Consider the principle:

"The wave front advances normal to itself at a speed given by the local medium velocity, c."

If $T$ gives the travel time, then the above statement is equivalent to $s \ell = \nabla T$. Here $s = 1/c$ and $\ell$ is a normal to the wavefront.

The Eikonal equation is just $s \ell = \nabla T$ dotted with itself:

$$s^2 = \nabla \cdot \nabla T$$

The ray equation is an equation involving only $s$ and $\ell$, not $T$. The vector field $\ell$ cannot be specified arbitrarily, since it must ultimately be related to $\nabla T$ by $s \ell = \nabla T$. Thus $s \ell$ is a conservative vector field and has no curl.

$$\epsilon_{ijk} (s \ell_k) j = 0$$

$$\epsilon_{ijk} s_j \ell_k + \epsilon_{ijk} s_k \ell_j = 0$$

$$s \nabla \times \ell = \ell \times \nabla s$$

$$\nabla \times \ell = \ell \times \frac{1}{s} \nabla s$$
(for a ray \( x(t) \))

The ray eqn is derived by taking \( \frac{dx}{dt} \) of the previous equation:

\[
\frac{dx}{dt} = \mathbf{e} \times (\mathbf{e} \times \mathbf{V})
\]

Note \( [\mathbf{e} \times (\nabla \times \mathbf{e})] \) = \( E_{ij} k \epsilon_{kmn} l_{ij} = E_{ij} k \epsilon_{kln} l_{ij} = (k \epsilon_{lmn} - k \epsilon_{mni}) l_{ij} l_{ln} = l_{mi} l_{mj} - l_{ml} l_{mj} = [\nabla \mathbf{l} - \mathbf{l} \cdot \nabla \mathbf{l}] \):

Note \( \nabla \cdot \mathbf{l} = l_{mi} l_{mj} = \frac{dx_n}{ds} \frac{dl_i}{ds} = \frac{dl_i}{ds} \)

Where \( s \) = arc length along ray

Note \( l \cdot \nabla l = 0 \) since \( l \cdot \nabla l \) follows from \( l \) being a unit vector:

\[
0 = (l \cdot l)_{ij} = 2 \epsilon_{ij} \epsilon_{kl} l_j = 2[l \cdot \nabla l]_i
\]

So ray eqn is:

\[\frac{dl}{ds} = \mathbf{e} \times (\mathbf{e} \times \mathbf{V})\]