\[ s_i = a_i - b_i T \] for species \( i \)

\[
\bar{s}_i = \int (a_i - b_i T(z)) p_i(z) \, dz = \]

\[ = q_i - b_i \int T(z) p_i(z) \, dz \]

Let \( T(z), p(z) \) be piecewise continuous with interval \( dz \)

\[
\bar{s}_i = q_i - b_i \sum T(z_j) p_i(z_j) \, dz_i \text{ if } T_j = T(z_j) \text{ and } p_{ij} = p_i(z_j) \, dz_j
\]

Let \( T(z) \) have an orthonormal expansion \( T(z) = \sum T_j j f_j(z) \)

\[ s_j = \text{and similar expansion for } p_j(z) = \sum p_{ij} j f_j(z) \]

\[
\bar{s}_i = q_i - b_i \sum \sum T_j p_{ik} \int f_j(z) f_k(z) \, dz
\]

\[ = q_i - b_i \sum \sum T_j p_{ik} s_{jk} \]

\[ \bar{s}_i = q_i - b_i \sum T_j p_{ij} \]

\[ \bar{q}_i = (a_i - \bar{s}_i) / b_i = \sum p_{ij} T_j \quad \bar{q} = \bar{p} T \]

Now consider that one has \( \bar{q}_i \) at many \( x_k \), \( \bar{Q}_{ik} \)

\[
\bar{Q}_{ik} = \sum p_{ij} T_{ik} \quad \text{or } \bar{Q} = \bar{p} \bar{T} \quad \bar{P} = \bar{Q} \bar{T}^{-1}
\]

Test \( \bar{p} \bar{q} \), \( q_{\text{obs}} \)

\[ \bar{P} = \bar{Q} \bar{T}^{-1} \quad \text{"Training" locations} \]

\[ \bar{P} = \bar{q} \quad \text{target location} \]