MRN036

Bob. Forgive the odd stationary. Here is something similar to your gravity problem using the method of Paulis & Booker. Let $\underline{\underline{G}}\,\underline{m} = \underline{d}$ be the gravity problem in matrix form, with $\underline{m}$ = density, $\underline{d}$ = gravity observations. Then let $\underline{m} = \bar{m}\underline{1} + \delta\underline{m}$ so that $\underline{\underline{G}}\,\underline{m} = \underline{d} \Rightarrow \underline{g}\,\bar{m} + \underline{\underline{G}}\,\delta\underline{m} = d$ with $\underline{g} = \underline{\underline{G}}\,\underline{1}$. Now $\underline{g}$ has singular value decomposition $\underline{g} = \underline{\underline{U}}\,\underline{\underline{\Lambda}}\,\underline{\underline{V}}^T$ where $\underline{\Lambda} = [\|\underline{g}\|_2, 0, 0 \ldots]^T$, $\underline{V} = [1]$ and $\underline{\underline{U}} = [\underline{\underline{U}}_p, \underline{\underline{U}}_0]$, where $\underline{\underline{U}}_p = \|g\|^{-1}[g_1, g_2, g_3 \cdots]^T$ and $\underline{\underline{U}}_0^T \underline{\underline{U}}_p = 0$. Now Paulis & Booker say to dot eqn by $\underline{\underline{U}}_0$:

$$\underline{g}\,\bar{m} + \underline{\underline{G}}\,\delta\underline{m} = \underline{d} = \underline{\underline{U}}_p \Lambda_p V_p^T \bar{m} + \underline{\underline{G}}\,\delta\underline{m} = \underline{d}$$

becomes $\underline{\underline{U}}_0^T \underline{\underline{G}}\,\delta\underline{m} = \underline{\underline{U}}_0^T \underline{d}$

which can be solved by minimizing $\delta\underline{m}$ that satisfies data exactly! $\delta\underline{m} = \underline{\underline{G}}^T \underline{\underline{U}}_0 [\underline{\underline{U}}_0 \underline{\underline{G}}\,\underline{\underline{G}}^T \underline{\underline{U}}_0]^{-1} \underline{\underline{U}}_0^T \underline{d}$ and then the original equation can be solved for $\bar{m}$: $\bar{m} = \frac{1}{\underline{g}^T\underline{g}}[\underline{g}^T \underline{\underline{G}}\,\delta m - \underline{g}^T \underline{d}]$. Question: how does your semi norm differ from problem of finding $\underline{m}$ that solves $\underline{\underline{G}}\,\underline{m} = \underline{d}$ with constraint that $\|\nabla m\|_2$ is minimum?

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Dear Bill,

I was clearing up my messy desk and found an undated letter of yours buried somewhere in the middle Cretaceous; I unclose a copy. My apologies for having overlooked it when it came. The answer to your question depends on what you mean by different. If you mean, is the seminorm minimization that we used on the seamount magnetization problem exactly equivalent to fitting a model with the minimum $\|\nabla m\|$ then the answer is no. Our models will not match models built that way although they will be similar for reasons I will get to in a moment. On the other hand, the Pavlis-Booker scheme is an example of seminorm minimization, but the penalty function they use is different from the seamount one.

A seminorm is a functional that satisfies all the conditions of a norm, except that some elements of the space besides the zero element can make it vanish. The idea in using them is put certain kinds of models into the subspace that is not seen by the seminorm because these are the models we want to favor; we put components of the model that are undesirable into the complementary subspace and allow the seminorm the chance to suppress them as far as possible. The easiest way to design a seminorm is to construct a projection operator P that singles out the desirable subspace and then to set up a minimization problem for $\|(I-P)m\|^2$. The Pavlis-Booker subspace and the seamount subspace of desirable solution turns out to be the same one, essentially, the subspace of completely uniform models (constant density or uniform magnetization). But the norm used to penalize deviations from that ideal state is different in the two cases: ours is just the regular L_2 norm while theirs involves a derivative operation and is technically speaking and example of a Sobolev norm. We chose L_2 because it made the integrals easier to do, not because it has any important intrinsic properties.

I hope this reply (however late it is) answers your question to your satisfaction.

Yours sincerely,

Bob

Robert L. Parker

Professor of Geophysics