

$$\delta_i = a_i - b_i T \quad \text{for species } i$$

$$\begin{aligned} \bar{\delta}_i &= \int (a_i - b_i T(z)) P_i(z) dz = \\ &= a_i - b_i \int T(z) P_i(z) dz \end{aligned}$$

let $T(z), P_i(z)$ be piecewise continuous with interval Δz

$$\bar{\delta}_i = a_i - b_i \sum_j T(z_j) P_i(z_j) \Delta z \quad \text{if } T_j = T(z_j) \text{ and } P_{ij} = P_i(z_j) \Delta z$$

$$\bar{\delta}_i = a_i - b_i \sum_j T_j P_{ij}$$

let $T(z)$ have an orthonormal expansion $T(z) = \sum_j T_j f_j(z)$

$\bar{\delta}_i =$ and similar expansion for $P_i(z) = \sum_j P_{ij} f_j(z)$

$$\bar{\delta}_i = a_i - b_i \sum_j \sum_k T_j P_{ik} \int f_j(z) f_k(z) dz$$

$$= a_i - b_i \sum_j \sum_k T_j P_{ik} \delta_{jk}$$

$$\bar{\delta}_i = a_i - b_i \sum_j T_j P_{ij}$$

$$\bar{q}_i \equiv (a_i - \bar{\delta}_i) / b_i = \sum_j P_{ij} T_j \quad \underline{q} = \underline{P} \underline{T}$$

now consider that one has \bar{q}_i at many x 's \bar{Q}_{ik}

$$\bar{Q}_{ik} = \sum_j P_{ij} T_{jk} \quad \text{or} \quad \underline{\bar{Q}} = \underline{P} \underline{T} \quad \underline{P} = \underline{\bar{Q}} \underline{T}^{-1}$$

$$\underline{T}^{\text{test}} = \underline{P}^{-1} \underline{q}^{\text{obs}}$$

"Training"
locations

target location