Duration Scaling Thoughts for Slow Earthquakes – Bill Menke – October 5, 2017

The seismic moment is $M = \mu LWS$ for length L, width W, slip S and shear modulus μ.

Assume a square fault, so $W = L$.

My model for a slow earthquake is that the fault is initially stressed and that during the slow earthquake, the whole fault slowly slides, with velocity $V = \dot{S}$, until the stress is relaxed. Assume that a critical stress level is needed for a rupture to initiate and that this level is independent of moment. The stress drop is independent of moment and equals $\Delta\sigma = cS/L$ (and where c is an elastic modulus. Inserting $S = \Delta\sigma L/c$ into the equation for moment yields the familiar $L \propto M^{1/3}$ length-moment scaling:

$$M = \left(\frac{\mu\Delta\sigma}{c}\right)L^3 \quad \text{so} \quad L = \left(\frac{c}{\mu\Delta\sigma}\right)^{1/3} M^{1/3}$$

Now suppose that the fault zone consists of a viscous fluid of thickness H and viscosity ν. The constitutive law for a viscous fluid is that the strain rate $\dot{\varepsilon} = \sigma/\nu$ where σ is the stress and ν is the viscosity. Assuming velocity varies linearly with depth in the fluid, $\dot{\varepsilon} = V/H$ is constant with depth in the viscous layer. Equating the stress to the stress drop, we have $V = \Delta\sigma H/\nu$.

If the thickness H is presumed to be independent of moment, then the duration of sliding needed to cause a slip S is $T = 2\,S/V$, where the factor of two is added because the velocity declines linearly to zero as the stress is relaxed. Then the duration scales with the cube-root of moment (the same behavior as predicted for normal earthquakes):

$$T = 2\frac{S}{V} = 2\frac{\Delta\sigma L}{c}\frac{\nu}{\Delta\sigma H} = \left(\frac{2\nu}{cH}\right)L = \left(\frac{2\nu}{cH}\right)\left(\frac{c}{\mu\Delta\sigma}\right)^{1/3} M^{1/3}$$

However, H may actually scale with the length of the fault, because the viscous layer may represent a wear zone that thickens with successive earthquakes. If we assume $H = bL$, the duration is independent of moment:

$$T = \frac{2\nu}{cb}$$

Hence any scaling in the $T \propto M^n$, with $0 \leq n < 1/3$, might be plausible.