

$$C_x = \sigma_x^2 \begin{pmatrix} 1 & a & a & \dots \\ a & 1 & a & \\ a & a & 1 & \\ \vdots & & & \ddots \end{pmatrix}$$

$$\bar{x} = \frac{1}{N} [1 \dots 1] x$$

$$\sigma_{\bar{x}}^2 = \frac{1}{N} [1 \dots 1] \sigma_x^2 \begin{pmatrix} 1 & a & a & \dots \\ a & 1 & a & \\ a & a & 1 & \\ \vdots & & & \ddots \end{pmatrix} \frac{1}{N} \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$$

$$\sigma_{\bar{x}}^2 = \frac{\sigma_x^2}{N^2} [1 \dots 1] \begin{pmatrix} 1 + (N-1)a \\ 1 + (N-1)a \\ \vdots \end{pmatrix}$$

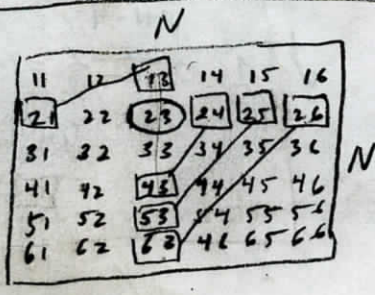
$$\sigma_{\bar{x}}^2 = \frac{\sigma_x^2}{N^2} N (1 + (N-1)a)$$

$$\lim_{a \rightarrow 0} \sigma_{\bar{x}}^2 = \frac{\sigma_x^2 N}{N^2}$$

$$\sigma_{\bar{x}} = \frac{\sigma_x}{N} \sqrt{N}$$

$$\lim_{N \rightarrow \infty} \sigma_{\bar{x}}^2 = a \sigma_x^2$$

$$\sigma_{\bar{x}} = \sqrt{a} \sigma_x$$

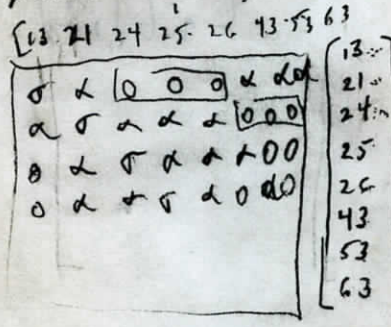


Row and col: omitting 2 diag and overlap
 $2N - 4$ elements, (no repeats)

$N - 2$ pairs = M
 corr per element: other elem in col or row
 $N - \text{self-diag} - \text{row} = N - 3$
 $+ \text{partner} \Rightarrow N - 2$

$N = 6$
 elements = 8
 pairs = 4

$$\sigma_{\bar{x}}^2 = \frac{\sigma_x^2}{(N-2)^2} (2N-4) (1 + (N-2)a)$$



$$\lim_{N \rightarrow \infty} \sigma_{\bar{x}}^2 = \frac{\sigma_x^2}{N^2} (2N) (2Na) = 2 \sigma_x^2 a$$

C_v v

My Schedule

target τ_{23} shows an index with all v 's. $\tau_{23}^A = Av$ with $A = \frac{1}{N-2} [1, 1, 1, \dots]$

let $u = \begin{bmatrix} v \\ \tau_{23} \end{bmatrix}$

$$C_u = \sigma^2 \begin{bmatrix} [C_v] & [\alpha] \\ [\alpha] & 1 \end{bmatrix}$$

$$\begin{bmatrix} \tau_{23}^A \\ \tau_{23}^{est} \end{bmatrix} = \begin{bmatrix} \frac{1}{N-2} [1, 1, \dots] & 0 \\ 0 & 1 \end{bmatrix} u$$

$$= \begin{bmatrix} \frac{1}{N-2} \mathbf{1}^T & 0 \\ 0 & 1 \end{bmatrix} \sigma^2 \begin{bmatrix} C_v & \alpha \\ \alpha^T & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{N-2} \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \sigma^2 \begin{bmatrix} \frac{1^T}{N-2} & 0 \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} \frac{C_v \mathbf{1}}{N-2} & \alpha \\ \frac{\alpha^T \mathbf{1}}{N-2} & 1 \end{bmatrix}$$

$$\sigma^2 \begin{bmatrix} \frac{1^T C_v \mathbf{1}}{(N-2)^2} & \frac{1^T \alpha}{N-2} \\ \frac{\alpha^T \mathbf{1}}{N-2} & 1 \end{bmatrix}$$

$$\alpha^T \mathbf{1} = \sum_{i=1}^{2N-4} \alpha = (2N-4)\alpha$$

$$C_u(1,2) = \frac{\sigma^2 (2N-4) \alpha}{N-2} = 2\sigma^2 \alpha$$

$$\sigma^2 = \begin{bmatrix} \alpha & (1-\alpha) \end{bmatrix} \begin{bmatrix} a & c \\ c & b \end{bmatrix} \begin{bmatrix} \alpha \\ (1-\alpha) \end{bmatrix}$$

$$= \begin{bmatrix} \alpha & (1-\alpha) \end{bmatrix} \begin{bmatrix} \alpha a + (1-\alpha)c \\ \alpha c + (1-\alpha)b \end{bmatrix}$$

$$= \alpha^2 a + (1-\alpha)\alpha c + (1-\alpha)\alpha c + (1-\alpha)^2 b \quad (1+\alpha^2-2\alpha)b$$

$$= \alpha^2(a+b-2c) + \alpha(2c-2b) + b$$

$$\frac{2\sigma^2}{2\alpha} = 0 = 2\alpha(a+b-2c) + 2(c-b) + 0$$

$$\alpha = \frac{b-c}{a+b-2c}$$

test $a=b$ $c=0$ $\alpha = \frac{1}{2}$ ✓

in our case $b=c$ so $\alpha=0$