Poles and Zeros Response of Geophones
Bill Menke, January 2017

The frequency-dependence of the geophone response is expressed as a Padé Approximant, that is as a ratio of two polynomials, the zeros polynomial $Z(\omega)$ (in the numerator) and poles polynomial $P(\omega)$ (in the denominator):

$$R(\omega) = F\,\frac{Z(\omega)}{P(\omega)} \quad \text{with } F = S_r\, S_g\, A_0 \text{ and } Z(\omega) = \prod_{n=1}^{N} (i\omega - z_n) \text{ and } P(\omega) = \prod_{m=1}^{M} (i\omega - p_m)$$

Here ω is angular frequency, measured in radian/s. The zeros z_n and poles p_m are complex constants. Because of the requirement that that time-domain response be purely real, poles and zeroes that lie off the real ω-axis must occur in complex conjugate pairs.

The normalization constant A_0 is defined such that $|A_0|\,|\,P(\omega_r)/Z(\omega_r)| = 1$ at the reference frequency ω_r.

The sensitivity constant S_g of the geophone is usually expressed with respect to ground velocity, e.g. in volts per meter/second. The sensitivity of the geophone is therefore exactly S_g at the reference frequency ω_r, since $|A_0|\,|\,P/Z| = 1$ at that frequency. To convert to a sensitivity with respect to displacement, consider a harmonic wave with velocity $v(t) = \exp(i\omega_r t)$, which has displacement$v(t) = \exp(i\omega_r t)/(i\omega_r)$. Thus the sensitivity for displacement, say S'_g, expressed in volts per meter, has an added factor of ω_r: $S'_g = \omega_r S_g$.

In this treatment, we assume that the recording process is frequency-independent, so that the conversion from volts to digital counts can be expresses by a sensitivity constant S_r measured in digital counts per volt.

Thus, the output of the recorder, $d(t)$, in digital counts, is related to the ground velocity $v(t)$, in m/s by:

$$d(t) = \mathcal{F}^{-1}\{R(\omega)\,\mathcal{F}\{v(t)\}\}$$

Here $\mathcal{F}$ is the Fourier transform and $\mathcal{F}^{-1}$ is its inverse.

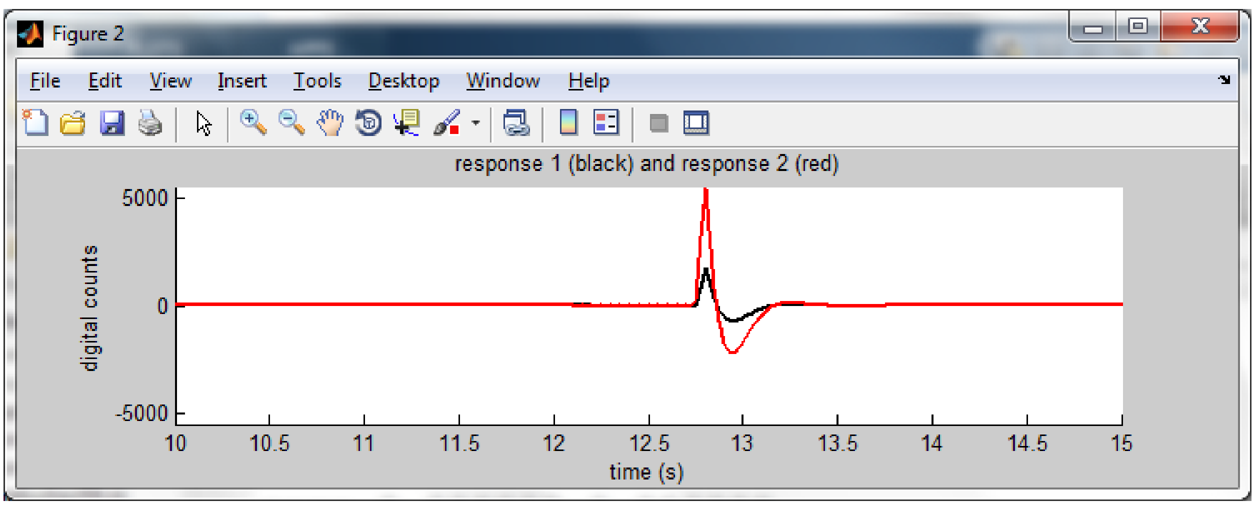
Sometimes, one wants to know how the output of the recorder, $d(t)$, in digital counts, is related to the ground displacement $u(t)$. This requires modifying the zero polynomial by adding a factor $(i\omega - z_{N+1})$ with $z_{N+1} = 0$, since a derivative in the time domain is equal to multiplication by $i\omega$ in the frequency domain:

$$Z'^{(\omega)} = \prod_{n=1}^{N+1} (i\omega - z_n) \quad \text{with} \quad z_{N+1} = 0$$

