Relationshup between velocity and traveltime perturbations

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Seismic velocity, written in terms of a reference level v_0 and a perturbation Δv:

$$v = v_0 + \Delta v = v_0 \left(1 + \frac{\Delta v}{v_0}\right)$$

The corresponding slowness, to first order:

$$s = \frac{1}{v} = v_0^{-1}\left(1 - \frac{\Delta v}{v_0}\right) = v_0^{-1} - \frac{\Delta v}{v_0^2} = s_0 + \Delta s$$

The corresponding travel time, for propagation distance H:

$$T = Hs = \frac{H}{v_0} - \frac{H\Delta v}{v_0^2} = T_0 + \Delta T$$

The ratio of P and S wave travel time deviations:

$$\frac{\Delta T^S}{\Delta T^P} = \left(-\frac{H\Delta v_s}{v_{0s}^2}\right)\left(-\frac{H\Delta v_p}{v_{0p}^2}\right)^{-1} = \left(\frac{\Delta v_s}{\Delta v_p}\right)\left(\frac{v_{0p}^2}{v_{0s}^2}\right) = \left(\frac{v_{0s}^{-1}\Delta v_s}{v_{0p}^{-1}\Delta v_p}\right)\left(\frac{v_{0p}}{v_{0s}}\right)$$

or:

$$\left(\frac{v_{0s}^{-1}\Delta v_s}{v_{0p}^{-1}\Delta v_p}\right) = \left(\frac{\Delta T^S}{\Delta T^P}\right)\Big/\left(\frac{v_{0p}}{v_{0s}}\right)$$

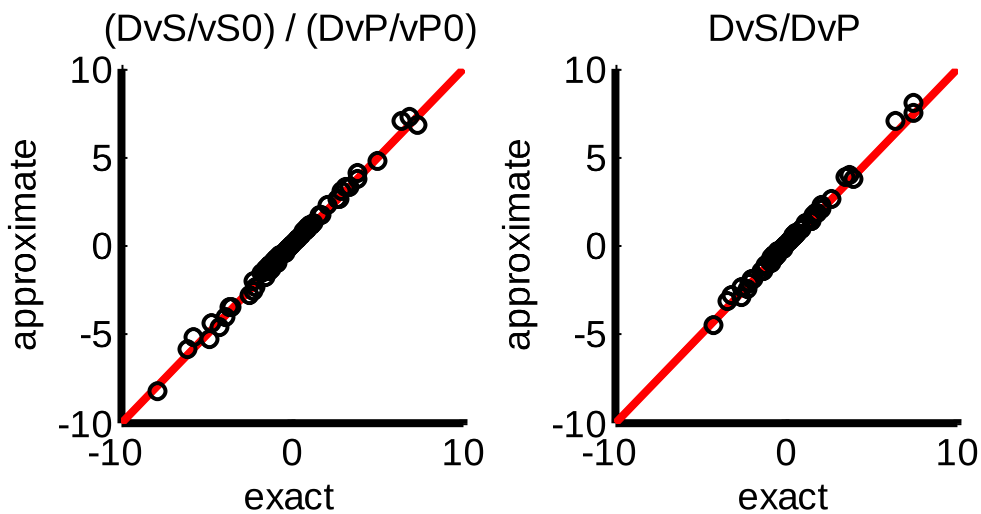
$$\left(\frac{\Delta v_s}{\Delta v_p}\right) = \left(\frac{\Delta T^S}{\Delta T^P}\right)\Big/\left(\frac{v_{0p}}{v_{0s}}\right)^2$$

For northeastern US, $\Delta T^S/\Delta T^P \approx 3.51 \pm 0.73$ (95%), and AK135 gives $v_p/v_s \approx 1.837$ (210 km), so

$$\left(\frac{v_{0s}^{-1}\Delta v_s}{v_{0p}^{-1}\Delta v_p}\right) \approx (3.51)/(1.837) \approx 1.91 \pm 0.40$$

$$\left(\frac{\Delta v_s}{\Delta v_p}\right) \approx (3.51)/(1.837)^2 \approx 1.04 \pm 0.22$$

Numerical Test




```
vS0 = 4.5;
vP0 = 1.83*vS0;
H = 100;
TP0 = H/vP0;
TS0 = H/vS0;

N = 100;
DvP = random('Uniform',-0.1*vP0, 0.1*vP0, N ,1);
DvS = random('Uniform',-0.1*vS0, 0.1*vS0, N, 1);

% Exact
vP = vP0 + DvP;
vS = vS0 + DvS;
TP = H./vP;
TS = H./vS;
DTP = TP - TP0;
DTS = TS - TS0;

% A: (DvS/vS0) / (DvP/vP0)
A_e = (DvS./vS0) ./ (DvP./vP0);
A_a = (DTS./DTP) ./ (vP0./vS0);

% B: DvS / DvP
B_e = (DvS) ./ (DvP);
B_a = (DTS./DTP) ./ ((vP0./vS0).^2);
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