Numerical Calculation of the Geometrical Spreading Function Given Ray Tangents

illustrated using the straight line case

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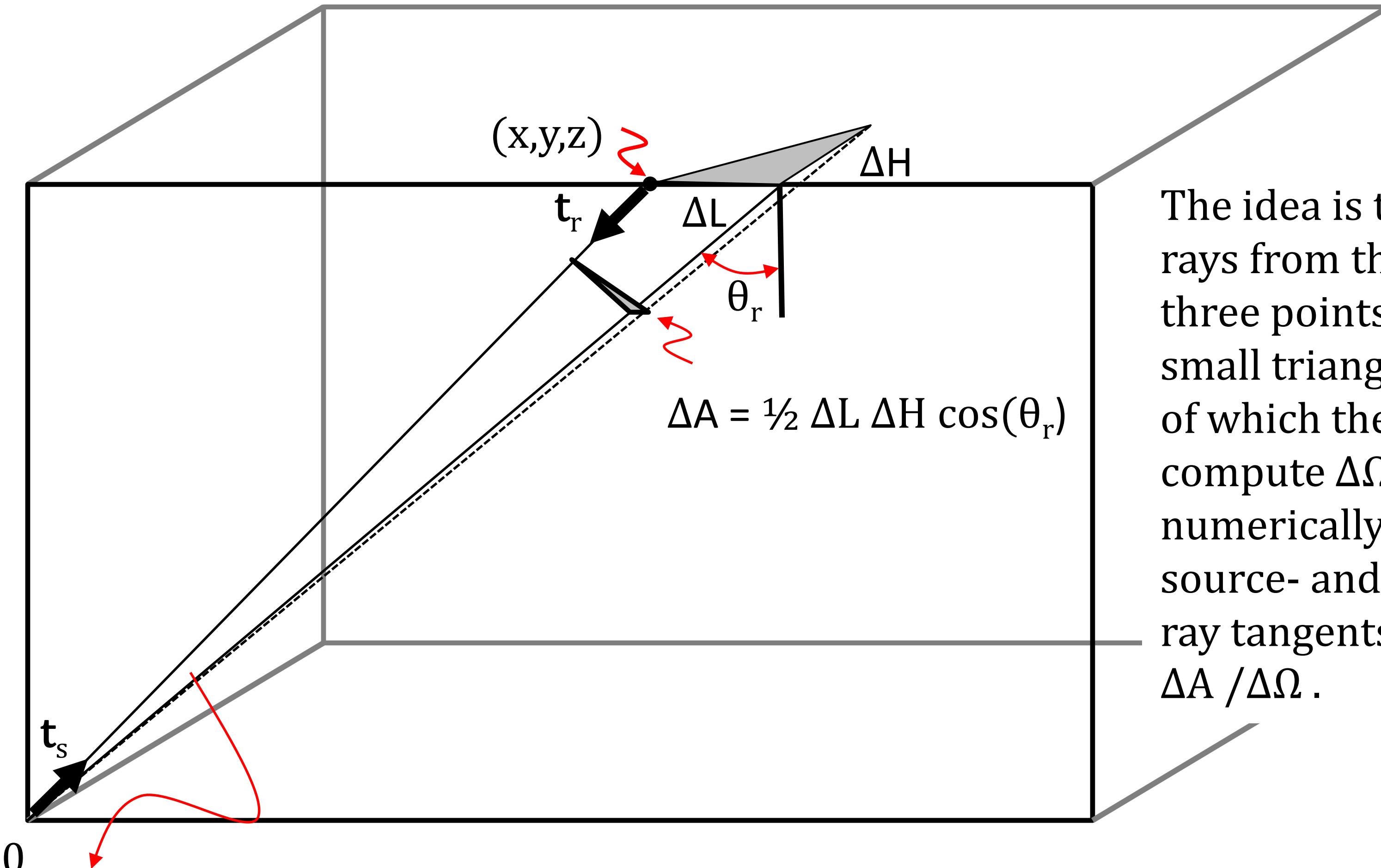
This approach is useful when the ray from a source to an arbitrary receiver (and hence source- and receiver-side ray tangents) can be computed.

The definition of geometrical spreading coefficient R_g is $\Delta A = R_g^2 \Delta\Omega$ (in analogy to the straight-line case $\Delta A = R^2 \Delta\Omega$ where R is distance)



The idea is to compute rays from the source to three points that form a small triangle, one vertex of which the station, to compute $\Delta\Omega$ and ΔA numerically,from the source- and receiver-side ray tangents, whence $R_g^2 = \Delta A / \Delta\Omega$.

