L_n norm straight line problem

05-01

MRN113

$\bar{X}_i, \bar{Y}_i, \sigma_{X_i}, \sigma_{Y_i}$

let $\tilde{X} = \frac{1}{N}\sum_i \bar{X}_i$ $\quad a = \max_i |X_i - \tilde{X}_i|$

let $\tilde{Y} = \frac{1}{N}\sum_i Y_i$ $\quad b = \max_i |\bar{Y}_i - \tilde{Y}_i|$

let $x_i = (X_i - \tilde{X}_i)/a$ $\quad X_i = a x_i + \tilde{X}$

let $y_i = (Y_i - \tilde{Y})/b$ $\quad Y_i = b y_i + \tilde{Y}$

$\sigma_{x_i} = \sigma_{X_i}/a$

$\sigma_{y_i} = \sigma_{Y_i}/b$

let $Y_i = M_1 + M_2 X_i$

$b y_i + \tilde{Y} = M_1 + M_2(a x_i + \tilde{X})$

$b y_i + \tilde{Y} = M_1 + a M_2 x_i + M_2 \tilde{X}$

$y_i = \frac{1}{b}\left(a M_2 x_i + (M_2 \tilde{X} - \tilde{Y} + M_1)\right)$

$y_i = \frac{1}{b}(M_2 \tilde{X} + M_1 - \tilde{Y}) + \left(\frac{a}{b} M_2\right) x_i$

$y_i = m_1 + m_2 x_i$

$m_1 = [M_1 + M_2 \tilde{X} - \tilde{Y}]/b$ $\quad M_1 = b m_1 - M_2 \tilde{X} + \tilde{Y}$

$= b m_1 - \frac{b}{a} m_2 \tilde{X} + \tilde{Y}$

$m_2 = \frac{a}{b} M_2$ $\quad M_2 = \frac{b}{a} m_2$

$\sigma_{m_2} = \frac{a}{b}\sigma_{M_2}$ $\quad \sigma_{m_1} = \left(\frac{\sigma_{M_1}^2}{b^2} + \frac{X^2}{b^2}\sigma_{M_2}^2\right)^{1/2}$

$$\phi = \left\{ + \frac{(m_1-\bar{m}_1)^2}{\sigma_{m_1}^2} + \frac{(m_2-\bar{m}_2)^2}{\sigma_{m_2}^2} + \sum_i \frac{(x_i-\bar{x}_i)^2}{\sigma_{x_i}^2} + \sum_i \frac{(m_1+m_2x_i-\bar{y}_i)^2}{\sigma_{y_i}^2} \right.$$

$$\frac{\partial\phi}{\partial m_1} = \frac{2(m_1-\bar{m}_1)}{\sigma_{m_1}^2} + \sum_i \frac{2(m_1+m_2x_i-\bar{y}_i)}{\sigma_{x_i}^2}$$

$$\frac{\partial\phi}{\partial m_2} = \frac{2(m_2-\bar{m}_2)}{\sigma_{m_2}^2} + \sum_i \frac{2x_i(m_1+m_2x_i-\bar{y}_i)^2}{\sigma_{x_i}^2}$$

$$\frac{\partial\phi}{\partial x_j} = \sum_i 2\delta_{ij}\frac{(x_i-\bar{x}_i)}{\sigma_{x_i}^2} + \sum_i 2m_2\delta_{ij}\frac{(m_1+m_2x_i-\bar{y}_i)}{\sigma_{x_i}^2}$$

$$= \frac{2(x_j-\bar{x}_j)}{\sigma_{x_j}^2} + 2\frac{m_2(m_1+m_2x_j-\bar{y}_j)}{\sigma_{x_j}^2}$$

$$\phi = \left\{ \frac{|m_1-\bar{m}_1|}{\sigma_{m_1}} + \frac{|m_2-\bar{m}_2|}{\sigma_{m_2}} + \sum_i \frac{|x_i-\bar{x}_i|}{\sigma_{x_i}^2} + \sum_i \frac{|m_1+m_2x_i-\bar{y}_i|}{\sigma_{y_i}} \right.$$

$$\frac{\partial\phi}{\partial m_1} = \frac{\mathrm{sgn}(m_1-\bar{m}_1)}{\sigma_{m_1}} + \sum_i \cdots$$

$$\frac{\partial\phi}{\partial m_2} = \frac{\mathrm{sgn}(m_2-\bar{m}_2)}{\sigma_{m_2}} + \sum_i \cdots$$

$$\frac{\partial\phi}{\partial x_j} = \sum_i \frac{\delta_{ij}\,\mathrm{sgn}(x_i-\bar{x}_i)}{\sigma_{x_i}} + \sum_i \frac{m_2\delta_{ij}\,\mathrm{sgn}(m_1+m_2x_i-\bar{y}_i)}{\sigma_{y_i}}$$

$$= \frac{\mathrm{sgn}(x_j-\bar{x}_j)}{\sigma_{x_j}} + \frac{m_2\,\mathrm{sgn}(m_1+m_2x_j-\bar{y}_j)}{\sigma_{y_i}}$$