product of gaussion is a gaussion 06/28/10

$$e^{-A(m+a)^2} e^{-B(m+b)^2}$$

$$= e^{-Am^2 - 2Aam - Aa^2 - Bm^2 - 2Bbm - Bb^2}$$

$$= e^{-(A+B)m^2 - 2(Aa+Bb)m - (Aa^2+Bb^2)}$$

$$= e^{-(A+B)\left(m^2 + 2\frac{Aa+Bb}{A+B}m + \frac{Aa^2+Bb^2}{A+B}\right)}$$

$$= e^{-d(m-c)^2 + D}$$

Choose D by complete square

$$d = (A+B) \qquad c = \frac{Aa+Bb}{A+B}$$

$$c^2 + D = \frac{A^2a^2 + B^2b^2 + ABab}{(A+B)^2} \qquad \frac{-ABab + AB(a^2+b^2)}{(A+B)(A+B)}$$

$$= \frac{(A+B)(Aa^2+Bb^2)}{(A+B)(A+B)} = \frac{Aa^2+Bb^2}{A+B}$$

matches

and $D = \frac{AB(a^2+b^2-ab)}{(A+B)^2} = \frac{AB}{(A+B)^2}(a-b)^2$

application to normal distribution $\frac{1}{\sqrt{2\pi}\sigma} \exp\left\{\frac{(m-\mu)^2}{2\sigma^2}\right\}$

$$\mu \rightarrow \frac{\mu_1\sigma_2^2 + \mu_2\sigma_1^2}{\sigma_1^2+\sigma_2^2}$$

$$\sigma^2 \rightarrow \frac{\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}$$

multi-dim case

$$(x+a)^T A (x+a) + (x+b)^T B (x+b)$$

$$x^T (A+B) x + 2 (a^T A + b^T B) x + a^T A a + b^T B b$$

$$(x+c)^T C (x+c) + D$$

$$x^T C x + 2 c^T C x + c^T C c + D$$

absorb non-perfect square terms into here, an overall amplitude

$$C = A+B \qquad c^T C = a^T A + b^T B$$

$$C c = A a + B b$$

$$c = (A+B)^{-1} (A a + B b)$$

$$c^T C c = (a^T A + b^T B)(A+B)^{-1}(A a + B b)$$

so $D = (a^T A + b^T A)(A+B)^{-1}(A a + B b) - a^T A a - b^T B b$

$$(m-s)^T W_s (m-s) + (Gm-d)^T W_d (Gm-d)$$

$$(x+a)^T A (x+a) + (Fx+b)^T B (Fx+b)$$

$$x^T (A + F^T B F) x + 2 (a^T A + b^T B F) x + a^T A a + b^T F^T B F b^T$$

$$C = (A + F^T B F) \qquad c = (A + F^T B F)^{-1} (A a + F^T B b)$$

$\Rightarrow$

$$c = (W_s + G^T W_d G)^{-1} (W_s s + G^T W_d d)$$

One D-Case
Bayes inference

$$P(M_i|D) = \frac{P(D|M_i)\,P(M_i)}{\sum_j P(D|M_i)M_i}$$

one-d case

d = fixed by obs.

$$P_A(m) = \frac{1}{\sqrt{2\pi}\,\sigma_A} \exp\left\{-\frac{1}{2}\sigma_A^{-2}(m-s)^2\right\}$$

$$P(d|m) = \frac{1}{\sqrt{2\pi}\,\sigma_d} \exp\left\{-\frac{1}{2}\sigma_d^{-2}(\cancel{\;}\,Gm-d)^2\right\}$$

G just a scalar

result is a gaussion ~~with~~.

$$P(d|m).p(m) \propto \exp\left\{-\frac{1}{2}\sigma_A^{-2}(m-s)^2 - \frac{1}{2}G^2\sigma_d^{-2}\left(m-\frac{d}{G}\right)^2\right\}$$

case1: no prior $\frac{\sigma_d}{\sigma_A} \to 0$ $\quad \hat{m} \to \frac{d}{G}$

new mean:

$$\hat{m} \to \frac{s\frac{\sigma_d^2}{G^2} + \frac{d}{G}\sigma_A^2}{\sigma_A^2 + \frac{\sigma_d^2}{G^2}}$$

case 2: no data $\frac{\sigma_A}{\sigma_d} \to 0$ $\quad \hat{m} \to s$

new variance:

$$\sigma \to \frac{\sigma_A^2\sigma_d^2}{G^2\left(\sigma_A^2 + \frac{\sigma_d^2}{G^2}\right)} = \frac{\sigma_A^2\sigma_d^2}{G^2\sigma_A^2 + \sigma_d^2}$$