\[
\left(\frac{x \cos \theta - y \sin \theta}{a^2}\right)^2 + \left(\frac{y \cos \theta + x \sin \theta}{b^2}\right)^2 = 1 = 0
\]

\[
\frac{x^2 \cos^2 \theta}{a^2} + \frac{y^2 \sin^2 \theta}{a^2} - \frac{2xy \cos \theta \sin \theta}{a^2} + \frac{y^2 \cos^2 \theta}{b^2} + \frac{x^2 \sin^2 \theta}{b^2} - \frac{2xy \cos \theta \sin \theta}{b^2} = 1 = 0
\]
\[
\frac{2y}{\Delta} = -\frac{\partial B}{\partial x} + (2B \frac{\partial B}{\partial x} - 4A \frac{\partial C}{\partial x}) (B^2 - 4AC)^{-1/2}
\]

\[
A = O(x^0)
\]

\[
B = O(x^1)
\]

\[
C = O(x^2)
\]

\[
\frac{(\frac{2B}{\partial x})^2 (B^2 - 4AC)}{2} = (2B \frac{\partial B}{\partial x} - 4A \frac{\partial C}{\partial x})^2
\]

\[
O(x^0) \quad O(x^2) = (0(x), 0(x))^2
\]

Quadratic in \(x\) if no \(x^1\) term

\[
0 = -\frac{B^2 (\frac{2B}{\partial x})^2 + 4AC (\frac{\partial B}{\partial x})^2 + \frac{3}{4} B^2 (\frac{\partial B}{\partial x})^2 + 16 A^2 (\frac{\partial C}{\partial x})^2}{16AB \frac{\partial B}{\partial x} \frac{\partial C}{\partial x}}
\]

\[
4AC (\frac{\partial B}{\partial x})^2 = 4 \left( \frac{\sin^2 \theta}{a^2} + \frac{\cos^2 \theta}{b^2} \right) \left( \frac{1}{b^2} - \frac{1}{a^2} \right)^2 \cos^2 \theta \sin^2 \theta \cdot \left\{ \frac{x^2 (\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2}) - 1}{x^2 (\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2}) - 1} \right\}
\]

\[
3 B^2 (\frac{\partial B}{\partial x})^2 = 3 \left( \frac{1}{b^2} - \frac{1}{a^2} \right)^2 \cos^4 \theta \sin^2 \theta \quad x^2
\]

\[
16 A^2 (\frac{\partial C}{\partial x})^2 = 16 \left( \frac{\sin^2 \theta}{a^2} + \frac{\cos^2 \theta}{b^2} \right) \left( \frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2} \right) x^2
\]
\[-16 A B \frac{\partial B}{\partial x} \frac{\partial c}{\partial x} = -16 \left( \frac{\sin^2 \theta}{a^2} + \frac{\cos^2 \theta}{b^2} \right) \cdot 4 \left( \frac{1}{b^2} - \frac{1}{a^2} \right) \cos \theta \sin \theta \cdot 2 \left( \frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2} \right) x^2\]

Yech.

\[A = A_0, \quad B = B_1 x, \quad \frac{\partial B}{\partial x} = B_1, \quad C = C_0 + C_2 x^2, \quad \frac{\partial C}{\partial x} = 2 C_2 x.\]

\[4A C \left( \frac{\partial B}{\partial x} \right)^2 = 4 A_0 (C_0 + C_2 x^2) B_1^2 = 4 A_0 B_1^2 C_0 + 4 A_0 B_1^2 C_2 x^2\]

\[3 B_1^2 \left( \frac{\partial C}{\partial x} \right)^2 = 3 B_1^2 x^2\]

\[16 A_0^2 \left( \frac{\partial B}{\partial x} \right)^2 = 16 A_0^2 + C_2 x^2 = 64 A_0^2 C_2 x^2\]

\[-16 A B \frac{\partial B}{\partial x} \frac{\partial c}{\partial x} = -16 A_0 B_1 x B_1 2 C_2 x = -32 A_0 B_1^2 C_2\]

\[
\left( 4 A_0 B_1^2 C_2 + 3 B_1^2 + 64 A_0^2 C_2^2 - 32 A_0 B_1^2 C_2 \right) x^2 = -4 A_0 B_1^2 C_0 = 1
\]

\[x^2 = \frac{4 A_0 B_1^2}{3 B_1^4 + 64 A_0^2 C_2^2 - 28 A_0 B_1^2 C_2}\]
\[ \frac{dy}{dx} = \infty \quad \text{when} \quad B^2 - 4AC = 0 \]

