MRN 080

Inference using Bayes Theorem

Bayes Thm $P(B|A)\,P(A) = P(A|B)\,P(B)$

or $$P(B|A) = \frac{P(A|B)\,P(B)}{P(A)}$$

~~now let~~ also note $P(A) = \sum_j P(A|B_j)\,P(B_j)$

now let B stand for a particular hypothesis H_i
and A stand for ~~data~~ D (or data)

$$P(H_i|D) = \frac{P(D|H_i)\,P(H_i)}{\sum_j P(D|H_j)\,P(H_j)}$$

where we understand

$P(H_i)$ is the prior probability of H_i (i.e. before measuring D.

$P(D|H_i)$ is the conditional probability for the data, presuming hypothesis H_i is true

$P(D) = \sum_j P(D, H_j)\,P(H_j)$ prior probability of data, given all possible hypotheses.

Example. fingerprint evidence in courtroom

H_1 = guilty = G H_2 = not guilty = NG

prior $P(H)$ $P(G) = 10^{-3}$ = 1/population of city in which crime was committed $P(NG) = 1 - P(G)$

D = fingerprint match, wrong once in a million

$P(\text{Match}\,|\,\text{Guilty}) = 1 - 10^{-6}$ $P(\text{Match}\,|\,NG) = 10^{-6}$

$$P(G|\text{Match}) = \frac{P(\text{Match}|\text{Guilty})\,P(\text{Guilty})}{P(\text{Match}|\text{Guilty})\,P(\text{Guilty}) + P(\text{Match}|\text{NotGuilty})\,P(\text{notguilty})}$$

$$= \frac{(0.999999)(0.001)}{(0.999999)(0.001) + (0.000001)(0.999)}$$

$$\approx 0.99900$$