Deviation of P-axes of earthquakes from the direction of maximum compression

Bill Menke, Feb 3, 2014

Issue at hand:

We observe that earthquake P-axes tend to have more-or-less consistent directions, and tend to be parallel what we expect the direction of maximum compression to be. Yet the P-axes are a kinematic property of the fault (that is; they are partly controlled by the orientation of the fault) and a wide range of fault orientations may slip for any given state of stress. How much deviation of P-axes from direction of maximum compression do we predict?

Assumptions:

Slip direction on fault is parallel to the shear traction on the fault plane

Fault slips when Mohr-Columb Failure Criteria met for a coefficient of friction $\sigma = 0.6$

Method

(x,y,z) = (E, N, Up) coordinate system

Fault parameterized in term of azimuth (E of N) and plunge (down from horizontal) of normal $\mathbf{n}$

Symmetric stress tensor $\mathbf{S}$ with compression stress positive

Direction of maximum compressive stress is eigenvector of $\mathbf{S}$ with largest eigenvalue

traction, $\mathbf{T} = \mathbf{S}\bullet\mathbf{n}$

normal traction, $\mathbf{T}_\mathrm{n} = (\mathbf{T}\bullet\mathbf{n})\,\mathbf{n}$ of magnitude $|\mathbf{T}_\mathrm{n}|$ and direction $\mathbf{t}_\mathbf{n} = \mathbf{T}_\mathrm{n}/|\mathbf{T}_\mathrm{n}|$

shear traction, $\mathbf{T}_\mathrm{s} = \mathbf{T} - \mathbf{T}_\mathrm{n}$ of magnitude $|\mathbf{T}_\mathrm{s}|$ and direction $\mathbf{t}_\mathbf{s} = \mathbf{T}_\mathrm{s}/|\mathbf{T}_\mathrm{s}|$

failure criterion $C = |\mathbf{T}_\mathrm{s}| - \sigma|\mathbf{T}_\mathrm{n}| > 0$

P-axis direction, in plane of fault normal and slip, halfway between: $\mathbf{t}_\mathrm{p} = (\mathbf{n} + \mathbf{t}_\mathbf{s})/|\mathbf{n} + \mathbf{t}_\mathbf{s}|$

as in the figure below:

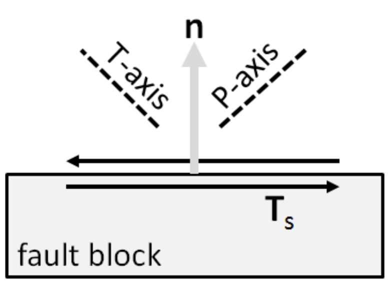

Figure: The P-axis is in the plane of outward-pointing normal $\mathbf{n}$ and the shear $\mathbf{T}_\mathrm{s}$, halfway between these two directions.

Results for

$$\mathbf{S} = \begin{bmatrix} 1.00 & 0.01 & 0.02 \\ 0.01 & 0 & 0 \\ 0.02 & 0 & 0 \end{bmatrix}$$

(maximum compressive stress is about east-west)

