New parameterization

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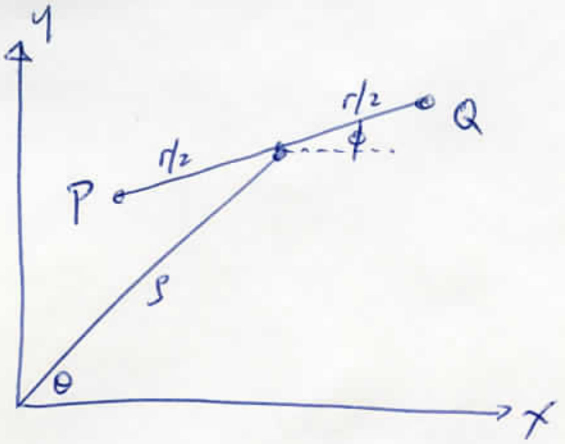
MRN027



$T(\rho, \theta, r, \phi)$

$r = |\underline{x}_p - \underline{x}_q|$

ϕ = dip of ray from horizontal

(ρ, θ) give ray midpoint

let $T(\rho, \theta, r, \phi) = \sum B_{nmpq} f_n(\rho)\, g_n(\theta)\, h_p(r)\, S_q(\phi)$

Then: $T(\underline{x}_p, \underline{x}_q) = T(\underline{x}_q, \underline{x}_p)$ means $T(\rho, \theta, r, \phi) = T(\rho, \theta, r, \phi + \pi)$

or $S_q(\phi) = e^{2iq\phi}$

also: $T(\rho_0, \theta_0, r, \phi_0) = O(r)$ when r is small

also:
$$\begin{pmatrix} x_p \\ y_p \end{pmatrix} = \begin{pmatrix} \rho \sin\theta - \frac{r}{2}\sin\phi \\ \rho\cos\theta - \frac{r}{2}\cos\phi \end{pmatrix} \qquad \begin{pmatrix} x_q \\ y_q \end{pmatrix} = \begin{pmatrix} \rho \sin\theta + \frac{r}{2}\sin\phi \\ \rho\cos\theta + \frac{r}{2}\cos\phi \end{pmatrix}$$

$$\rho = \left[\left(\frac{x_p + x_q}{2}\right)^2 + \left(\frac{y_p + y_q}{2}\right)^2 \right]^{1/2}$$

$$\theta = \tan^{-1}\left(\frac{y_p + y_q}{x_p + x_q}\right)$$

$$r = \left((x_p - x_q)^2 + (y_p - y_q)^2 \right)^{1/2}$$

$$\phi = \tan^{-1}\left(\frac{y_p - y_q}{x_p - x_q}\right)$$

and $\nabla_p T = \frac{\partial}{\partial x_p} T + \frac{\partial}{\partial y_p} T$ is computed as

$$\frac{\partial}{\partial x_p} = \frac{\partial \phi}{\partial x_p}\frac{\partial}{\partial \phi} + \frac{\partial \theta}{\partial x_p}\frac{\partial}{\partial \theta} + \frac{\partial r}{\partial x_p}\frac{\partial}{\partial r} + \frac{\partial \rho}{\partial x_p}\frac{\partial}{\partial \rho} \quad \text{etc.}$$