Duration Scaling Thoughts for Slow Earthquakes – Bill Menke – October 5, 2017

The seismic moment is \( M = \mu L W S \) for length \( L \), width \( W \), slip \( S \) and shear modulus \( \mu \).

Assume a square fault, so \( W = L \).

My model for a slow earthquake is that the fault is initially stressed and that during the slow earthquake, the whole fault slowly slides, with velocity \( V = \dot{S} \), until the stress is relaxed. Assume that a critical stress level is needed for a rupture to initiate and that this level is independent of moment. The stress drop is independent of moment and equals \( \Delta \sigma = c S / L \) (and where \( c \) is an elastic modulus. Inserting \( S = \Delta \sigma L / c \) into the equation for moment yields the familiar \( L \propto M^{1/3} \) length-moment scaling:

\[
M = \left( \frac{\mu \Delta \sigma}{c} \right) L^3 \quad \text{so} \quad L = \left( \frac{c}{\mu \Delta \sigma} \right)^{1/3} M^{1/3}
\]

Now suppose that the fault zone consists of a viscous fluid of thickness \( H \) and viscosity \( \nu \). The constitutive law for a viscous fluid is that the strain rate \( \dot{\varepsilon} = \sigma / \nu \) where \( \sigma \) is the stress and \( \nu \) is the viscosity. Assuming velocity varies linearly with depth in the fluid, \( \dot{\varepsilon} = V / H \) is constant with depth in the viscous layer. Equating the stress to the stress drop, we have \( V = \Delta \sigma H / \nu \).

If the thickness \( H \) is presumed to be independent of moment, then the duration of sliding needed to cause a slip \( S \) is \( T = 2 S / V \), where the factor of two is added because the velocity declines linearly to zero as the stress is relaxed. Then the duration scales with the cube-root of moment (the same behavior as predicted for normal earthquakes):

\[
T = 2 \frac{S}{V} = 2 \frac{\Delta \sigma L}{c} \frac{\nu}{\Delta \sigma H} = \left( \frac{2 \nu}{c H} \right) L = \left( \frac{2 \nu}{c H} \right) \left( \frac{c}{\mu \Delta \sigma} \right)^{1/3} M^{1/3}
\]

However, \( H \) may actually scale with the length of the fault, because the viscous layer may represent a wear zone that thickens with successive earthquakes. If we assume \( H = b L \), the duration is independent of moment:

\[
T = \frac{2 \nu}{c b}
\]

Hence any scaling in the \( T \propto M^n \), with \( 0 \leq n < 1/3 \), might be plausible.