Discrete Events

Suppose that there are m intervals, I, each n years long, and the probability of a year being declared an "event year" is p. The probability a year not being an event year is $(1-p)$ and, assuming independence, of the interval n not containing an event year is $(1-p)^n$. The probability of one or more event years in a single interval is $P_I = 1-(1-p)^n$. The probability of one or more events occuring in m such intervals is $P_I^m = (1-(1-p)^n)^m$, again assuming independence.

Oct 1, 2014, for Dallas

Scenario: time intervals of n years $(n=2)$
total of m intervals $(m=6)$
a year is determined to be an "event year" or not by some measurement process.

Observation: All m intervals contain event years.

Null Hypothesis: Event-years are independent of one another and occur randomly with probability $p \approx \frac{\text{total event years}}{\text{total years}} = 0.2$.

Probability of Observation under the null Hypothesis

$$(1-(1-p)^n)^m \approx 0.002$$

Frost Ring Calculation Menke Oct 1, '14

1. A year is declared an 'event-year' (frost ring, low growth), or it isn't.

2. On average, the average probability per year is

$$p = \frac{\text{total number of event-years}}{\text{total number of years}}$$

3. We identify $m=6$ intervals each $n=2$ years long

4. What is the probability that all M intervals will have at least one event, under the null hypothesis that the events occur randomly.

5. The probability of exactly k events in one interval is binomial distributed:

$$\binom{n}{k} p^k (1-p)^{n-k} \qquad \binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

7. The probability of exactly zero events in an interval is:

$$(1-p)^n \text{ since } p^0=1,\ n-0=n \text{ and } \binom{n}{0} = \frac{n!}{0!\,(n-0)!} = \frac{n!}{1 \cdot n!} = 1$$

8. The probability of more than zero events in an interval is $1-(1-p)^n$

9. The probability of all m intervals having more than zero events is

$$P_T = \boxed{\left(1-(1-p)^n\right)^m}$$

with $n=2$, $m=6$
$p = 32/(1610-1450) = 0.200$ $P_T = 0.0022$

$n=2$ $(1-p)^2 = (1-p)(1-p) = 1-2p+p^2$ so $1-(1-p)^2 = 2p-p^2 = p(2-p)$

$p^m (2-p)^m$