Cammarano et al.'s (2003) Variation of Vp and Vs with Temperature
Bill Menke, July 29, 2019

I reproduce here part of Figure 3 (and its caption) of Cammarano et al. (2003).

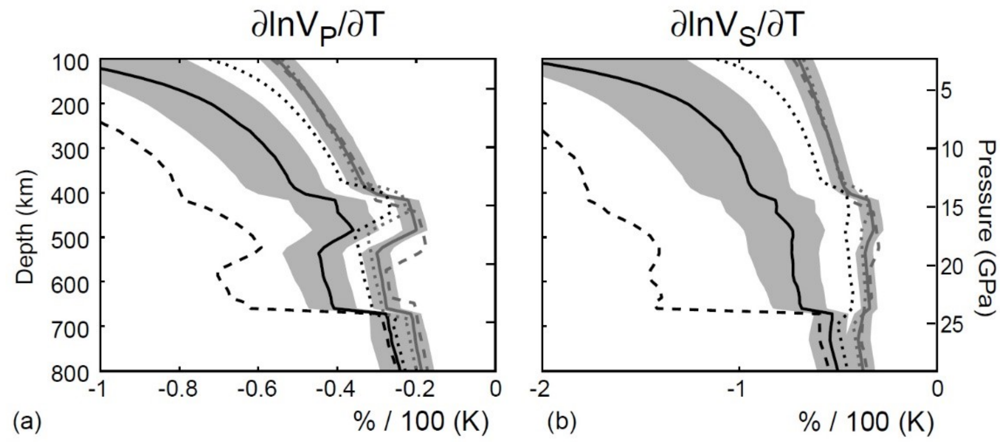

Figure 1. From Cammarano et al. (2003), their Figure 3. Temperature derivatives of VP, VS, … are shown as a function of depth. Dotted, solid and dashed lines are for 1000, 1300 and 1600 °C adiabats, respectively; gray lines correspond to purely anharmonic derivatives. Light gray fields illustrate uncertainties in anharmonic and full derivatives along the 1300 °C adiabat. Anelasticity (model Q5 is applied) makes the derivatives of the seismic wave velocities temperature dependent (black lines) and significantly increases temperature sensitivity in the upper-mantle.

I digitized the $C_P(z)$ curve:

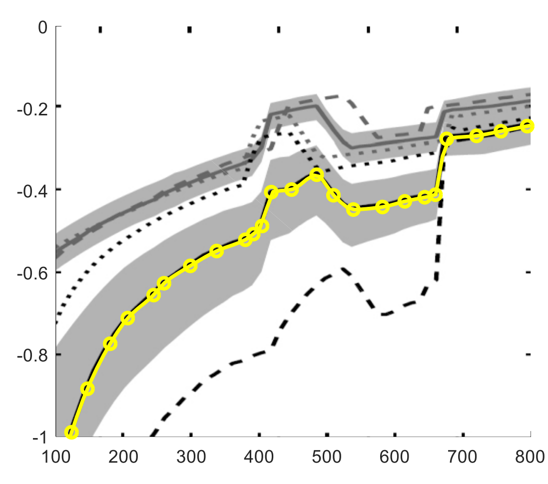

Figure 2. My digitization (yellow) of the 1300 degC adiabat in Figure 3a of Cammarano et al. (2003).

And I digitize the $C_S(z)$ curve:

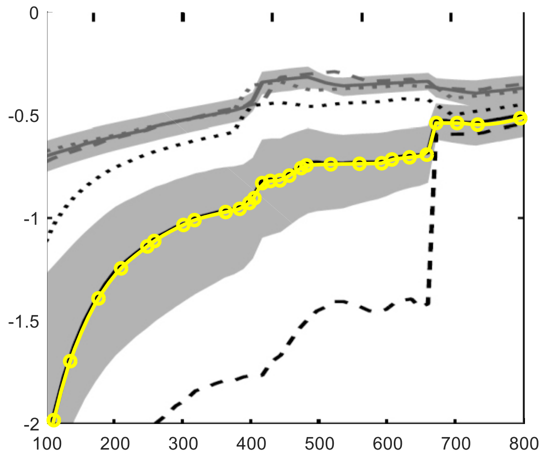

Figure 3. My digitization (yellow) of the 1300 degC adiabat in Figure 3b of Cammarano et al. (2003).