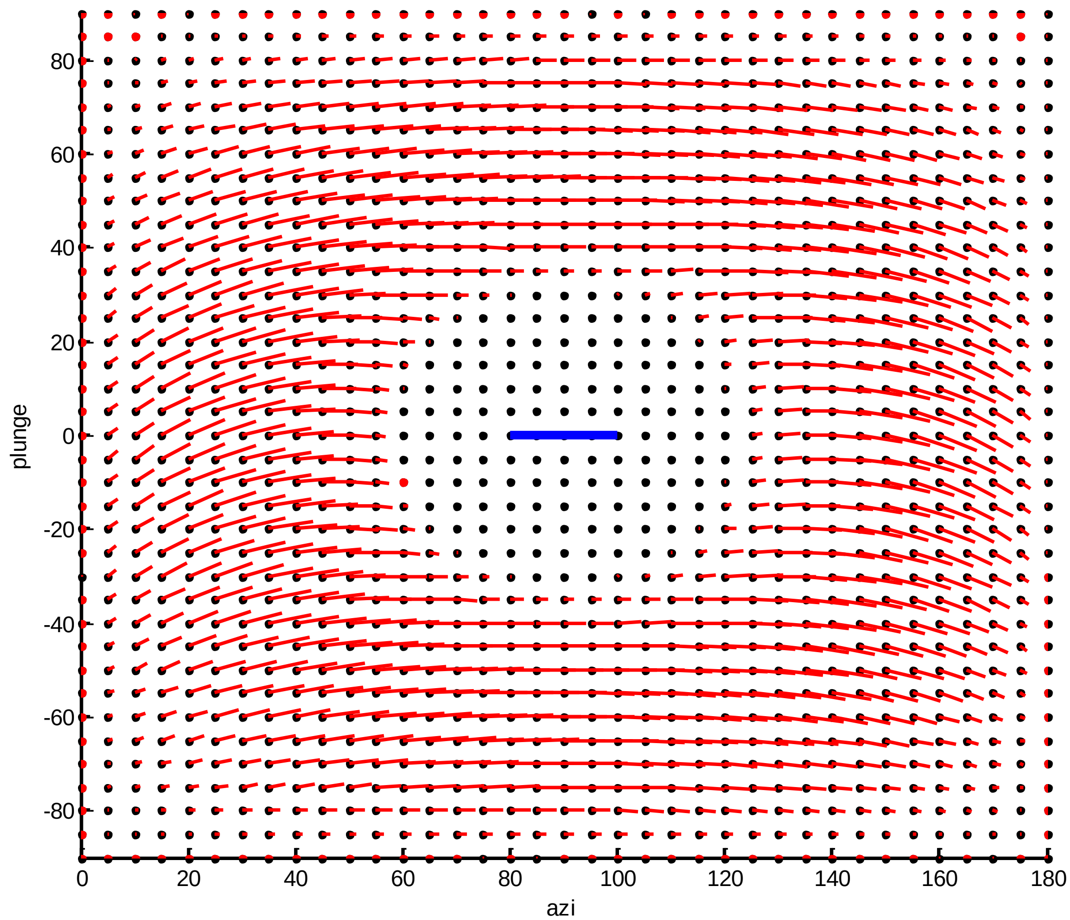

Figure. Blue bar gives direction of maximum compressive stress. Black dots show various combinations of azimuth (abcissa) and plunge (ordinate) of fault normals. Red bar has a direction that indicates the projection of the P-axis onto the (x,y) plane and a length that scales with the magnitude of the Mohr-Columb Failure Criterion, C, but are shown only for faults predicted to slip (that is, with $C > 0$).

Note that center of diagram (plunge=0, azimuth=90) is a vertical fault striking N/S. This has only a large normal fault acting on it, and so it not predicted to slip. Faults with moderate dip slip. Those left/right of the center are vertical strike-slip faults with strikes oblique to north. They slip in a horizontal direction that is oblique to east-west. Those above/below the center are north-striking dip-slip faults that slip in the east-west direction. Faults at the top and bottom of the diagram are sub-horizontal faults that don't slip because the overall traction on their surface is small.

