An attempt to reconcile Sarto's and Singh et al.'s source discontinuities.

Sarto:
\[ S_0 = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega t} dw \int_{0}^{2\pi} dk \sum_{m=0}^{\infty} \left[ f_1^L L_m + \text{etc} \right] \]

\[ S_0 = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega t} dw \int_{0}^{2\pi} dk \sum_{m=0}^{\infty} \left[ f_2^L L_m + \text{etc} \right] \]

Singh:
\[ S_0 = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega t} dw \int_{0}^{2\pi} dk \sum_{m=0}^{\infty} \left[ 2\pi L_0 C_m + \text{etc} \right] \]

\[ S_0 = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega t} dw \int_{0}^{2\pi} dk \sum_{m=0}^{\infty} \left[ 2\pi \mu L_0 k \bar{C}_m + \text{etc} \right] \]

So we have:
\[ f_1^L = \frac{2\pi L_0 C_m}{2\pi} \]

\[ f_2^L = \frac{2\pi \mu L_0 k \bar{C}_m}{2\pi} \]

Since \( L_0 = \frac{m_o}{4\pi \mu} \), we have
\[ f_1^L = \frac{m_o}{2\mu} C_m \]

\[ f_2^L = \frac{m_o}{2} k \bar{C}_m \]

Now, I will convert Singh's c's into Sarto's f's. Both authors agree that only the following Love modes are non-zero:

- \( m = 1 \), cosine
- \( m = 1 \), sine
- \( m = 2 \), cosine
- \( m = 2 \), sine
\[
\begin{align*}
\begin{pmatrix}
C_i^c \\
\bar{C}_i^c
\end{pmatrix} &= 
\begin{pmatrix}
-b_1 \\
0
\end{pmatrix} \Rightarrow 
\begin{pmatrix}
f_1^u \\
f_2^u
\end{pmatrix} &= 
\begin{pmatrix}
\frac{m_0}{2\mu} b_1 \\
0
\end{pmatrix} \\
\begin{pmatrix}
C_i^s \\
\bar{C}_i^s
\end{pmatrix} &= 
\begin{pmatrix}
a_1 \\
0
\end{pmatrix} \Rightarrow 
\begin{pmatrix}
f_1^l \\
f_2^l
\end{pmatrix} &= 
\begin{pmatrix}
\frac{m_0}{2\mu} a_1 \\
0
\end{pmatrix} \\
\begin{pmatrix}
C_i^c \\
\bar{C}_i^c
\end{pmatrix} &= 
\begin{pmatrix}
0 \\
2b_2
\end{pmatrix} \Rightarrow 
\begin{pmatrix}
f_1^l \\
f_2^l
\end{pmatrix} &= 
\begin{pmatrix}
0 \\
M_0 k b_2
\end{pmatrix} \\
\begin{pmatrix}
C_i^s \\
\bar{C}_i^s
\end{pmatrix} &= 
\begin{pmatrix}
0 \\
-2a_2
\end{pmatrix} \Rightarrow 
\begin{pmatrix}
f_1^l \\
f_2^l
\end{pmatrix} &= 
\begin{pmatrix}
0 \\
-M_0 K a_2
\end{pmatrix}
\end{align*}
\]

\[a_1 = 2 \cos \lambda \cos 8 \]
\[a_2 = \frac{1}{2} \sin \lambda \sin 28 \]
\[b_1 = 2 \sin \lambda \cos 28 \]
\[b_2 = -\cos \lambda \sin 8 \]
Now I plug in the definitions of the $a$s and $b$s:

\[
\begin{pmatrix}
(f_1^L) \\
(f_2^L)
\end{pmatrix}
= 
\frac{M_0}{\mu}
\begin{pmatrix}
\sin \lambda \cos 2\theta & 0 \\
0 & 0
\end{pmatrix}
\begin{pmatrix}
(f_1^L) \\
(f_2^L)
\end{pmatrix}
\]

\[
\begin{pmatrix}
(f_1^L) \\
(f_2^L)
\end{pmatrix}_{m=1}^s = 
\frac{M_0}{\mu}
\begin{pmatrix}
\cos \lambda \cos \delta \\
0
\end{pmatrix}
\]

\[
\begin{pmatrix}
(f_1^L) \\
(f_2^L)
\end{pmatrix}_{m=1}^s = 
\begin{pmatrix}
0 \\
- \frac{M_0 K \cos \lambda \sin \delta}{\mu}
\end{pmatrix}
\]

\[
\begin{pmatrix}
(f_1^L) \\
(f_2^L)
\end{pmatrix}_{m=2}^s = 
\begin{pmatrix}
0 \\
- \frac{M_0 K \sin 2\lambda \sin 2\delta}{\mu^2}
\end{pmatrix}
\]
new sense gives these discontinuities in terms of fault normal and fault slip parameters. These are:

\[
\begin{pmatrix}
N_x \\
N_y \\
N_z
\end{pmatrix} =
\begin{pmatrix}
-\sin \delta \sin \phi_s \\
+\sin \delta \cos \phi_s \\
-\cos \delta
\end{pmatrix}
\Rightarrow
\begin{pmatrix}
0 \\
+\sin \delta \\
-\cos \delta
\end{pmatrix}
\]

Paul's

Paul's notation is with $z^+$ downward. Singh's is $z^+$ upward, so we must change the sign of the $z$ components. But now the coord system is left handed.

