MRN052

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The Radon Transformation on a family of curves in the plane
Proc Am Mat Soc 83, 325-330, 1981.

considers class of curves in polar coordinate
fixed (p,ϕ) $\quad r^\alpha \cos|\alpha(\theta-\phi)| = p^\alpha$

generalized radon transformation:

$$\hat{f}(p,\phi) = \int_\alpha f(r,\theta)\, ds_\alpha$$

$\alpha > 0$ curves symmetrical about line $\theta = \phi$
$\to \infty$ as $|\theta - \phi| \to \pi/2\alpha$
intersects itself at least once if $0<\alpha<1/2$
no intersections if $\alpha \geq 1/2$
parabola, st. line, one branch hyperbola
$1/2$ $\quad 1$ $\quad 2$

$\alpha < 0$ tend to origin as $|\theta-\phi| \to -\pi/2\alpha$
intersect $0 < -\alpha < 1/2$
don't intersect $-\alpha \geq 1/2$
cardioids, circles, one branched lemniscate
$-1/2$ $\quad -1$ $\quad -2$

define $\quad f_\ell(r) = \frac{1}{2\pi}\int_0^{2\pi} f(r,\theta)\, e^{-i\ell\theta}\, d\theta$

$$F_\ell(s) = \frac{1}{\alpha} f_\ell(s^{1/\alpha})\, s^{1/\alpha - 1}$$

$$\hat{F}_\ell(q) = \hat{f}(q^{1/\alpha})$$

$$\hat{f}_\ell(p) = \frac{1}{2\pi}\int_0^{2\pi} f(p,\phi)\, e^{i\ell\phi}\, d\phi$$

case $\alpha > 0$

$$\hat{F}_\ell(q) = 2\int_q^\infty F_\ell(s) \frac{\cos\left\{ \frac{\ell}{\alpha} \cos^{-1}\left(\frac{q}{s}\right)\right\}}{\left(1 - \left(\frac{q}{s}\right)^2\right)^{1/2}}\, ds$$

$$F_\ell(t) = -\frac{1}{\pi t}\int_t^\infty \frac{d\hat{F}_\ell(q)}{dq} \frac{\cosh\left\{\left(\frac{\ell}{\alpha}\right)\cosh^{-1}\left(\frac{q}{t}\right)\right\}}{(q^2 - t^2)^{1/2}}\, dq$$

HOLE Theorem $\alpha < 0$

only necessary to know $\hat{f}(p, \phi)$ for $p \geq r_0$

in order to determine $f(r_0, \theta)$

$\alpha > 0$ $p \leq r_0$

$\alpha = -1$ $\frac{p}{r} = \cos(\theta - \phi)$. circles thru origin