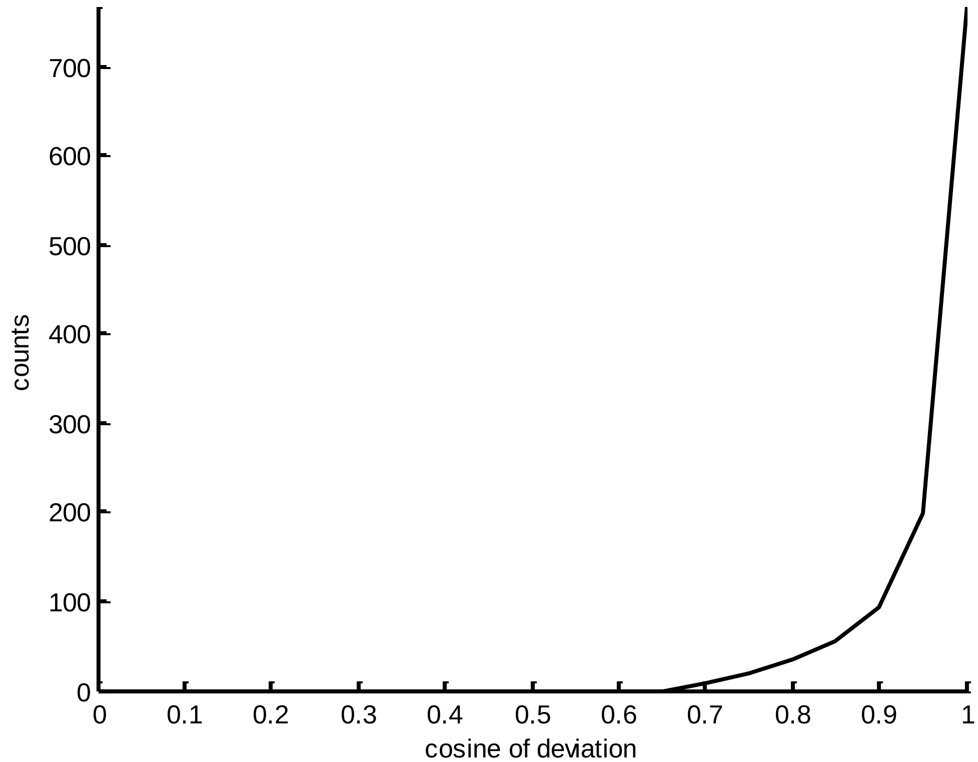

Figure. Histogram of the cosine of the deviation of the projection onto the (x,y) plane of the P-axes and the direction of maximum compressive stress. Only for faults predicted to slip (that is, with $C > 0$) are used.

Interpretation: While there a certainly some faults that have large deviations of P-axes from the direction of maximum compression. But the histogram is strongly peaked at a cosine of deviation of unity, which is to say a deviation of zero. I suspect that the assumption that the slip is parallel to the direction of shear stress on the fault places a strong constraint on the orientation of the P-axis.

MatLab Code

```
clear all;
DTOR = pi/180;

% stress tensor, but work in system where compressive stress is positive
S= [1, 0.001, 0.002; 0.001, 0, 0; 0.002, 0, 0];

% principle stress directions
[V,D] = eig(S);
tmp = [V(1,3), V(2,3)]';
Ph = tmp / sqrt(tmp'*tmp); % maximum compressive stress direction
tmp = [V(1,1), V(2,1)]';
Th = tmp / sqrt(tmp'*tmp);  % minimum compressive stress direction

sigma = 0.6; % coefficient of friction

azi = [0:5:180]'; % degrees E of N
plunge = [-90:5:90]'; % degrees from horizontal

k=0;
for ia = azi'
for ip = plunge'
    k=k+1;
    a(k) = ia;
    p(k) = ip;
end
end
N=k;

figure(1);
clf;
set(gca,'LineWidth',2);
hold on;
axis( [min(azi), max(azi), min(plunge), max(plunge) ] );
plot( a, p, 'k.', 'LineWidth', 2 );
scale = 10;
plot( [-scale*Ph(1)+90,scale*Ph(1)+90]', [-scale*Ph(2),scale*Ph(2)]', 'b-',
'LineWidth', 4 );
% plot( [-scale*Th(1)+90,scale*Th(1)+90]', [-scale*Th(2),scale*Th(2)]', 'g-',
'LineWidth', 4 );
xlabel( 'azi' );
ylabel( 'plunge' );

% x is east, y is north, z is up
% normal to fault plane
nx = cos(DTOR*p).*sin(DTOR*a);
ny = cos(DTOR*p).*cos(DTOR*a);
nz = -sin(DTOR*p);

NC=0;
for i = [1:N]
    n = [nx(i), ny(i), nz(i)]'; % normal to fault
    T = S*n; % traction on fault
    Tn = (T'*n)*n; % normal traction
```

```
    mTn = sqrt(Tn'*Tn); % magnitude of normal traction
    dTn = Tn/mTn; % direction of normal traction
    Ts = T - Tn; % shear traction
    mTs = sqrt(Ts'*Ts); % magnitude of shear traction
    dTs = Ts / mTs; % direction of shear traction
    tmp = n+dTs;
    paxis = tmp/sqrt(tmp'*tmp);
    C = mTs - sigma*mTn;
    [mTs, mTn, C]

    if( C>0 )
        NC=NC+1;
        scale = 30.0*C;
        plot( [a(i), a(i)+scale*paxis(1)], [p(i), p(i)+scale*paxis(2)], 'r-',
'LineWidth', 2 );
        tmp = sqrt( paxis(1)^2 + paxis(2)^2 );
        costheta(NC) = (paxis(1)*Ph(1)+paxis(2)*Ph(2))/tmp;
    end

end


    bins = [-100:5:100]/100;
    h = hist( costheta, bins );
    figure(2);
    clf;
    set(gca,'LineWidth',2);
    axis( [0, 1, 0, max(h) ] );
    hold on;
    plot( bins, h, 'k-', 'LineWidth', 2);
    xlabel('cosine of deviation');
    ylabel('counts');
```