Consider a three dimensional diffusion equation with temperature $\theta$ and diffusivity $D$:

$$D^{-1} \frac{\partial \theta}{\partial t} - \frac{\partial^2 \theta}{\partial x^2} - \frac{\partial^2 \theta}{\partial y^2} - \frac{\partial^2 \theta}{\partial z^2} = 0$$

with boundary conditions: $\lim_{t \to 0} \theta(x, y, z, t) \to 0$ and $\int \theta(x, y, z, t) \, dx \, dy \, dz = Q$. We conjecture that the solution is separable:

$$\theta(x, y, z) = X(x, t) \, Y(y, t) \, Z(z, t)$$

Substituting this form of the solution into the diffusion equation yields:

$$\left( D^{-1} \frac{1}{X} \frac{\partial X}{\partial t} - \frac{1}{X} \frac{\partial^2 X}{\partial x^2} \right) + \left( D^{-1} \frac{1}{Y} \frac{\partial Y}{\partial t} - \frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} \right) + \left( D^{-1} \frac{1}{Z} \frac{\partial Z}{\partial t} - \frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} \right) = 0$$

The first term in parenthesis is not a function of $(y, z)$; the second not a function of $(x, z)$; the third not a function of $(x, y)$. In order for their sum to equal zero, they must individually equal zero. Hence:

$$D^{-1} \frac{\partial X}{\partial t} = \frac{\partial^2 X}{\partial x^2} \quad \text{and etc.}$$

Each equation is a one-dimensional diffusion equation, whose solution is well known to be a Gaussian with variance $\sigma(t) = 2Dt$:

$$X(x, t) \propto (2\pi)^{-1/2} (2Dt)^{-1/2} \exp\left(-\frac{1}{2} \frac{x^2}{2Dt}\right) \quad \text{and etc.}$$

With amplitude normalized so that:

$$\int X(x, t) \, dx = 1 \quad \text{and etc.}$$

The overall solution is therefore:

$$\theta(r, t) = Q \, (2\pi)^{-3/2} (2Dt)^{-3/2} \exp\left(-\frac{1}{2} \frac{r^2}{2Dt}\right) \quad \text{with} \quad r^2 = x^2 + y^2 + z^2$$

Note that this equation has the same spatial variance as the one-dimensional solution. Now suppose we use $\sqrt{\sigma}$ the as a measure of half width $w$ of the temperature distribution:

$$w^2 = \sigma = 2Dt$$
The time $\tau_1$ need to reach half width $w$ is $\tau_1 = w^2 / 2D$, the time $\tau_2$ need to reach half width $2w$ is $\tau_1 = 4w^2 / 2D$, and the doubling time is $\tau = \tau_2 - \tau_1 = 4w^2 / 2D - w^2 / 2D = 3w^2 / 2D$. From the point of view of amplitudes, a doubling of width implies a decrease in amplitude by a factor of $2^3 = 8$, since the heat spreads out in three dimensions. The change in half-width associated with a halving of amplitude is $\sqrt{2} \approx 1.26$ and the characteristic time is $\tau = 2^{2/3}w^2 / 2D - w^2 / 2D = (2^{2/3} - 1) w^2 / 2D \approx 0.59 w^2 / 2D$. For a hot (>1000 K) upper mantle, the thermal diffusivity is in the range $D \approx 5 - 12 \times 10^{-7}$ m$^2$/s (Gibert & Seipold 2003), leading to a prediction of time needed for 50% decay as shown in the figure.


Figure. Decay times for $D \approx 5 \times 10^{-7}$ m$^2$/s (upper curve) and $D \approx 12 \times 10^{-7}$ m$^2$/s (lower curve).