$$\Delta\Omega = 2 \tan^{-1}(N/D)$$

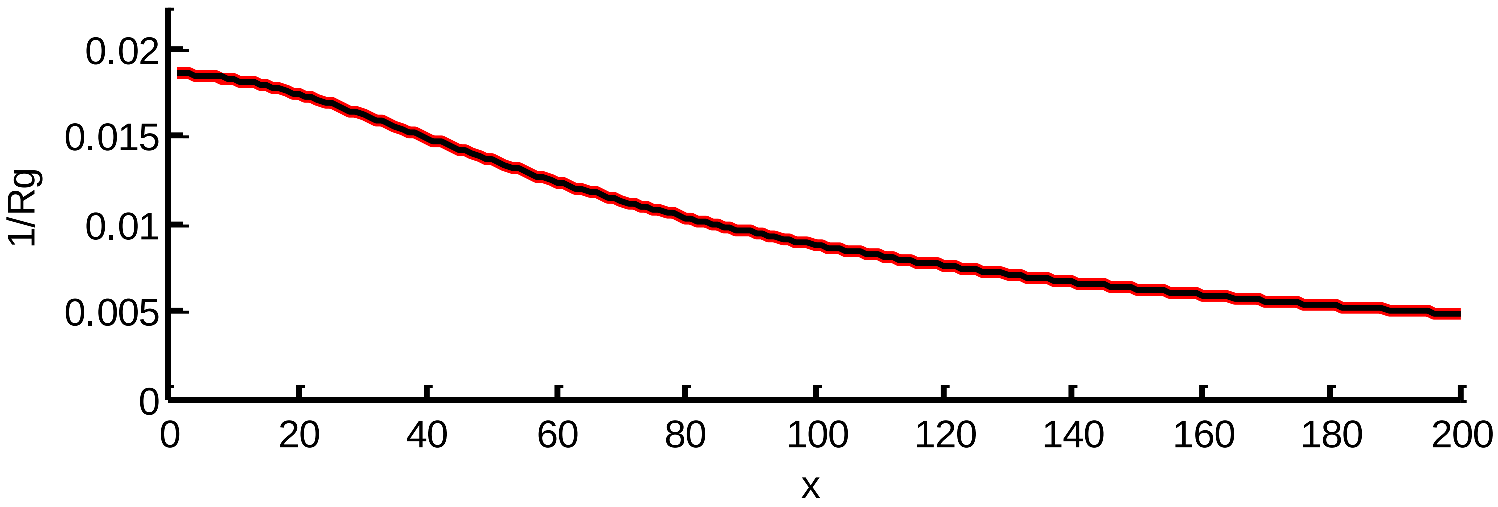
with $N = |\mathbf{t}_s \cdot (\mathbf{t}_{sL} \times \mathbf{t}_{sH})|$

$$D = 1 + \mathbf{t}_s \cdot \mathbf{t}_s + \mathbf{t}_s \cdot \mathbf{t}_{sH} + \mathbf{t}_{sL} \cdot \mathbf{t}_{sH}$$

($\mathbf{t}_{sL}$, $\mathbf{t}_{sH}$ are the source-side tangents of the rays going to the other vertices of the triangle)

see en.wikipedia.org/wiki/Solid_angle#Tetrahedron for formula to compute solid angle $\Delta\Omega$

The amplitude of a body wave is proportional to 1/Rg



Red: Numerical approximation
Black: Exact result

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% Geometrical spreading illustrated with straight line rays
clear all

N=200; % number of receivers
Dx = 1; % x spacing
X = Dx*[1:N]; % x array

Rg = zeros(N,1); % numerical result
R = zeros(N,1);  % eact result for straight line rays

for i=[1:N]

% source at (0,0,0)

% receiver location
x=X(i);
y=20;
z=50;

% small perturbation of x, y of receiver
DL = 0.01;
DH = 0.01;
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```
% begin straight line "raytrace"
% to determine tangents to rays at source and receiver

ts = [x, y, z]'/sqrt(x^2+y^2+z^2);
tsL = [x+DL, y, z]'/sqrt((x+DL)^2+y^2+z^2);
tsH = [x, y+DH, z]'/sqrt(x^2+(y+DH)^2+z^2);
tr = -[x, y, z]'/sqrt(x^2+y^2+z^2);

% end of straight line "raytrace"

% angle of incidence at receiver
qr = atan( sqrt((tr(1)^2)+(tr(2))^2) / tr(3) );
% area of the ray tube
DA = 0.5*DL*DH*cos(qr);

% solid angle at source
% see en.wikipedia.org/wiki/Solid_angle#Tetrahedron
numerator = abs( ts'*cross(tsL,tsH) );
denominator = 1 + ts'*tsL + ts'*tsH + tsL'*tsH;
DW = 2*atan( numerator / denominator );

% geometrical spreading coefficient
Rg(i) = sqrt( DA/DW );
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% in the case of straight line rays, this should
% equal distance R
R(i) = sqrt(x^2 + y^2 + z^2 );

end

figure(1);
clf;
set(gca,'LineWidth',2);
hold on;
axis( [0, X(end), 0, 1.2*(Rg(1)^(-1))] );
plot( X, Rg.^(-1), 'r-', 'LineWidth', 3 );
plot( X, R.^(-1), 'k-', 'LineWidth', 2 );
xlabel('x');
ylabel('1/Rg');
```