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HERE is an interesting application of polarization tomography:

suppose the space-time metric is written

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$$

$$\text{where } \underline{\eta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{ and } \|h\| \ll \|\eta\|$$

eg. "almost flat space-time". Then

if $\phi = -\frac{1}{2} h_{00}$, then the equation

for a the tangent, $\underline{\hat{x}}$ (a 3-vector), to a light ray is:

$$\frac{d}{dt} \underline{\hat{x}} = \underline{\hat{x}} \times (\underline{\hat{x}} \times \nabla \phi)$$

see Weinberg, Gravitation & cosmology, p222, Wiley, 1972.

But compare with Menke & Abbott's formula for the tangent to an acoustic ray (M&A 8.2.11)

$$\frac{d}{ds} \underline{\hat{t}} = \underline{\hat{t}} \times [\underline{\hat{t}} \times (-c \nabla c^{-1})]$$

hence light rays act as acoustic rays with

$$-c \nabla c^{-1} = -\nabla \phi = -\nabla \frac{h_{00}}{2}$$

Hence the existing polarization tomography method can be used in Astronomy, to study the curvature of space-time.