Perturbation Series for Inverse of Operator 2014

$$(A+\varepsilon B)\, m = d$$

$$(A+\varepsilon B)(m_0+\varepsilon m_1+\varepsilon^2 m_2 \cdots) = d$$

$$\varepsilon^0 A m_0 = \varepsilon^0 d \quad \therefore \quad m_0 = A^{-1} d$$

$$\varepsilon^1 (B m_0 + A m_1) = 0 \quad \therefore \quad m_1 = -A^{-1} B m_0 = -A^{-1} B A^{-1} d$$

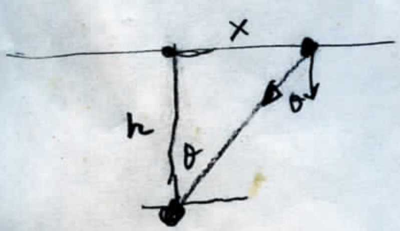
$$\varepsilon^2 (B m_1 + A m_2) = 0 \quad \therefore \quad m_2 = -A^{-1} B m_1 = A^{-1} B A^{-1} B A^{-1} d$$

$$m = (A^{-1} - \varepsilon A^{-1} B A^{-1} + \varepsilon^2 A^{-1} B A^{-1} B A^{-1} + \cdots)\, d$$

$$= (A+\varepsilon B)^{-1} d$$

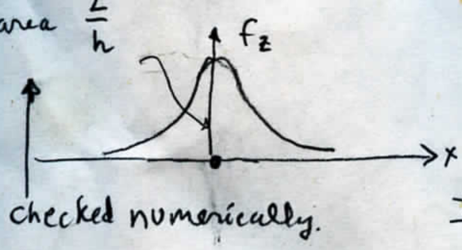
so $(A+\varepsilon B)^{-1} = A^{-1} + \varepsilon A^{-1} B A^{-1} + \varepsilon^2 A^{-1} B A^{-1} B A^{-1} + \cdots)$

Gravity Operator and its normalization



$$f = \frac{1}{h^2+x^2} \qquad f_z = f\cos\theta \qquad \cos\theta = \frac{h}{(h^2+x^2)^{1/2}}$$

$$f\cos\theta = \frac{h}{(h^2+x^2)^{3/2}}$$



checked numerically.

CRC 615

$$I_1 = \int_0^\infty \frac{x^a\, dx}{(m+x^b)^c}$$

$a=0$	$a>-1$ ✓
$b=2$	$b>0$ ✓
$m=h^2$	$m>0$ ✓
$c=\frac{3}{2}$	$c>\frac{a+1}{b}=0$ ✓

$$D = \frac{a+1-bc}{b} = \frac{0+1-2\cdot\frac{3}{2}}{2} = \frac{1-3}{2} = \frac{-2}{2} = -1$$

$$E = \frac{a+1}{b} = \frac{0+1}{2} = \frac{1}{2} \qquad F = c - \frac{a+1}{b} = \frac{3}{2} - \frac{1}{2} = \frac{2}{2} = 1$$

$$\Gamma(1) = 1$$

$$I_1 = \frac{m^D}{b}\frac{\Gamma(E)\Gamma(F)}{\Gamma(c)} = \frac{m^{-1}}{2}\frac{\Gamma(\frac{1}{2})\Gamma(1)}{\Gamma(\frac{3}{2})}$$

$$\Gamma(n+1) = n\Gamma(n)$$

$$\Gamma(\tfrac{1}{2}+1) = \Gamma(\tfrac{3}{2}) = \tfrac{1}{2}\Gamma(\tfrac{1}{2})$$

$$I_1 = \frac{1}{2} m^{-1}\, 2 = m^{-1} = h^{-2} \qquad \frac{\Gamma(\frac{1}{2})}{\Gamma(\frac{3}{2})} = \frac{\Gamma(\frac{1}{2})}{\frac{1}{2}\Gamma(\frac{1}{2})} = 2$$