Menke 09-12-12 Schuler Thesis Corrections - musings on Variance & Resolution

[page 125] ... and **G** is a matrix containing the Green Function of for each moment tensor element. [Insert] The elements of **m** can be independently resolved as long as **G** has no near-zero singular values. We first write **G** in terms of its singular value decomposition, $\mathbf{G} = \mathbf{U\Sigma V}^T$, where $\boldsymbol{\Sigma}$ is a diagonal matrix of singular values and **U** and **V** are matrices of eigenvectors. When some eigenvectors are near-zero, it is convenient to write $\boldsymbol{\Sigma} = \mathrm{diag}(\boldsymbol{\Sigma}_p, \boldsymbol{\Sigma}_0)$, where the subscript zero refers to near-zero eigenvalues and the subscript p refers to the rest and to partition the corresponding eigenvector matrices as $\mathbf{U}=[\mathbf{U}_p, \mathbf{U}_0]$ and $\mathbf{V}=[\mathbf{V}_p, \mathbf{V}_0]$. The model resolution matrix is then $\mathbf{R} = \mathbf{V}_p\mathbf{V}_p^T$ and the model covariance matrix is $\mathbf{C}_m = \sigma^2_d\,[\mathbf{G}^T\mathbf{G}]^{-1} = \sigma^2_d\,\mathbf{V\Sigma}^{-2}\mathbf{V}^T$, where σ^2_d is the variance of the data. Note that $\mathbf{C}_m \approx \sigma^2_d\,\mathbf{V}_0\boldsymbol{\Sigma}_0^{-2}\mathbf{V}_0^T$, since by supposition the elements of $\boldsymbol{\Sigma}_p$ are much larger than the elements of $\boldsymbol{\Sigma}_0$. With the identification $\boldsymbol{\Lambda} = \sigma^2_d\boldsymbol{\Sigma}^{-2}$, the covariance matrix has eigenvalue - eigenvector decomposition $\mathbf{C}_m = \mathbf{V\Lambda V}^T$. We can study the issue of resolution through examining the properties of either **R** or $\mathbf{C}_m$; we choose the latter. A near-zero singular value of **G** implies a large corresponding eigenvalue of $\mathbf{C}_m$, since the two are related by a reciprocal. In our case, a plot of eigenvalues of $\mathbf{C}_m$ indicates that exactly one is unusually large, implying that one singular value of **G** is near-zero and that $\mathbf{V}_0$ contains only one column, say $\mathbf{v}_0$. Calling that eigenvalue λ_0, the covariance matrix is then $\lambda_0\,\mathbf{v}_0\mathbf{v}_0^T$. As we will show below, when **m** is parameterized so that the isotropic and pure vertical-CLVD components of the moment tensor are its first two elements, then $\mathbf{v}_0 \approx [-0.87, +0.49, 0, 0, 0, 0]^T$, which implies that these two components cannot be independently resolved.

[Delete rest of first paragraph]

[Retain second paragraph, but replace $\mathbf{A}^{-1}$ with $\mathbf{C}_m$]

[Delete paragraph on page 126 that starts "We calculate the ..."]

[Delete first sentence of last paragraph on page 126, and modify second sentence to read:]

By examining the covariance matrix $\mathbf{C}_m$, we find (as expected) that the isotropic and pure vertical-CLVD components of the moment tensor have the largest relative standard deviation.