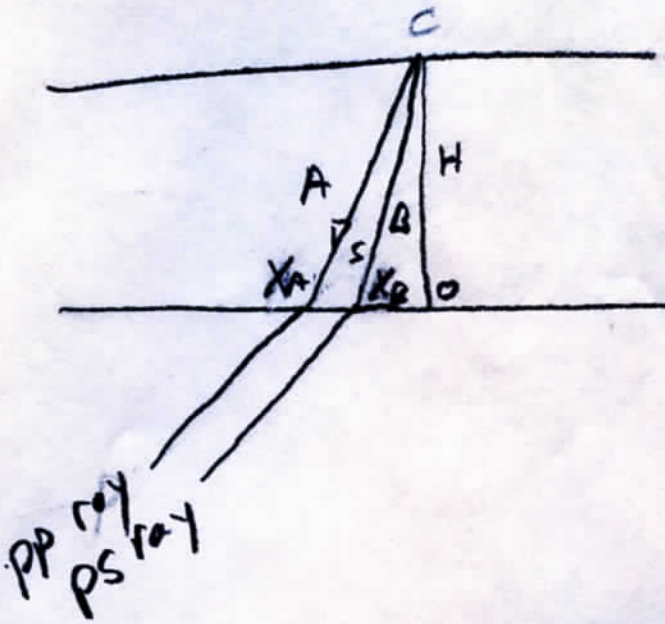
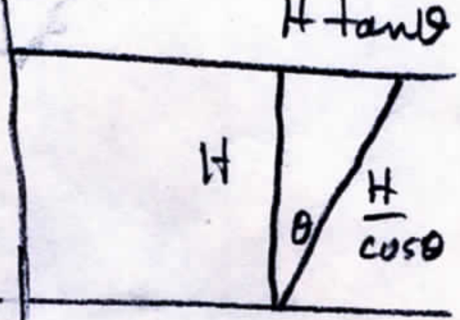
Receiver Function times.



Time along ray is just $\underline{x}\cdot\underline{p}$



$$p_x = \frac{\sin\theta}{V_p}$$

$$p_z = \frac{\cos\theta}{V_p}$$

$$\frac{H}{V_p\cos\theta} = H\tan\theta\, p_x + H p_z$$

$$= \frac{H\sin^2\theta}{V_p\cos\theta} + \frac{H\cos\theta}{V_p}$$

$$= \frac{H}{V_p\cos\theta}\left(\sin^2\theta + \cos^2\theta\right)$$

$$\stackrel{\checkmark}{=} \frac{H}{V_p\cos\theta}$$

$$T_p = \frac{A}{V_p} = X_A P_x + H P_z^p$$

$$T_s = \frac{B}{V_s} + (X_A - X_B)\cdot P_x$$

← correction for different rays

$$= X_B P_x + H P_z^s + X_A P_x - X_B P_x$$

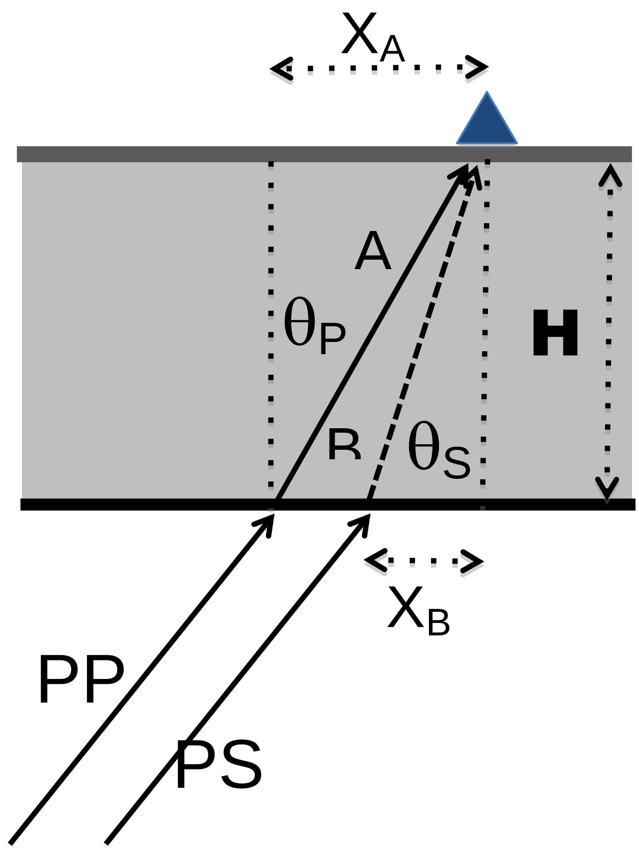
$$= H P_z^s + X_A P_x$$

$$T_s - T_p = H p_z^s - H p_z^p$$

⟸ just difference of vertical slownesses.

proof that "vertical slowness method" is equivalent to a ray calculation that <u>includes</u> the effect of the PP and PS rays being different.

Derivation of formulae for predicting times of P-to-S converted phases from a single interface. Author – Bill Menke. Typed up by Vadim Levin



Travel time along the ray is a vector product of the slowness vector **p** and distance vector **x**.

Distance vector for the PP ray is **x**=(XA,H), slowness vector for the PP wave is **p**=(px, pz).

Horizontal slowness of a PP wave in the upper layer:

$$p_x = \frac{sin\theta_P}{V_P}$$

Vertical slowness of a PP wave in the upper layer:

$$p^p{}_z = \frac{cos\theta_P}{V_P}$$

Length of the PP ray in the upper layer is

$$A = H/cos\theta_P$$

Travel time T_P along ray PP that is a vector product **p·x** evaluates to

$$T_P = X_A p_x + Hp^p{}_z = Htan\theta_P p_x + Hp^p{}_z$$

For PS ray the horizontal slowness $p_x = \frac{sin\theta_S}{V_S} = \frac{sin\theta_P}{V_P}$ (Snell's Law), while the vertical slowness is $p^S{}_z = \frac{cos\theta_S}{V_S}$.

Travel time for the PS phase will include the travel along ray B in the crust, and a fraction of travel along the ray below the boundary that propagates there while PP wave has already crossed into the crust:

$$T_S = \frac{B}{V_S} + (X_A - X_B)p_x$$

$$= X_B p_x + H p^S{}_z + X_A p_x - X_B p_x$$
$$= H p^S{}_z + X_A p_x$$

Consequently, the difference in travel time (the *delay* of the phase) in a receiver function will be

$$T_P - T_S = H p^S{}_z - H p^P{}_z$$