$G_0 m_0 = d_0$ (p=0 case)

inverse problem with G, m fcns of p.

MRN040

12/01

$G_p m_p = d_0$ ($p \neq 0$)

$\left(G_0 + p\frac{\partial G}{\partial p}\right)\left(m_0 + p\frac{\partial m}{\partial p}\right) = d_0$ (first order approx)

$G_0 m_0 + G_0 p \frac{\partial m}{\partial p} + p\frac{\partial G}{\partial p} m_0 = d_0$ (apply zero-order eqn)

$$G_0 \frac{\partial m}{\partial p} = -\frac{\partial G}{\partial p} m_0$$

$$\frac{\partial m}{\partial p} = -G_0^{-1}\frac{\partial G}{\partial p} m_0$$

$\min \|m_p - \bar{m}\|^2$ w.r.t. p.

$$\|m_0 + p\frac{\partial m}{\partial p} - \bar{m}\|^2 = 0 = \|\underbrace{(m_0 - \bar{m})}_{=\Delta m} + p\frac{\partial m}{\partial p}\|^2$$

$$= \|\Delta m + p\frac{\partial m}{\partial p}\|^2$$

$$= \Delta m \Delta m + p^2 \frac{\partial m}{\partial p}\frac{\partial m}{\partial p} + 2\Delta m\, p\frac{\partial m}{\partial p}$$

$\frac{\partial}{\partial p}$: $0 = 2p\frac{\partial m}{\partial p}\cdot\frac{\partial m}{\partial p} + 2\Delta m \cdot \frac{\partial m}{\partial p}$

$$p = \frac{\Delta m \cdot \frac{\partial m}{\partial p}}{\frac{\partial m}{\partial p}\cdot\frac{\partial m}{\partial p}} = \frac{-\Delta m\, G_0^{-1}\frac{\partial G}{\partial p} m_0}{m_0 \frac{\partial \tilde{G}}{\partial p}\tilde{G}_0^{-1} G_0^{-1}\frac{\partial G}{\partial p} m_0}$$

$$= \frac{-\Delta m\, G_0^{-1}\frac{\partial G}{\partial p} m_0}{\tilde{m}_0 \frac{\partial \tilde{G}}{\partial p}\left[G_0 \tilde{G}_0\right]^{-1}\frac{\partial G}{\partial p} m_0}$$