

Define

fourier transform  $f(\omega) = \int_t f(t) e^{\pm i\omega t} dt$

inverse f. t.  $f(t) = \frac{1}{2\pi} \int_\omega f(\omega) e^{\mp i\omega t} d\omega$

Note

$$\int_\omega e^{i\omega(t-t')} d\omega = 2\pi \delta(t-t')$$

because  $2\pi \int_t \delta(t-t') e^{-i\omega t} dt = 2\pi e^{-i\omega t'}$  is a fourier tr.

so  $\frac{1}{2\pi} \int_\omega 2\pi e^{-i\omega t'} e^{i\omega t} d\omega = \int_\omega e^{i\omega(t-t')} d\omega$  is its inv. tr.

Parseval's Thm

$$\int_t f^2(t) dt = \frac{1}{2\pi} \int_\omega |f(\omega)|^2 d\omega$$

proof.  $\int_\omega |f(\omega)|^2 d\omega = \int_\omega f(\omega) f^*(\omega) d\omega =$

$$\int_\omega \int_t f(t) e^{i\omega t} dt \int_{t'} f(t') \bar{e}^{i\omega t'} dt' d\omega =$$

$$\int_t f(t) \int_{t'} f(t') \int_\omega e^{i\omega(t-t')} d\omega dt' dt =$$

$$\int_t f(t) \int_{t'} f(t') 2\pi \delta(t-t') dt' dt =$$

$$2\pi \int_t f(t) f(t) dt = 2\pi \int_t f^2(t) dt$$

Discrete version.

Define Discrete fourier transform

$$\tilde{f}(\omega_j) = \sum_i f(t_i) e^{i\omega_j t_i}$$

Then  $f(\omega_i) \approx \Delta t \tilde{f}(\omega_i)$ 

also  $f_{ny} = \frac{1}{2\Delta t} = \frac{N}{2} \Delta f = \frac{N}{2} \frac{\Delta\omega}{2\pi}$  so  $\Delta t \Delta\omega = \frac{2\pi}{N}$

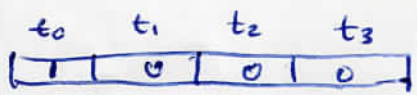


$$\int_t f^2(t) dt = \frac{1}{2\pi} \int_{\omega} |f(\omega)|^2 d\omega$$

$$\Delta t \sum_i f^2(t_i) = \frac{1}{2\pi} \Delta \omega \sum_i |f(\omega_i)|^2 = \frac{1}{2\pi} \Delta \omega (\Delta t)^2 \sum_i |\tilde{f}(\omega_i)|^2$$

$$\sum_i f^2(t_i) = \frac{1}{2\pi} \Delta \omega \Delta t \sum_i |\tilde{f}(\omega_i)|^2 = \frac{1}{N} \sum_i |\tilde{f}(\omega_i)|^2$$

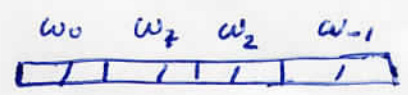
Example



$N=4$

$$\sum_i f^2(t_i) = 1$$

DFT  
→



$$\sum_i f^2(\omega_i) = 4$$

$$\frac{1}{N} \sum_i f^2(\omega_i) = 1$$