7/7/94 notes on fitting a differential equation to data.

let $\underline{d}$ be a discretized field.

then let this field satisfy $\underline{\underline{A}}(\underline{m})\,\underline{d}(\underline{m}) = \underline{b}(\underline{m})$ (ie a discretized diff. eqn), where $\underline{m}$ is a vector of parameters.

given observations $\underline{d}_o$, The error is

$$E(m) = \| \underline{d}(m) - d_o \|_2 = (\underline{d}(m) - \underline{d}_o)^T (\underline{d}(m) - \underline{d}_o)$$

and we then want to minimize E.

This requires the derivative $\frac{\partial E}{\partial m}$

$\min E = (\underline{d}(\underline{m}) - \underline{d}_0)^T (\underline{d}(\underline{m}) - \underline{d}_0)$ w/c $\underline{\underline{A}}^{(m)} \underline{d}^{(m)} = b(\underline{m})$ w.r.t. $\underline{m}$

$$\frac{\partial E}{\partial m} = 2(d - d_0)^T \frac{\partial d}{\partial m}$$

$$\lambda^T \frac{\partial A}{\partial m} d + \lambda^T A \frac{\partial d}{\partial m} = \lambda^T \frac{\partial b}{\partial m}$$ $\underline{\lambda}$ = lagrange mult

$$\frac{\partial E}{\partial m} = 2(d - d_0)^T \frac{\partial d}{\partial m} + \lambda^T \frac{\partial b}{\partial m} - \lambda^T \frac{\partial A}{\partial m} d - \lambda^T A \frac{\partial d}{dm}$$

$$= \left(2(d - d_0)^T - \lambda^T A\right) \frac{\partial d}{\partial m} + \lambda^T \frac{\partial b}{\partial m} - \lambda^T \frac{\partial A}{\partial m} d$$

so choose λ so $2(d - d_0)^T - \lambda^T A = 0$

which is to say $\underline{\underline{A}}^T \underline{\lambda} = 2(\underline{d} - \underline{d}_0)$

and then $$\frac{\partial E}{\partial m} = \lambda^T \frac{\partial b}{\partial m} - \lambda^T \frac{\partial A}{\partial m} d$$

which doesn't involve the nasty quantity

$\frac{\partial d}{\partial m}$.