I define the sensitivity of velocity V as $S \equiv \partial V / \partial \theta$ where θ is potential temperature. I converted $C_P(z)$ and $C_S(z)$ to sensitivity using the formulas:

$$S_P(z) = \frac{V_{P0}(z)\, C_P(z)}{100 \times 100} \quad \text{and} \quad S_S(z) = \frac{V_{S0}(z)\, C_S(z)}{100 \times 100}$$

The two factors of 100 arises because $C_{P0}(z)$ and $C_{S0}(z)$ are in percent per 100 K. I used estimates of compressional velocity $V_P(z)$ and shear velocity $V_S(z)$ from the AK135 Earth Model (Kennett et al. 1995) to achieve numerical values for the sensitivity (Figure 4).

For instance, at 210 km depth, the velocities are:

$$V_P(200) = 8.301 - (5.838 \times 10^{-4})(\theta - 1300)$$

$$V_S(200) = 4.518 - (5.596 \times 10^{-4})(\theta - 1300)$$

Here θ is the potential temperature in °C. See also Table 1. The ratio of sensitivities, $R = S_P(z)/S_S(z)$ is close to unity throughout the 100-400 km depth range; that is, $0.9385 < R < 1.0534$.

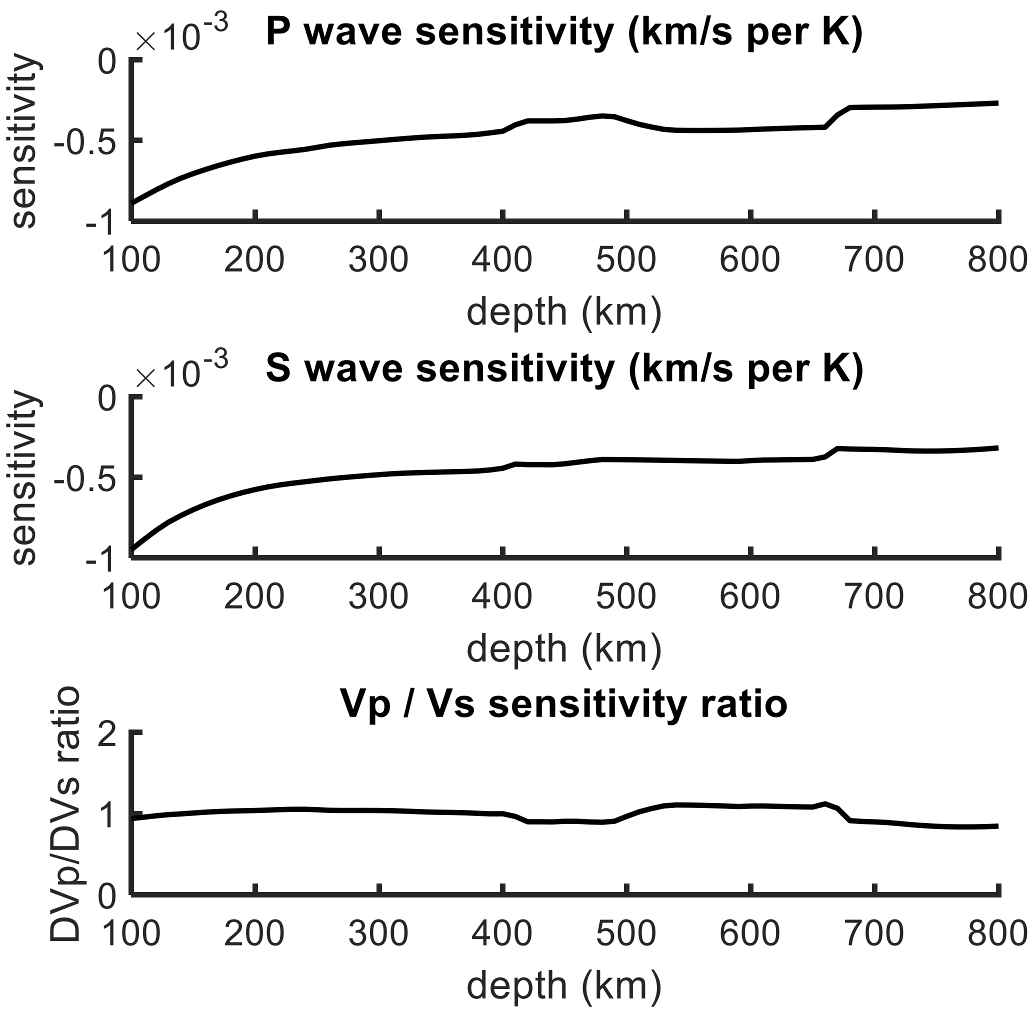

Figure 4. Compressional and shear wave sensitivities and their ratios

References

Cammarano, F., S. Goes, P. Vacher, and D. Giardini (2003), Inferring upper-mantle temperatures from seismic velocities, Phys. Earth Planet. Int., 138, 197–222, doi:10.1016/0012-821X(95)00238-8.

Kennett, B. L. N., E. R. Engdahl, and R. Buland (1995), Constraints on seismic velocities in the Earth from travel times, Geophys. J. Int., 122, 108–124, doi:10.1111/j.1365-246X.1995.tb03540.x.

