

Example of Finite Frequency Travel Times

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In this example we construct a “ringy” pulse by bandpass filtering a spike with a filter that has a narrow bandwidth. We then consider a two time series, $u_0(t)$, which has one copy of the pulse with an amplitude of unity, and $u(t) = u_0(t) + \delta u(t)$, which is the sum of two copies of this pulse, the main pulse with unit amplitude, and a secondary pulse, with an amplitude of 0.2 (Figure 1). The time between the two pulses is varied.

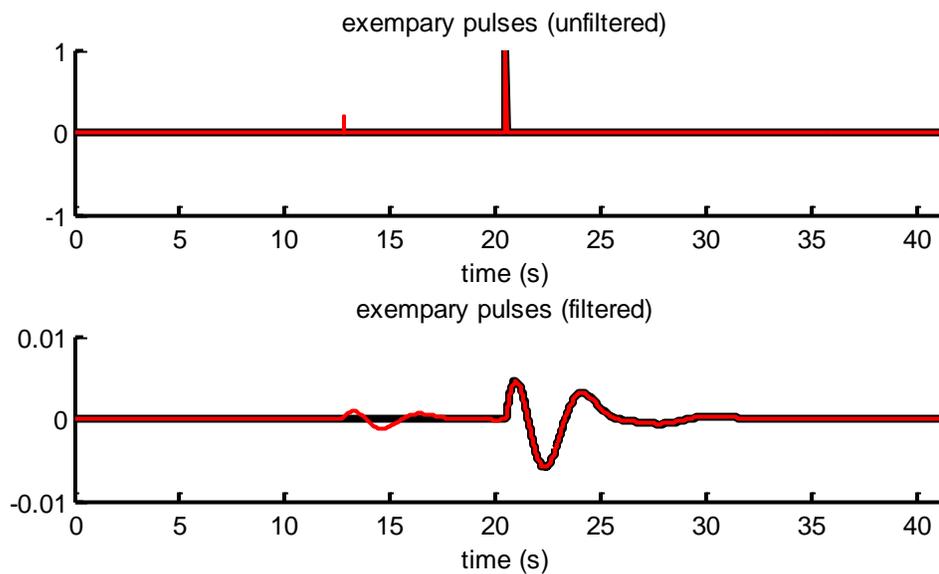


Figure 1. (Top) A small secondary pulse δu leads a main pulse u_0 . The main pulse u_0 is shown in black, the combined pulse $u = u_0 + \delta u$ in red. (Bottom) Band-pass filtered versions u_0 (black) is and $u = u_0 + \delta u$ (red), for a Chebychev bandpass filter with low and high corners of 0.2 Hz and 0.3 Hz, respectively.

We then ask whether the presence of the small pulse has the effect of advancing or delaying u with respect to u_0 , when the delay is determined by cross-correlation. Intuitively, we would expect an advance when the small pulse occurs prior to the main pulse, and a delay when it occurs after it. This would indeed be the case in the low frequency limit, however, because of wave interference, the intuitive result does not hold at higher frequencies, except when the separation between the two pulses is small (Figure 2).

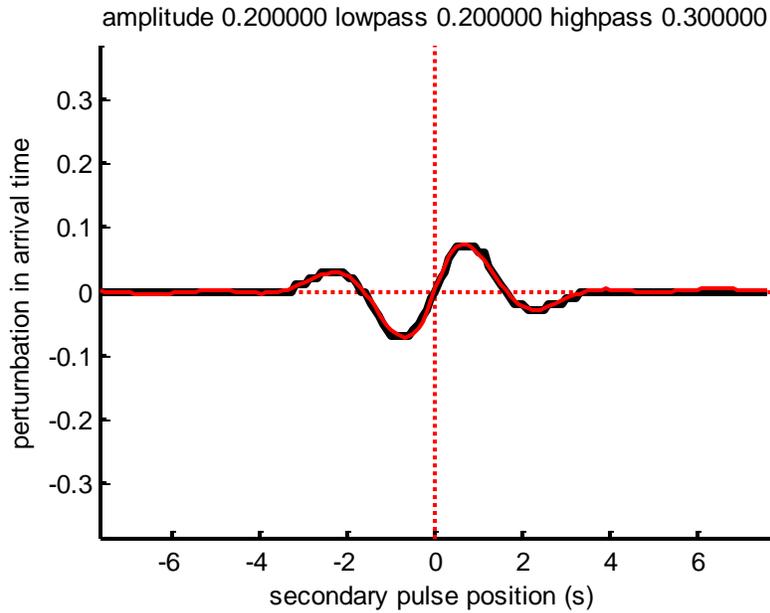


Figure 2. Advance (negative times) or delay of the pulse $u = u_0 + \delta u$ with respect to u_0 for a suite of positions of the secondary pulse, measured with respect to the main pulse. The result is computed by cross-correlating u and u_0 (black) and using Marquering et al.'s (1999) formula.

The delay τ between the two time series is found using cross-correlation, with MATLAB code:

```
% cross correlate to find time lag
c = xcorr(u0f, uf);
Nc = length(c);
Ncenter = (Nc+1)/2;
[cmax, icmax] = max(c);
tau(j) = -Dt * (icmax-N);
```

The delay is also estimated using Marquering et al.'s (1999) formula:

```
% formula from Marquering et al, GJI 137, 805-815, 1999
u0fd = [diff(u0f); 0]/Dt; % 1st derivative
u0fdd = [0; diff(u0f,2); 0]/(Dt^2); % 2nd derivative
tau2(j) = (Dt*u0fd'*duf)/(Dt*u0fdd'*u0f);
```

Marquering, H, FA Dahlen, G. Nolet, Three-dimensional sensitivity kernels for finite-frequency traveltimes: the banana-doughnut paradox, *Geophys. J. Int.* 137, 805-815. doi: 10.1046/j.1365-246x.1999.00837.x, 1999.