Olivine, the most abundant mineral in the upper mantle, is a solid solution between Mg and Fe end members. Perturbations in the Fe/Mg ratio change the density of the mineral but have minimal effect on its elastic properties (which are mainly controlled by the overall crystal structure).

In an elastic material in which only the density is perturbed, the ratio of compressional to shear wave velocities is:

\[
\begin{align*}
V_p &= \frac{(\lambda + 2\mu)^{1/2}}{(\rho_0 + \Delta \rho)^{1/2}} = \frac{(\lambda + 2\mu)^{1/2}}{\rho_0^{1/2}} (1 + \Delta \rho / \rho_0)^{-1/2} \approx V_p^0 (1 - \frac{1}{2} \Delta \rho / \rho_0) = V_p^0 - \frac{1}{2} V_p^0 \Delta \rho / \rho_0 \\
\Delta V_p &= V_p - V_p^0 \approx -\frac{1}{2} V_p^0 \Delta \rho / \rho_0 \\
V_s &= \frac{\mu^{1/2}}{(\rho_0 + \Delta \rho)^{1/2}} = \frac{\mu^{1/2}}{\rho_0^{1/2}} (1 + \Delta \rho / \rho_0)^{-1/2} \approx V_s^0 (1 - \frac{1}{2} \Delta \rho / \rho_0) = V_s^0 - \frac{1}{2} V_s^0 \Delta \rho / \rho_0 \\
\Delta V_s &= V_s - V_s^0 \approx -\frac{1}{2} V_s^0 \Delta \rho / \rho_0 \\
\frac{\Delta V_p}{\Delta V_s} &\approx -\frac{1}{2} \frac{V_p^0 \Delta \rho / \rho_0}{V_s^0 \Delta \rho / \rho_0} = \frac{V_p^0}{V_s^0} \approx 1.73
\end{align*}
\]

Here is a numerical check of this result:

\[
\begin{array}{c|c}
L & 3 \\
mu & 3 \\
L+2mu & 9 \\
rho & 2 \\
Drho & 0.1 \\
Vp0 & 2.12132 \\
Vs0 & 1.224745 \\
Vp & 2.070197 \\
Vs & 1.195229 \\
DVp & -0.05112 \\
DVs & -0.02952 \\
DVp/DVz & 1.732051 \\
\sqrt{3} & 1.732051
\end{array}
\]