MRN 069

Covariance of slowness + travel times 12-1-00 Menke

let $C_{\delta t}(\Delta x) = \langle \delta t(x) \delta t(x+\Delta x) \rangle$

$C_{\delta s}(\Delta x, \Delta y) = \langle \delta s(x,y)\, \delta s(x+\Delta x, y+\Delta y) \rangle$

$\delta t(x, H) = \int_0^H \delta s(x,y)\, dy$

(x,y) = position

t = time

s = slowness

$\langle\,\rangle$ = expected value

Then

$$C_{\delta t}(\Delta x) = \left\langle \int_0^H \delta s(x,y)\, dy \int_0^H \delta s(x+\Delta x, y')\, dy' \right\rangle$$

$$= \int_0^H dy \int_0^H dy' \langle \delta s(x,y)\, \delta s(x+\Delta x, y') \rangle$$

$$= \int_0^H dy \int_{-y}^{H-y} d\Delta y \langle \delta s(x,y)\, \delta s(x+\Delta x, y+\Delta y) \rangle$$

$$= \int_0^H dy \int_{-y}^{H-y} d\Delta y \; C_{\delta s}(\Delta x, \Delta y)$$

where we have employed transformation of variables

$y' = y + \Delta y$ so $dy' = d\Delta y$

$y' = 0 \quad \Delta y = -y$

$y' = H \quad \Delta y = H - y$

example

$$C_{\delta s}(\Delta x, \Delta y) = s_0^2\, e^{-a|\Delta x|} e^{-a|\Delta y|}$$

$$C_{\delta t}(\Delta x, H) = 2 S_0^2\, e^{-a|\Delta x|} \left[Ha - (1 - e^{-aH}) \right]$$

$$C_{\delta s}(\Delta x, \Delta y) = e^{-a|\Delta x|} e^{-a|\Delta y|}$$

$$\int_0^H dy \int_{-y}^{H-y} d\Delta y \; C_{\delta s}(\Delta x, \Delta y) =$$

$$\int_0^H dy \int_{-y}^{H-y} d\Delta x \; e^{-a|\Delta x|} e^{-a|\Delta y|} =$$

$$e^{-a|\Delta x|} \int_0^H dy \left[\int_{-y}^{0} e^{+a\Delta y} d\Delta y + \int_0^{H-y} e^{-a\Delta y} d\Delta y \right] =$$

$$e^{-a|\Delta x|} \int_0^H dy \left[\frac{1}{a} e^{a\Delta y} \Big|_{-y}^{0} - \frac{1}{a} e^{-a\Delta y} \Big|_0^{H-y} \right]$$

$$\frac{1}{a} e^{-a|\Delta x|} \int_0^H dy \left[1 - e^{-ay} - e^{-aH} e^{+ay} + 1 \right]$$

$$\frac{1}{a} e^{-a|\Delta x|} \int_0^H \left[2 - e^{-ay} - e^{-aH} e^{ay} \right] dy =$$

$$\frac{1}{a} e^{-a|\Delta x|} \left[2y\Big|_0^H + \frac{1}{a} e^{-ay}\Big|_0^H - \frac{1}{a} e^{-aH} e^{+ay}\Big|_0^H \right]$$

$$\frac{1}{a} e^{-a|\Delta x|} \left[2H - \frac{1}{a}(1 - e^{-aH}) - \frac{1}{a}(e^{-aH} e^{+aH} - e^{-aH}) \right] =$$

$$\frac{1}{a} e^{-a|\Delta x|} \left[2H - \frac{1}{a}(1 - e^{-aH}) - \frac{1}{a}\left[1 - e^{-aH}\right] \right] =$$

$$\frac{1}{a} e^{-a|\Delta x|} \left[2H - \frac{2}{a}(1 - e^{-aH}) \right] =$$

$$\frac{2}{a} e^{-a|\Delta x|} \left[H - \frac{1}{a}(1 - e^{-aH}) \right] =$$

$$2 e^{-a|\Delta x|} \left[Ha - (1 - e^{-aH}) \right]$$