Effect of Perturbation in Density on Vp/Vs Ratio

Bill Menke, September 2016

Olivine, the most abundant mineral in the upper mantle, is a solid solution between Mg and Fe end members. Perturbations in the Fe/Mg ratio change the density of the mineral but have minimal effect on its elastic properties (which are mainly controlled by the overall crystal structure.

In an elastic material in which only the density is perturbed, the ratio of compressional to shear wave velocities is:

$$V^P = \frac{(\lambda + 2\mu)^{1/2}}{(\rho_0 + \Delta\rho)^{1/2}} = \frac{(\lambda + 2\mu)^{1/2}}{\rho_0^{1/2}} (1 + \Delta\rho/\rho_0)^{-1/2} \approx V_0^P (1 - ½\Delta\rho/\rho_0) = V_0^P - ½V_0^P \Delta\rho/\rho_0$$

$$\Delta V^P = V^P - V_0^P \approx -½V_0^P \Delta\rho/\rho_0$$

$$V^S = \frac{\mu^{1/2}}{(\rho_0 + \Delta\rho)^{1/2}} = \frac{\mu^{1/2}}{\rho_0^{1/2}} (1 + \Delta\rho/\rho_0)^{-1/2} \approx V_0^S (1 - ½\Delta\rho/\rho_0) = V_0^S - ½V_0^S \Delta\rho/\rho_0$$

$$\Delta V^S = V^S - V_0^S \approx -½V_0^S \Delta\rho/\rho_0$$

$$\frac{\Delta V^P}{\Delta V^S} \approx \frac{-½V_0^P \Delta\rho/\rho_0}{-½V_0^S \Delta\rho/\rho_0} = \frac{V_0^P}{V_0^S} \approx 1.73$$

Here is a numerical check of this result:

L	3
mu	3
L+2mu	9
rho	2
Drho	0.1
Vp0	2.12132
Vs0	1.224745
Vp	2.070197
Vs	1.195229
DVp	-0.05112
DVs	-0.02952
DVp/DVz	1.732051
sqrt(3)	1.732051