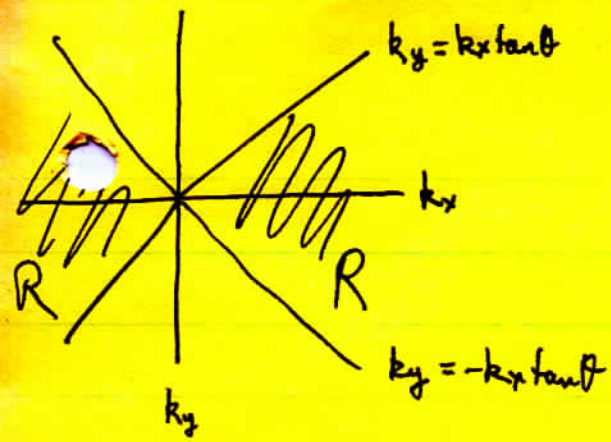


Space domain response of low filter

M R N 058



$$f(x, y) = \iint_R dk_x dk_y \exp(i k_x x + i k_y y)$$

$$= \int_{-\infty}^0 dk_x \int_{k_x \tan \theta}^{-k_x \tan \theta} dk_y \exp(i k_x x + i k_y y)$$

$$+ \int_0^{\infty} dk_x \int_{-k_x \tan \theta}^{k_x \tan \theta} dk_y \exp(i k_x x + i k_y y)$$

$$= \int_{-\infty}^0 dk_x e^{i k_x x} \frac{1}{i y} \left[\frac{e^{i k_y y}}{k_y} \right]_{k_x \tan \theta}^{-k_x \tan \theta}$$

$$+ \int_0^{\infty} dk_x e^{i k_x x} \frac{1}{i y} \left[\frac{e^{i k_y y}}{k_y} \right]_{-k_x \tan \theta}^{k_x \tan \theta}$$

$$\int_{-\infty}^{\infty} dk_x e^{ik_x x} \frac{1}{iy} e^{iky y} \left| \begin{matrix} -k_x \tan \theta \\ k_x \tan \theta \end{matrix} \right.$$

only $\omega = \dots$
 $(\theta^2 \text{ not } \omega^2 = \dots)$

$$= \int_{-\infty}^{\infty} dk_x e^{ik_x x} \frac{1}{iy} [e^{-iy k_x \tan \theta} - e^{+iy k_x \tan \theta}]$$

only $\omega = \dots$

$$= \frac{1}{iy} \int_{-\infty}^{\infty} dk_x \left\{ \exp[ik_x (x - y \tan \theta)] - \exp[ik_x (x + y \tan \theta)] \right\}$$

$$= \frac{1}{iy} \int_0^{\infty} dk_x \left\{ \exp[ik_x (x + y \tan \theta)] - \exp[ik_x (x - y \tan \theta)] \right\}$$

$$= \frac{1}{iy} \int_0^{+\infty} dk_x \left\{ \exp[-ik_x (x - y \tan \theta)] - \exp[-ik_x (x + y \tan \theta)] \right\}$$

$$\int_0^{\infty} dk_x e^{ik_x x} \frac{1}{iy} e^{iky y} \left| \begin{matrix} k_x \tan \theta \\ k_x \tan \theta \end{matrix} \right.$$

$$= \frac{1}{iy} \int_0^{\infty} dk_x \left\{ \exp[ik_x (x + y \tan \theta)] - \exp[ik_x (x - y \tan \theta)] \right\}$$

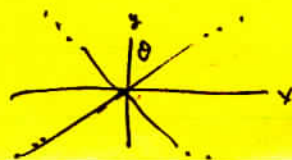
$$f(x, y) = \frac{1}{iy} \int_0^{\infty} \left\{ \exp[ik_x (x + y \tan \theta)] - \exp(-ik_x (x + y \tan \theta)) \right\} dk_x$$

$$- \frac{1}{iy} \int_0^{\infty} \left\{ \exp[ik_x (x - y \tan \theta)] - \exp(-ik_x (x - y \tan \theta)) \right\} dk_x$$

