relationship between 3D Fourier & cosine transform MRN 040

$$F(k_r) = \int_{-\infty}^{\infty} d^3\underline{x}\; f(x)\, e^{i\underline{k}\cdot\underline{x}}$$

see Bracewell p 253

$$= \frac{4\pi}{k_r}\int_0^{+\infty} r\, f(r) \sin(k_r r)\, dr$$

$$= \frac{4\pi}{k_r}\frac{\partial}{\partial k_r}\int_0^{k_r} dk_r' \int_0^{\infty} r\, f(r) \sin(k_r' r)\, dr$$

$$= \frac{4\pi}{k_r}\frac{\partial}{\partial k_r}\int_0^{\infty} r\, f(r) \int_0^{k_r} \sin(k_r' r)\, dr$$

$$= -\frac{4\pi}{k_r}\frac{\partial}{\partial k_r}\int_0^{\infty} r\, f(r) \left.\frac{\cos(k_r' r)}{r}\right|_{k_r'=0}^{k_r'=k_r} dr$$

$$= -\frac{4\pi}{k_r}\frac{\partial}{\partial k_r}\int_0^{\infty} f(r)\left[\cos(k_r r) - 1\right] dr$$

$\leftarrow \frac{\partial}{\partial k_r}$ on $1 = 0$

$$= -\frac{4\pi}{k_r}\frac{\partial}{\partial k_r}\underbrace{\int_0^{\infty} f(r) \cos(k_r r)\, dr}_{P(k_r)}$$

$$= -\frac{4\pi}{k_r}\frac{\partial}{\partial k_r} P(k_r)$$