7/7/94 notes on fitting a differential equation to data.

Let $\mathbf{d}$ be a discretized field.

Then let this field satisfy $A(m) \mathbf{d}(m) = \mathbf{b}(m)$ (i.e., a discretized diff. eqn.) where $m$ is a vector of parameters.

given observations $\mathbf{d}_0$, the error is

$$E(m) = \| \mathbf{d}(m) - \mathbf{d}_0 \|^2 = (\mathbf{d}(m) - \mathbf{d}_0)^T (\mathbf{d}(m) - \mathbf{d}_0)$$

and we then want to minimize $E$.

This requires the derivative $\frac{2E}{2m}$. 
\[
\begin{align*}
\min E &= (d^{(m)} - d_0)^T (d^{(m)} - d_0) \quad \text{w/} \quad A^{(m)} d^{(m)} = b^{(m)} \quad \text{w/} \quad m \\
2E &= 2 \quad \text{Rate of change of} \\
\frac{2E}{2m} &= \frac{2(d-d_0)^T 2d}{2m} \\
\lambda^T 2A d + \lambda^T A \frac{2d}{2m} &= \lambda^T \frac{2b}{2m} \\
2 &= \text{degenerate matrix} \\
\frac{2E}{2m} &= 2(d-d_0)^T \frac{2d}{2m} + \lambda^T \frac{2b}{2m} - \lambda^T \frac{2A d}{2m} \\
&= (2(d-d_0)^T - \lambda^T A) \frac{2d}{2m} + \lambda^T \frac{2b}{2m} - \lambda^T \frac{2A d}{2m} \\
\text{So choose } \lambda \text{ so } \quad 2(d-d_0)^T - \lambda^T A > 0 \\
\text{which is to say} \quad A^T \lambda = 2(d-d_0) \\
\text{and then} \quad \frac{2E}{2m} = \lambda^T \frac{2b}{2m} - \lambda^T \frac{2A d}{2m} \\
\text{which doesn't involve the nasty quantity} \quad \frac{2d}{2m}.
\end{align*}
\]