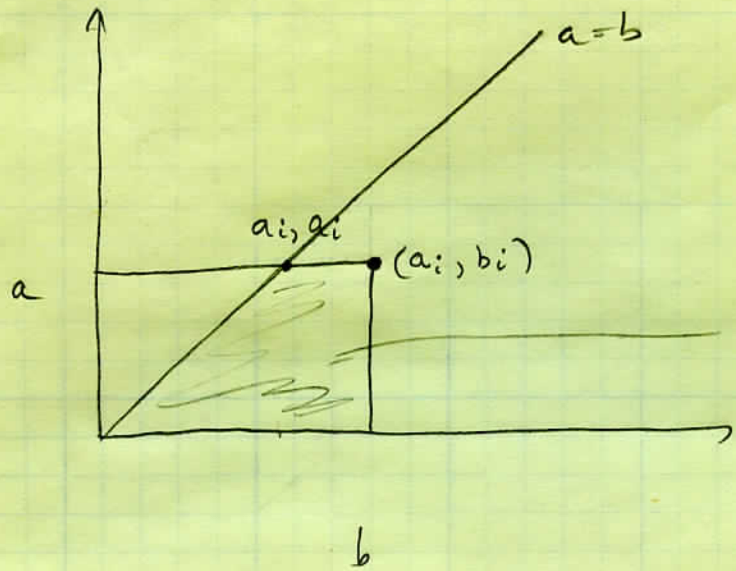
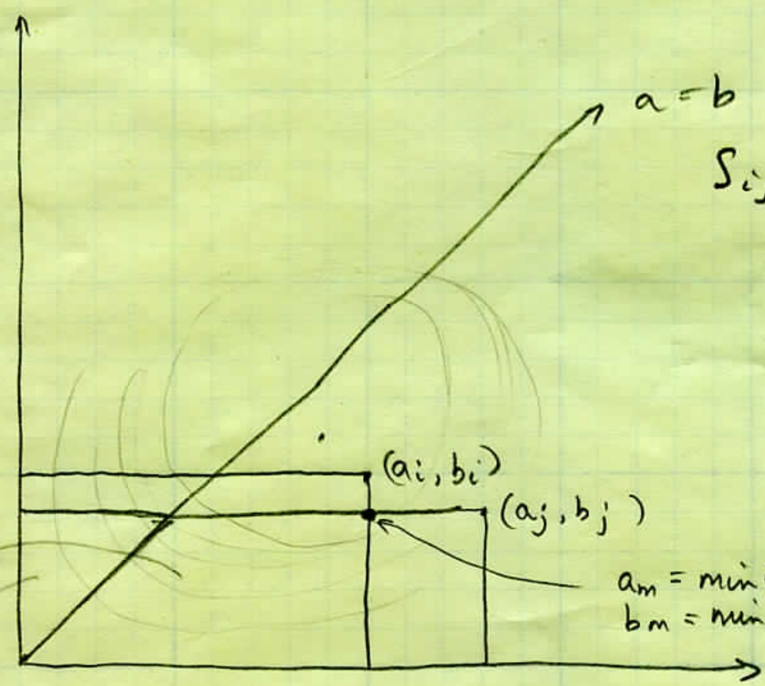
Backus - Gilbert Theory of elliptical sieve



$$u_i = \int G_i(r)\,dr$$

$$\text{Area} = a_i b_i - \tfrac{1}{2} a_i^2$$



$$S_{ij} = \int (r-r_0)^2 G_i(r) G_j\,dr$$

= RECTANGLE
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TRIANGLE

$a_m = \min(a_i, a_j)$
$b_m = \min(b_i, b_j)$

$$\int_{a_1}^{a_2} (a-a_0)^2\,da = \int_{a_1-a_0}^{a_2-a_0} a'^2\,da = \frac{(a_2-a_0)^3 - (a_1-a_0)^3}{3}$$

TRIANGLE

$$\int_{a=0}^{a=a_m} da \int_{b=a}^{b=a_m} db \left\{ (a-a_0)^2 + (b-b_0)^2 \right\} =$$

RECTANGLE

$$\int_{a=0}^{a=a_m} da \int_{b=0}^{b=b_m} db \left\{ (a-a_0)^2 + (b-b_0)^2 \right\}$$

$$\int_{a=0}^{a=a_m} da \quad a\,(a-a_0)^2 =$$

$a' = a - a_0 \qquad a = a' + a_0 \qquad da = da'$

$a = a_m \qquad a = 0$

$a' = a_m - a_0 \qquad a' = -a_0$

$$\int_{a'=-a_0}^{a'=a_m-a_0} (a'+a_0)\,a'^2\,da' =$$

$$\int_{-a_0}^{a_m-a_0} \{a'^3 + a_0 a'^2\}\,da =$$

$$\frac{a'^4}{4} + \frac{a_0 a'^3}{3} \Big|_{-a_0}^{a_m-a_0}$$

$$\frac{(a_m-a_0)^4}{4} + \frac{a_0(a_m-a_0)^3}{3} - \left[\frac{3a_0^4}{4\cdot 3} - \frac{4a_0^4}{3\cdot 4}\right]$$

$$-\frac{a_0^4}{12}$$

RECTANGLE

$$\int_0^{a_M}\int_0^{b_m}\left\{(a-a_0)^2+(b-b_0)^2\right\}da\,db$$

$$= b_m\left\{\frac{(a_m-a_0)^3}{3}+\frac{a_0^3}{3}\right\}+a_m\left\{\frac{(b_m-b_0)^3}{3}+\frac{b_0^3}{3}\right\}$$

TRIANGLE

$$\int_{a=0}^{a=a_m}da\int_{b=a}^{b=a_m}db\left\{(a-a_0)^2+(b-b_0)^2\right\}=$$

$$\int_{a=0}^{a=a_m}da\left\{a_m(a-a_0)^2-a(a-a_0)^2+\frac{1}{3}(a_m-b_0)^3-\frac{1}{3}(a-b_0)^3\right\}=$$

$$-\left\{\frac{(a_m-a_0)^4}{4}+a_0\frac{(a_m-a_0)^3}{3}+\frac{a_0^4}{12}\right\}$$

$$+a_m\left\{\frac{(a_m-a_0)^3}{3}+\frac{a_0^3}{3}\right\}$$

$$+\left\{\frac{a_m}{3}(a_m-b_0)^3\right\}$$

$$-\frac{1}{3}\left\{\frac{(a_m-b_0)^4}{4}-\frac{b_0^4}{4}\right\}$$