Notes on Cond. probabilities SAVE MRN024

1. let a, M be sets of events

2. $P(a|M)$ conditional probability of a given M

$$P(a|M) = \frac{P(aM)}{P(M)}$$ part of a included in M

3. Special cases A: $a \cap M = 0$ then $P(a|M) = 0$

~~P(aM)~~

B: $a \subset M$ Then $aM = a$ and $P(a|M) = \frac{P(a)}{P(M)}$

C. $a \supset M$ Then $aM = M$ and $P(a|M) = 1$

4. Dice example probability that the throw will be 3 if it is odd

$$P(3|odd) = P(3 \cap odd)/P(odd) = \frac{1}{6}/\frac{1}{2} = \frac{1}{3}$$

total probability Theorem:

given a_n such that $a_i a_j = 0$ (mutually exclusive) and $\sum_i^N a_i = S$ (everything) Then

$$P(B) = \sum_i^N P(B, a_i)$$

6. Bayes Theorem, Given a_i as in 5

$$P(a_i|B) = \frac{P(B|a_i)\,P(a_i)}{\sum_j P(B|a_j)\,P(a_j)}$$

7. Special case Suppose $P(a_i)/P(a_j) \approx 1$ for all i, j

and That $P(B|a_1) >> \sum_{n=2}^{N} P(B|a_i)$

Then

$$P(a_1|B) = \frac{P(B|a_1)\,P(a_1)}{P(B|a_1)\,P(a_1) + \cdots} \approx 1$$

8. Example of Bayes Theorem

2 Boxes, ~~Box 2 Boxes~~ each w/ 1000 cards. Box 1 has 999 white and 1 red, box 2 has 999 red and one white. A random card is extracted from a random ~~card~~ box, and its ~~blue~~ white what is prob it came from box 1.

B_1 and B_2 are elements of Box 1 and Box 2 respectively.
W and R are white and red, respectively

$P(B_1 | W) = \frac{999}{1000}$ $\quad P(B_1 | R) = \frac{1}{1000}$ $\quad P(B_1) = \frac{1}{2} = P(B_1)$

$P(B_2 | W) = \frac{1}{1000}$ $\quad P(B_2 | R) = \frac{999}{1000}$ $\quad P(W) = \frac{1}{2}$ $\quad P(R) = \frac{1}{2}$

$$P(W | B_1) = \frac{P(B_1 | W)\, P(W)}{P(B_1 | R) P(R)\ P(B_1 | W) P(w)} = \frac{\frac{999}{1000} \cdot \frac{1}{2}}{\frac{1}{1000} \cdot \frac{1}{2} + \frac{999}{1000} \cdot \frac{1}{2}} \approx 1$$

Random variable $\hat{x}_{M}$ where M is a condition on x

$P(x / M)$ probability that r.v. $\hat{x}$ is between $(x, x+\delta x)$ and that $x \subset M$.

10. Cumulative probability

$$F_x(x / M) = P(\hat{x} \leq x \,|\, M) = P(\hat{x} \leq x, M) / P(M)$$

note $P(x|m) = \frac{d}{dx} F(x, M)$

11. Special case $M : \{\hat{x} \leq a\}$ $\quad$ given that its less than a, what's the probability its less than x ?

$$F(x / M) = P(\hat{x} \leq x \,|\, \hat{x} \leq a\} = \frac{P(\hat{x} \leq x,\ x \leq a\}}{P(x \leq a)}$$

but assume $x<a$ so $P(\hat{x} \leq x, \hat{x} \leq a) = P(\hat{x} \leq x)$

~~for~~ x $\quad P(\hat{x} \leq x \,|\, \hat{x} \leq a) = \frac{P(\hat{x} < x)}{P(\hat{x} \leq a)} = \frac{F(x)}{F(a)}$

Then density ~~is~~ f is

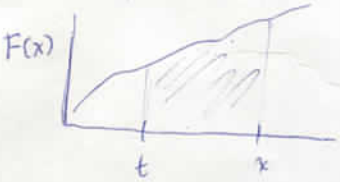
$$f(x \,|\, x \leq a) = \begin{cases} f(x) / F(a) = \dfrac{f(x)}{\int_{-\infty}^{a} f(x)\, dx} & x < a \\ 0 & x > a \end{cases}$$

12. Failure rates.

$\hat{x}$ is a r.v. that gives the time of failure of a system, and has density $f(x)$.

A. compute $f(\hat{x}=x \mid \hat{x} \geq t)$, the conditional distribution assuming that nothing failed up to time t $(t \leq x)$

$$F(x \mid \hat{x} \geq t) = \frac{P(\hat{x} \leq x \cap x \geq t)}{P(\hat{x} \geq t)} = \frac{F(x) - F(t)}{1 - F(t)}$$



computing density gives

$$f(x \mid \hat{x} \geq t) = \frac{f(x)}{\int_t^\infty f(x)\, dx} = \frac{F'(x)}{1 - F(t)}$$

B. conditional failure rate $\beta(t) = f(t \mid \hat{x} \geq t)$

$$\beta(t) = \frac{F'(t)}{1 - F(t)}$$ integrates to

$$-\ln[1 - F(t)] = \int_0^t \beta(x)\, dx$$

$$f(t) = \beta(t)\, e^{-\int_0^t \beta(x)\, dx}$$

note ~~$\beta(t) = a e^{-at}$ Then $f(t) =$~~

13. Bayes Thm for r.v.'s

$$f(x \mid a) = \frac{P(a \mid \hat{x}=x)\, f(x)}{\int_{-\infty}^\infty P(a \mid \hat{x}=x)\, f(x)\, dx}$$