

Wu's Apparent Attenuation Formula with an Exponential Autocorrelation Function

Bill Menke, August 20, 2017

Wu [1982] gives the following formula for the apparent quality factor Q of compressional waves in a 3D heterogeneous medium:

$$Q^{-1} = \gamma^2 k [P(\sqrt{2}k) - P(2k)]$$

Here γ is the r.m.s. relative velocity perturbation, $k = \omega/c = 2\pi f/c$ is wavenumber, ω is angular frequency, f is frequency, c is wave speed and $P(k)$ is the Fourier cosine transform of the normalized autocorrelation function $N(r)$, where r is offset. Menke [1984] puts forward the autocorrelation function $N(r) = \exp(-r/a)$ and its transform $P(k) = 2a/(1 + k^2 a^2)$.

Inserting this transform into Wu's [1982] formula yields:

$$Q^{-1} = \frac{4\gamma^2 a^3 k^3}{(1 + 2a^2 k^2)(1 + 4a^2 k^2)}$$

The choices $a = 50,000\text{m}$, $c = 8,200\text{ m/s.}$, $\gamma = 0.05$, $f = 0.1\text{ Hz}$, which might be appropriate for long-period P waves in the upper mantle, yield $Q = 3,221$. With these choices of a , c and γ , $1,000 < Q < 100,000$ for the 0.01 – 1.00 Hz frequency band (Figure 1).

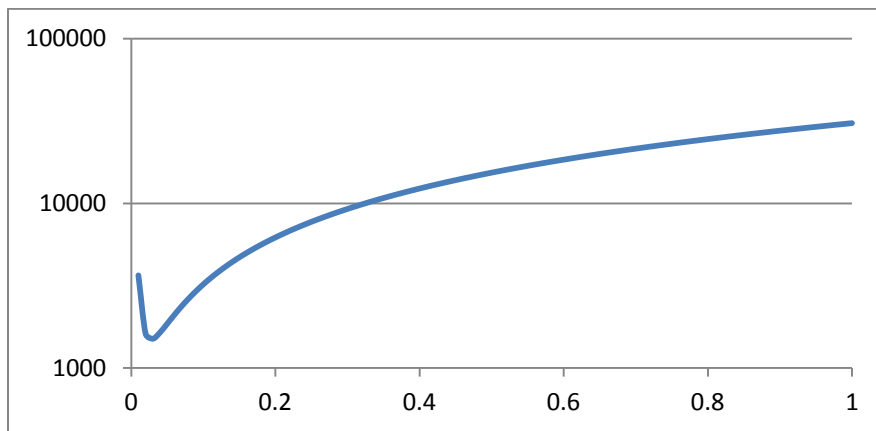


Figure 1. Q vs Frequency for $a = 50,000\text{ m}$, $c = 8,200\text{ m/s.}$, $\gamma = 0.05$.

References.

Menke, W., Asymptotic formulae for the apparent Q of weakly scattering three dimensional media, Bull. Seism. Soc. Am. 74, 1079-1081, 1984.

Wu, R.S., Attenuation of short period seismic waves due to scattering Geophys. Res. Lett. 9, 9-12, 1982.