Solve \(x^2\)

\[ B_1^2 x^2 - 4A_0 (c_0 x^2) = 0 \]

\[ B_1^2 x^2 - 4A_0 c_0 x^2 = 0 \]

\[ (B_1^2 - 4A_0 c_0) x^2 = 4A_0 c_0 \]

\[ x^2 = \frac{4A_0 c_0}{(B_1^2 - 4A_0 c_0)} \]

\[ = \frac{4}{b^2} \left( \frac{\sin^2 \theta}{a^2} + \frac{\cos^2 \theta}{b^2} \right) \]

\[ + \frac{1}{a^2} \left( \frac{\sin^2 \theta}{a^2} + \frac{\cos^2 \theta}{b^2} \right) \left( \frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2} \right) \]

\[ - \left( \frac{1}{b^2} \right) \cos^2 \theta \sin^2 \theta \]

\[ \text{checked by computer: seems to work} \]

\[ x^2 = \frac{\left( \frac{\sin^2 \theta}{a^2} + \frac{\cos^2 \theta}{b^2} \right) \left( \frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2} \right)}{\left( \frac{1}{b^2} \right) \cos^2 \theta \sin^2 \theta} \]
\[
\frac{dy}{dx} = \infty \quad \text{when} \quad -\frac{1}{4} B^2 + AC = 0 \\
\sin^2 \theta = 1 - \cos^2 \theta
\]

\[-\frac{1}{4} B^2 = -\frac{1}{4} \left( \frac{1}{b^2} - \frac{1}{a^2} \right)^2 \cos^2 \theta \left( 1 - \cos^2 \theta \right) \times^2\]

\[= -\left( \frac{1}{b^2} - \frac{1}{a^2} \right)^2 \times^2 \cos^2 \theta + \left( \frac{1}{b^2} - \frac{1}{a^2} \right)^2 \times^2 \cos^4 \theta\]

\[A = \frac{1}{a^2} - \frac{\cos^2 \theta}{a^2} + \frac{\cos^2 \theta}{b^2} = \left( \frac{1}{b^2} - \frac{1}{a^2} \right) \cos^2 \theta + \frac{1}{a^2}\]

\[C = \left( \frac{\cos^2 \theta}{a^2} + \frac{1}{b^2} - \frac{\cos^2 \theta}{b^2} \right) \times^2 - 1\]

\[= \left( \frac{1}{a^2} - \frac{1}{b^2} \right) \times^2 \cos^2 \theta + \left( \frac{x^2}{b^2} - 1 \right)\]

\[AC = \left\{ \left( \frac{1}{b^2} - \frac{1}{a^2} \right) \cos^2 \theta + \frac{1}{a^2} \right\} \left\{ \left( \frac{1}{b^2} - \frac{1}{a^2} \right) \times^2 \cos^2 \theta + \left( \frac{x^2}{b^2} - 1 \right) \right\}\]

\[+ \left( \frac{1}{b^2} - \frac{1}{a^2} \right) \left( \frac{x^2}{b^2} - 1 \right) + \frac{1}{a^2} \left( \frac{x^2}{b^2} - 1 \right) \times^2 \cos^2 \theta\]

\[\left( \frac{1}{b^2} - \frac{1}{a^2} \right) \left\{ \frac{x^2}{b^2} - \frac{x^2}{a^2} - 1 \right\} \cos^2 \theta\]
Terms multiplying
\( \cos^4 \theta : \)
\[
\left( \frac{1}{b^2} - \frac{1}{a^2} \right)^2 x^2 - \left( \frac{1}{b^2} - \frac{1}{a^2} \right)^2 x^2
\]
\[
= 0
\]
\( \cos^2 \theta : \)
\[
- \left( \frac{1}{b^2} - \frac{1}{a^2} \right)^2 x^2 + \left( \frac{1}{b^2} - \frac{1}{a^2} \right) \left\{ \frac{x^2}{b^2} - \frac{x^2}{a^2} - 1^2 \right\}
\]
\[
= \frac{1}{a^2} \left( \frac{x^2}{b^2} - 1 \right)
\]
\[
x^2 \left( \frac{1}{b^2} - \frac{1}{a^2} \right) \left\{ -\frac{1}{b^2} + \frac{1}{a^2} + \frac{1}{b^2} - \frac{1}{a^2} - \frac{1}{x^2} \right\}
\]
\[
= -\left( \frac{1}{b^2} - \frac{1}{a^2} \right)
\]

\( \cos^2 \theta = \)
\[
\frac{\frac{1}{a^2} \left( \frac{x^2}{b^2} - 1 \right)}{-\left( \frac{1}{b^2} - \frac{1}{a^2} \right)}
\]

checked by computer, seems to work.
\[ \theta = 30^\circ \quad x_{mf} = 1.3228 \]
\[ \theta = 10^\circ \quad x_{mf} = 1.04425 \]
\[ \theta = 20^\circ \quad x_{mf} = 1.16230 \]