delta\lambda)^4 + C(\Delta\lambda)^2(\Delta L)^2}_{\cos\bar\lambda}}$$

$$(a+b)^n = a^n + nba^{n-1} + \frac{n(n-1)}{2} b^2 a^{n-2}$$

$$6(1-\tfrac{q}{3})^{1/2} = 1 - \frac{1}{2}\frac{q}{3} + \frac{\frac{1}{2}(-\frac{1}{2})}{2}\frac{q^2}{9}$$

$$= 1 - \frac{q}{6} - \frac{q^2}{72}$$

$$6 - 6(1-\tfrac{q}{3})^{1/2} = \cancel{6-6} + q + \frac{q^2}{12}$$

$$= q + \frac{q^2}{12}$$

$$\frac{q^2}{12} = \frac{(\Delta\lambda)^4}{12} + \frac{(\Delta L)^4 \cos^2\bar{\lambda}}{12} + \frac{2(\Delta\lambda)^2 \cdot (\Delta L)^2 \cos^2\bar{\lambda}}{12}$$

$$r^2 = q + q^2/12 = (\Delta\lambda)^2 + (\Delta L)^2\cos^2\bar{\lambda} - \frac{(\Delta\lambda)^2(\Delta L)^2}{4} + \frac{(\Delta\lambda)^2(\Delta L)^2\cos^2\bar{\lambda}}{6}$$

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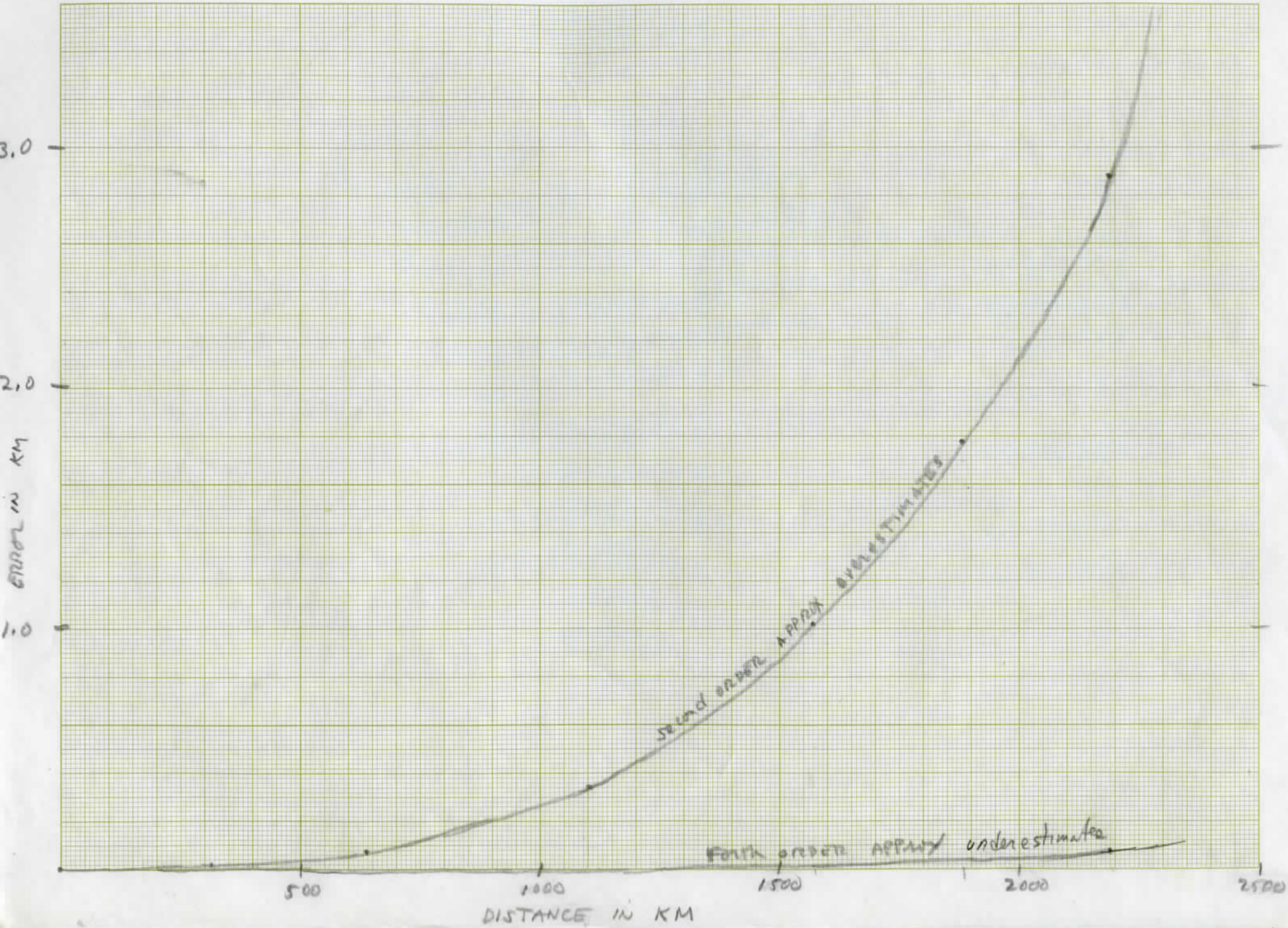

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