-\[
\begin{pmatrix}
X_1 \\
X_2 \\
X_3
\end{pmatrix}
\]

-\[
\begin{pmatrix}
X_1 \\
X_2 \\
X_3
\end{pmatrix}
\]

So we must change the sign of the $x_3$ components and change $\phi$ to $-\phi$.
The then gives:

\[
\begin{pmatrix}
\eta_\alpha \\
\eta_\beta \\
\eta_\gamma \\
\eta_\lambda
\end{pmatrix} =
\begin{pmatrix}
\cos \phi \cos \psi + \sin \psi \\
\cos \phi \sin \psi - \sin \phi \cos \psi \\
- \sin \phi \\
\cos \phi
\end{pmatrix}
\Rightarrow
\begin{pmatrix}
0 \\
- \sin \phi \\
- \sin \phi \\
- \sin \phi
\end{pmatrix}
\phi_5 = 0
\]

\[
\begin{pmatrix}
\eta_\alpha \\
\eta_\beta \\
\eta_\gamma \\
\eta_\lambda
\end{pmatrix} =
\begin{pmatrix}
\cos \lambda \cos \phi_5 - \cos \lambda \sin \phi_5 \\
\cos \lambda \sin \phi_5 + \cos \phi_5 \sin \lambda \cos \phi_5 \\
\sin \lambda \sin \phi_5 \\
\sin \lambda \sin \phi_5
\end{pmatrix}
\phi_5 = 0
\]

This is now identical to

\[
\begin{pmatrix}
\cos \phi_5 \\
- \cos \phi_5 \\
- \cos \phi_5 \\
\sin \phi_5
\end{pmatrix}
\]

\[
\text{This is now identical to }
\]

\[
\text{This is correct, except that the sign of } T \text{ is wrong. But this just means we have the field normal, so we simply switch the sign.}
\]
\[
\begin{pmatrix}
N_x \\
N_y \\
N_z
\end{pmatrix} = 
\begin{pmatrix}
0 \\
-\sin S \\
\cos S
\end{pmatrix} 
\]

\[
\begin{pmatrix}
V_x \\
V_y \\
V_z
\end{pmatrix} = 
\begin{pmatrix}
\cos \lambda \\
\sin \lambda \cos S \\
\sin \lambda \sin S
\end{pmatrix} 
\]
so we can write Sato's $f^L$'s.

\[
\begin{pmatrix}
    f_1^L \\
    f_2^L
\end{pmatrix}
= \begin{pmatrix}
    \frac{1}{2\pi\mu} \left( -\eta_y \mathbf{v}_z - \eta_z \mathbf{v}_y \right) \\
    0
\end{pmatrix} = \begin{pmatrix}
    \frac{1}{2\pi\mu} \sin \lambda \left( \sin^2 \theta - \cos^2 \theta \right) \\
    0
\end{pmatrix}
\]

\[
\begin{pmatrix}
    f_2^L \\
    f_1^L
\end{pmatrix}
= \begin{pmatrix}
    \frac{1}{2\pi\mu} \left( \eta_z \mathbf{v}_x + \eta_x \mathbf{v}_z \right) \\
    0
\end{pmatrix} = \begin{pmatrix}
    \frac{1}{2\pi\mu} \cos \delta \cos \lambda \\
    0
\end{pmatrix}
\]

\[
\begin{pmatrix}
    f_2^L \\
    f_1^L
\end{pmatrix}
= \begin{pmatrix}
    \frac{1}{2\pi\mu} \left( \eta_x \mathbf{v}_y + \eta_y \mathbf{v}_x \right) \\
    0
\end{pmatrix} = \begin{pmatrix}
    \frac{1}{2\pi\mu} \sin \delta \cos \lambda \\
    0
\end{pmatrix}
\]

\[
\begin{pmatrix}
    f_2^L \\
    f_1^L
\end{pmatrix}
= \begin{pmatrix}
    \frac{1}{2\pi\mu} \left( -\eta_x \mathbf{v}_y + \eta_y \mathbf{v}_x \right) \\
    0
\end{pmatrix} = \begin{pmatrix}
    \frac{1}{2\pi\mu} \sin \delta \cos \delta \sin \lambda \\
    0
\end{pmatrix}
\]

\[
\begin{pmatrix}
    f_2^L \\
    f_1^L
\end{pmatrix}
= \begin{pmatrix}
    \frac{1}{2\pi\mu} \left( -\eta_y \mathbf{v}_z + \eta_z \mathbf{v}_y \right) \\
    0
\end{pmatrix} = \begin{pmatrix}
    \frac{1}{2\pi\mu} \sin 2\lambda \sin \theta \cos \theta \\
    0
\end{pmatrix}
\]

Note: There is a mistake. Sato's This is the corrected form.
Differences:

Factor of $M_0$ is assumed unit moment. Remedy is to multiply Saito by $M_0$.

Factor of $\frac{1}{2\pi}$. I was incorrect in pulling a $(2\pi)^{-1}$ out of Sngsh's integrand. The $(2\pi)^{-1}$ is implicitly inside the Fourier transform. Remedy is to multiply Sngsh by $2\pi$.

Conclusions:

\[ M_0 f_1^L = L_0 C_m^5 = \frac{M_0 C_m^5}{4\pi \mu} \]

\[ M_0 f_2^L = k \mu L_0 C_m^5 = \frac{k M_0 C_m^5}{4\pi} \]

\[ \text{Saito} = \sin \theta \cos \theta \]

So that

\[ \frac{f_{m=1}^{L}}{c} = \begin{pmatrix} \frac{-M_0 b_1}{4\pi \mu} \\ 0 \\ \frac{M_0 s_1}{4\pi \mu} \\ 0 \\ 0 \\ \frac{2k M_0 b_2}{4\pi} \end{pmatrix} \]

\[ f_{m=1}^{L} = \begin{pmatrix} 0 \\ \frac{M_0 s_1}{4\pi \mu} \\ 0 \\ \frac{2k M_0 b_2}{4\pi} \\ -2k M_0 b_2 \end{pmatrix} \]