$$C_x = \sigma_x^2 \begin{pmatrix} 1 & a & a & \cdots \\ a & 1 & a & \\ a & a & 1 & \\ \vdots & & & \end{pmatrix} \qquad \bar{x} = \frac{1}{N}\left[1\ 1\ 1\ 1\ 1\right] x$$

$$\sigma_{\bar{x}}^2 = \frac{1}{M}\overbrace{\left[1\ 1\ 1\ 1\right]}^{N}\ \sigma_x^2 \begin{bmatrix} 1 & a & a & \cdots \\ a & 1 & a & \\ a & a & 1 & \\ \vdots & & & \end{bmatrix} \frac{1}{M}\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$$

$$\sigma_{\bar{x}}^2 = \frac{\sigma_x^2}{M^2}\left[1\ 1\ 1\ 1\ \cdots\right]\begin{bmatrix} 1+(N-1)a \\ 1+(N-1)a \\ \vdots \end{bmatrix}$$

$$\sigma_{\bar{x}}^2 = \frac{\sigma_x^2}{M^2} N\left(1+(N-1)a\right)$$

$$\lim\ a \to 0 \qquad \sigma_{\bar{x}}^2 = \frac{\sigma_x^2 N}{M^2} \qquad \sigma_{\bar{x}} = \frac{\sigma_x}{M}\sqrt{N}$$

$$\lim\ N \to \infty \qquad \sigma_{\bar{x}}^2 = a\,\sigma_x^2 \qquad \sigma_{\bar{x}} = \sqrt{a}\,\sigma_x$$

$$\begin{array}{c} N \\ \begin{bmatrix} 11 & 12 & \boxed{13} & 14 & 15 & 16 \\ \boxed{21} & 22 & 23 & \boxed{24} & \boxed{25} & \boxed{26} \\ 31 & 32 & 33 & 34 & 35 & 36 \\ 41 & 42 & \boxed{43} & 44 & 45 & 46 \\ 51 & 52 & \boxed{53} & 54 & 55 & 56 \\ 61 & 62 & \boxed{63} & 46 & 65 & 66 \end{bmatrix} N \end{array}$$

row and col; omiting 2 diag and overlap

$2N-4$ elements, (no repeats)

$N-2$ pairs $= M$

corr per element: other elem in col or row
N − self − diag − twin $= N-3$
+partner → $N-2$

$N=6$

elements $= 8$

pairs $= 4$

$$\left[13\ 21\ 24\ 25\ 26\ 43\ 53\ 63\right]$$

$$\begin{bmatrix} \sigma & \alpha & \boxed{0 \quad 0 \quad 0} & \alpha & \alpha & \\ \alpha & \sigma & \alpha & \alpha & \alpha & \boxed{0\ 0\ 0} \\ 0 & \alpha & \sigma & \alpha & \alpha & \alpha\ 0\ 0 \\ 0 & \alpha & \alpha & \sigma & \alpha & 0\ \alpha\ 0 \\ & & & & & \end{bmatrix} \begin{bmatrix} 13 \\ 21 \\ 24 \\ 25 \\ 26 \\ 43 \\ 53 \\ 63 \end{bmatrix}$$

C_v $\qquad$ V

$$\sigma_{\bar{x}}^2 = \frac{\sigma_x^2}{(N-2)^2}(2N-4)\left(1+(N-2)a\right)$$

$$\lim_{N\to\infty} \sigma_{\bar{x}}^2 = \frac{\sigma_x^2}{N^2}(2N)(2Na) = 2\,\sigma_x^2\,\alpha$$

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target τ_{23} shows an index with all v's. $\tau^A_{23} = Av$ with $A = \frac{1}{N-2}[1, 1, 1, 1, \cdots]$

let $u = \begin{bmatrix} [v] \\ \tau_{23} \end{bmatrix}$

$$C_u = \sigma^2 \begin{bmatrix} [Cv] & [\alpha] \\ [\alpha] & 1 \end{bmatrix}$$

$$\begin{bmatrix} \tau^A_{23} \\ \tau^{est}_{23} \end{bmatrix} = \begin{bmatrix} \frac{1}{N-2}[1\,1\,1\,1\,1] & 0 \\ 0 & 1 \end{bmatrix} u$$

$$= \begin{bmatrix} \underline{1}^T/(N-2) & 0 \\ \underline{0}^T & 1 \end{bmatrix} \sigma^2 \begin{bmatrix} \underline{\underline{C}}_v & \alpha \\ \alpha^T & 1 \end{bmatrix} \begin{bmatrix} \frac{\underline{1}}{N-2} & \underline{0} \\ 0 & \underline{1} \end{bmatrix}$$

$$= \sigma^2 \begin{bmatrix} \underline{1}^T/(N-2) & 0 \\ \underline{0}^T & 1 \end{bmatrix} \begin{bmatrix} \frac{\underline{C}_v \underline{1}}{(N-2)} & \alpha \\ \frac{\alpha^T \underline{1}}{N-2} & 1 \end{bmatrix}$$

$$\sigma^2 \begin{bmatrix} \frac{1^T C_v 1}{(N-2)^2} & \frac{1^T \alpha}{N-2} \\ \frac{\underline{\alpha}^T \underline{1}}{N-2} & 1 \end{bmatrix}$$

$$\underline{\alpha}^T \underline{1} = \sum_{i=1}^{2N-4} \alpha = (2N-4)\alpha$$

$$C_u(1,2) = \frac{\sigma^2 (2N-4)\alpha}{N-2} = 2\sigma^2\alpha$$

$$\sigma^2 = \begin{bmatrix} \alpha & (1-\alpha) \end{bmatrix} \begin{bmatrix} a & c \\ c & b \end{bmatrix} \begin{bmatrix} \alpha \\ (1-\alpha) \end{bmatrix}$$

$$= \begin{bmatrix} \alpha & (1-\alpha) \end{bmatrix} \begin{bmatrix} \alpha a + (1-\alpha) c \\ \alpha c + (1-\alpha) b \end{bmatrix}$$

$(1+\alpha^2 - 2\alpha) b$

$$= \alpha^2 a + (1-\alpha)\alpha c + (1-\alpha)\alpha c + (1-\alpha)^2 b$$

$$= \alpha^2 (a+b-2c) + \alpha(2c-2b) + b$$

$$\frac{\partial \sigma^2}{\partial \alpha} = 0 = 2\alpha(a+b-2c) + 2(c-b) + 0$$

$$\alpha = \frac{b-c}{a+b-2c}$$

test $a=b$ $c=0$ $\alpha = \frac{1}{2}$ ✓

in our case $b=c$ so $\alpha=0$