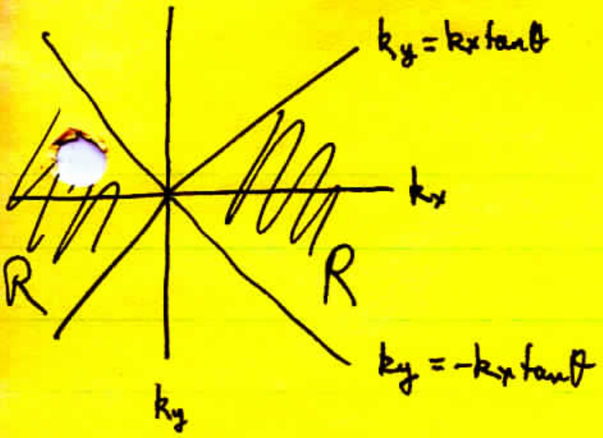
Space domain response of fan filter

MRN 058



$$f(x,y) = \iint_R dk_x\, dk_y \, \exp(\, i k_x x + i k_y y)$$

$$= \int_{-\infty}^{0} dk_x \int_{k_x \tan\theta}^{-k_x \tan\theta} dk_y \, \exp(i k_x x + i k_y y)$$

$$+ \int_{0}^{\infty} dk_x \int_{-k_x \tan\theta}^{k_x \tan\theta} \exp(i k_x x + i k_y y)$$

$$= \int_{-\infty}^{0} dk_x \, e^{i k_x x} \, \frac{1}{iy} \, e^{i k_y y} \Big|_{k_x \tan\theta}^{-k_x \tan\theta}$$

$$+ \int_{0}^{\infty} dk_x \, e^{i k_x x} \, \frac{1}{iy} \, e^{i k_y y} \Big|_{-k_x \tan\theta}^{k_x \tan\theta}$$

$$\int_{-\infty}^{0} dk_x\, e^{ik_x x} \frac{1}{iy} e^{ik_y y}\Big|_{k_x \tan\theta}^{-k_x \tan\theta}$$

$$= \int_{-\infty}^{0} dk_x\, e^{ik_x x} \frac{1}{iy} \left[e^{-iyk_x \tan\theta} - e^{+iyk_x\tan\theta} \right]$$

$$= \frac{1}{iy}\int_{-\infty}^{0} dk_x \left\{ \exp\left[ik_x(x - y\tan\theta)\right] - \exp\left[ik_x(x + y\tan\theta)\right] \right\}$$

$$= \frac{1}{iy}\int_{0}^{-\infty} dk_x \left\{ \exp\left[ik_x(x + y\tan\theta)\right] - \exp\left[ik_x(x - y\tan\theta)\right] \right\}$$

$$= \frac{1}{iy}\int_{0}^{+\infty} dk_x \left\{ \exp\left[-ik_x(x - y\tan\theta)\right] - \exp\left[-ik_x(x + y\tan\theta)\right] \right\}$$

$$\int_{0}^{\infty} dk_x\, e^{ik_x x} \frac{1}{iy} e^{ik_y y}\Big|_{-k_x \tan\theta}^{k_x \tan\theta}$$

$$= \frac{1}{iy}\int_{0}^{\infty} dk_x \left\{ \exp\left[ik_x(x + y\tan\theta)\right] - \exp\left[ik_x(x - y\tan\theta)\right] \right\}$$

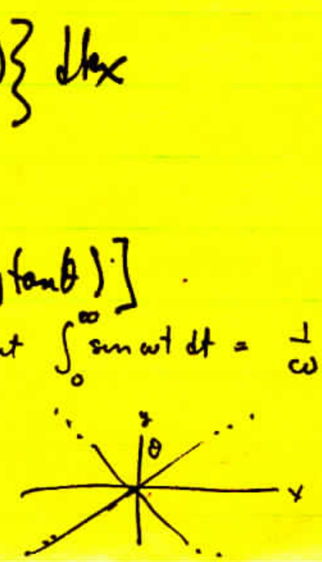
$$f(x,y) = \frac{1}{iy}\int_{0}^{\infty} \left\{ \exp\left[ik_x(x + y\tan\theta)\right] - \exp\left(-ik_x(x + y\tan\theta)\right) \right\} dk_x$$

$$- \frac{1}{iy}\int_{0}^{\infty} \left\{ \exp\left[ik_x(x - y\tan\theta)\right] - \exp\left(-ik_x(x + y\tan\theta)\right) \right\} dk_x$$

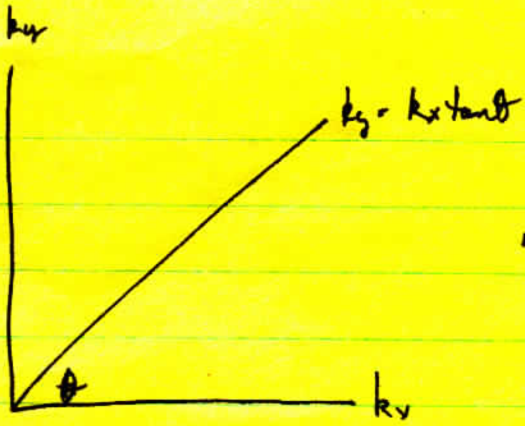
$$= \frac{2}{y}\int_{0}^{\infty} dk_x \sin\left[k_x(x + y\tan\theta)\right] - \frac{2}{y}\int_{0}^{\infty} dk_x \sin\left[k_x(x - y\tan\theta)\right]$$

but $\int_0^\infty \sin\omega t \, dt = \frac{1}{\omega}$

$$= \frac{2}{y}(x + y\tan\theta)^{-1} - \frac{2}{y}(x - y\tan\theta)^{-1}$$



Alternate derivation



$$4\int_0^\infty dk_x \int_0^{k_x\tan\theta} dk_y \quad \cos(k_x x)\cos(k_y y)$$

$$= 4\int_0^\infty dk_x \cos(k_x x) \int_0^{k_x\tan\theta} \cos(k_y y)$$

$$= 4\int_0^\infty dk_x \cos(k_x x) \quad \frac{1}{y}\sin(k_y y)\Big|_{y=0}^{y=k_x\tan\theta}$$

$$= \frac{4}{y}\int_0^\infty \sin(y\tan\theta\, k_x)\cos(x k_x)\quad dk_x$$

$$= \frac{4}{y}\left[-\frac{\cos[(y\tan\theta - x)k_x]}{2(y\tan\theta - x)} - \frac{\cos[(y\tan\theta + x)k_x]}{2(y\tan\theta + x)}\right]_0^\infty$$

$$= \frac{4}{2y}\left[\frac{1}{y\tan\theta - x} + \frac{1}{y\tan\theta + x}\right]$$

$$= \frac{2}{y}\left[\frac{1}{x+y\tan\theta} - \frac{1}{x-y\tan\theta}\right]$$

$$= \frac{2}{y} \cdot \frac{-2\, y \tan\theta}{(x^2 - y^2 \tan^2\theta)}$$

$$= \frac{-4 \tan\theta}{x^2 - y^2 \tan^2\theta}$$