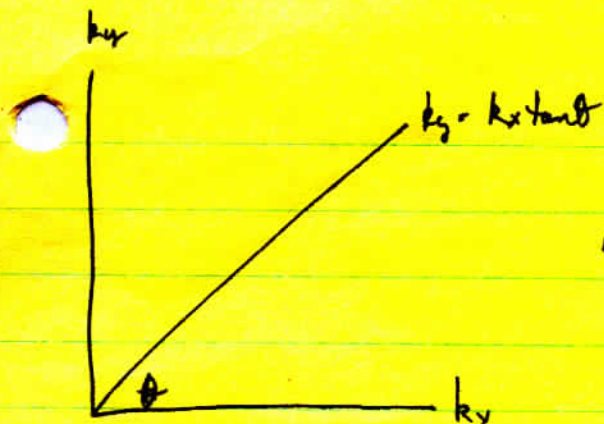
$$= \frac{2}{y} \int_0^{\infty} dk_x \sin[k_x (x + y \tan \theta)] - \frac{2}{y} \int_0^{\infty} dk_x \sin[k_x (x - y \tan \theta)]$$

but $\int_0^{\infty} \sin \omega t dt = \frac{1}{\omega}$

$$= \frac{2}{y} (x + y \tan \theta)^{-1} - \frac{2}{y} (x - y \tan \theta)^{-1}$$



Alternate derivation



$$4 \int_0^{\infty} dk_x \int_0^{k_x \tan \theta} dk_y \cos(k_x x) \cos(k_y y)$$

$$= 4 \int_0^{\infty} dk_x \cos(k_x x) \int_0^{k_x \tan \theta} \cos(k_y y) dk_y$$

$$= 4 \int_0^{\infty} dk_x \cos(k_x x) \left. \frac{1}{y} \sin(k_y y) \right|_{y=0}^{y=k_x \tan \theta}$$

$$= \frac{4}{y} \int_0^{\infty} \sin(y \tan \theta k_x) \cos(x k_x) dk_x$$

$$= \frac{4}{2} \left[-\frac{\cos[(y \tan \theta - x) k_x]}{2(y \tan \theta - x)} - \frac{\cos[(y \tan \theta + x) k_x]}{2(y \tan \theta + x)} \right]_0^{\infty}$$

$$= \frac{4}{2y} \left[\frac{1}{y \tan \theta - x} + \frac{1}{y \tan \theta + x} \right]$$

$$= \frac{2}{y} \left[\frac{1}{x + y \tan \theta} - \frac{1}{x - y \tan \theta} \right]$$

$$= \frac{z}{y} \cdot \frac{-2y \tan \theta}{(x^2 - y^2 \tan^2 \theta)}$$

$$= \frac{-4 \tan \theta}{x^2 - y^2 \tan^2 \theta}$$

$$x^2 - y^2 \tan^2 \theta$$

$$\left\{ \left[(\tan \theta y - x) x \right] y \cos \theta - \left[(\tan \theta y + x) x \right] y \sin \theta \right\} x \sin \theta \cos \theta =$$

$$\left\{ \left[(\tan \theta y - x) x \right] y \cos \theta - \left[(\tan \theta y + x) x \right] y \sin \theta \right\} x \sin \theta \cos \theta =$$

$$\left\{ \left[(\tan \theta y - x) x \right] y \cos \theta - \left[(\tan \theta y + x) x \right] y \sin \theta \right\} x \sin \theta \cos \theta =$$

$$x \sin \theta \left\{ \left[(\tan \theta y - x) x \right] y \cos \theta - \left[(\tan \theta y + x) x \right] y \sin \theta \right\} = (y \cos \theta)$$

$$x \sin \theta \left\{ \left[(\tan \theta y - x) x \right] y \cos \theta - \left[(\tan \theta y + x) x \right] y \sin \theta \right\} =$$

$$\left[(\tan \theta y - x) x \right] y \cos \theta \frac{5}{6} - \left[(\tan \theta y + x) x \right] y \sin \theta \frac{5}{6} =$$

$$\left[(\tan \theta y - x) \right] \frac{5}{6} - \left[(\tan \theta y + x) \right] \frac{5}{6} =$$

L = 10 hours