The new normalization constant, say A'_0 must have an added factor of ω_r in its denominator so that $P(z)$ has unit modulus at the reference frequency. Thus, $A'_0 = A_0/\omega_r$. Note, however, that $S'_g A'_0 = S_g A_0$; thus in practice no explicit modification of the geophone sensitivity and normalization factor is required. The displacement response is therefore:

$$R'(\omega) = F\,\frac{Z'(\omega)}{P(\omega)}$$

Examples:



Response of the geophone/recorder system to a unit spike in displacement, for the BHE channels of seismic stations TA/J59E (black) and PO/LATQ (red). The time series are of length $N = 1024$ and have sampling interval $\Delta t = 0.025\ s$. The displacement time series is a unit spike, that is, it is everywhere zero, except for the value of 1×10^{-6} m at sample $N/2$.

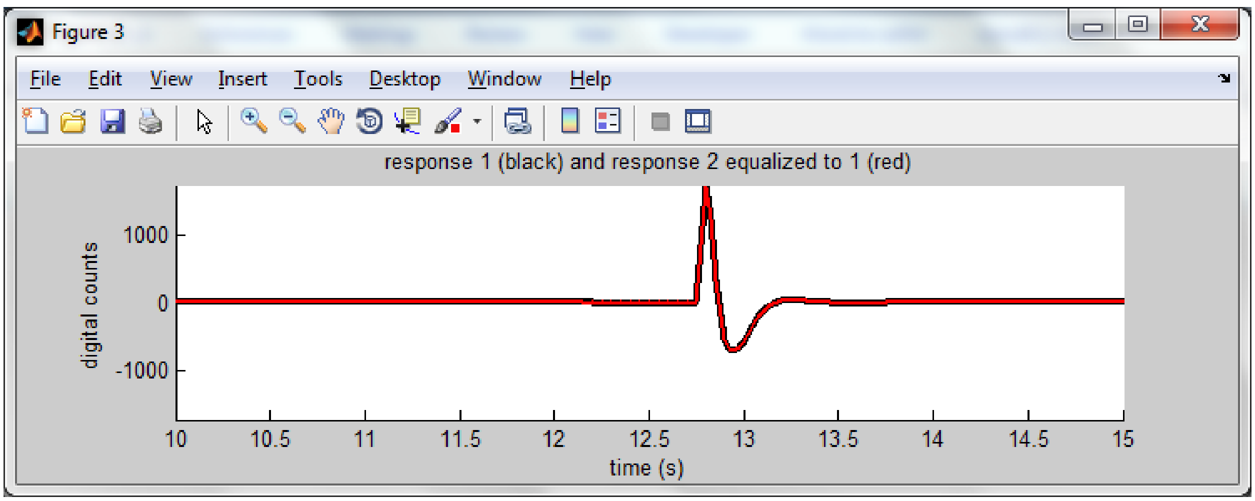
An instrument response can be deconvolved from the recorded data $d(t)$ to produce an estimate of ground displacement $u(t)$:

$$u(t) = \mathcal{F}^{-1}\left\{\frac{\mathcal{F}\{d(t)\}}{R(\omega)}\right\}$$

However, because $Z'(\omega)$ has at least one, and usually several, zeroes on the real axis (typically at $\omega = 0$), this process is unstable and leads to amplification of low-frequency noise. *Array equalization*, in which the response of several stations is adjusted to a reference response $R_r(\omega)$, is often a better option. In this case, one works in corrected digital counts, $d_c(t)$:

$$d_c(t) = \mathcal{F}^{-1}\left\{\frac{R_r(\omega)}{R(\omega)}\ \mathcal{F}\{d(t)\}\right\} = \left\{\frac{Z_r(\omega)P(\omega)}{Z'(\omega)P_r(\omega)}\ \mathcal{F}\{d(t)\}\right\}$$

The calculation can be made stable by analytically cancelling common factors in the numerator and denominator of $Z_r(\omega)/Z'(\omega)$. For example, both numerator and denominator contain the factor $(i\omega - 0)$, and these common factors cancel. (Common factors can also be cancelled from $P(\omega)/P_r(\omega)$, however these polynomials seldom have poles on the real axis, so cancellation is usually unnecessary).



Corrected data $d_c(t)$ for the BHE channels of seismic stations TA/J59E (black) and PO/LATQ (red), array-equalized to the response of TA/J59E. Note that the two time series exactly overlay.