Table 1: Velocities, sensitivities and the sensitivity ratio as a function of depth

depth	Vp	Vs	Sp	Ss	ratio
170	8.1887	4.5099	-6.564E-04	-6.400E-04	1.0255
180	8.2156	4.5116	-6.347E-04	-6.155E-04	1.0312
190	8.2426	4.5132	-6.148E-04	-5.941E-04	1.0349
200	8.2706	4.5153	-5.973E-04	-5.755E-04	1.038
210	8.3007	4.5184	-5.838E-04	-5.596E-04	1.0432
220	8.3339	4.5274	-5.737E-04	-5.468E-04	1.0491
230	8.3698	4.5446	-5.650E-04	-5.366E-04	1.0529
240	8.4072	4.5663	-5.557E-04	-5.275E-04	1.0534
250	8.445	4.5891	-5.430E-04	-5.181E-04	1.048
260	8.4822	4.6094	-5.300E-04	-5.094E-04	1.0404
270	8.5187	4.6271	-5.214E-04	-5.020E-04	1.0387
280	8.5552	4.6445	-5.144E-04	-4.953E-04	1.0386
290	8.5918	4.6618	-5.082E-04	-4.892E-04	1.0388
300	8.6284	4.6791	-5.018E-04	-4.834E-04	1.038
310	8.665	4.6964	-4.953E-04	-4.780E-04	1.0361
320	8.7015	4.7138	-4.889E-04	-4.742E-04	1.0311
330	8.7381	4.7311	-4.831E-04	-4.714E-04	1.025
340	8.7746	4.7485	-4.783E-04	-4.691E-04	1.0195
350	8.8111	4.7658	-4.747E-04	-4.672E-04	1.0162
360	8.8476	4.7832	-4.717E-04	-4.652E-04	1.014
370	8.8753	4.7962	-4.677E-04	-4.631E-04	1.0101
380	8.8942	4.8049	-4.618E-04	-4.602E-04	1.0034
390	8.9175	4.8158	-4.525E-04	-4.537E-04	0.99723
400	8.9585	4.8354	-4.426E-04	-4.434E-04	0.99831
410	9.0302	4.8702	-4.033E-04	-4.179E-04	0.96507
420	9.4007	5.1062	-3.787E-04	-4.210E-04	0.89947
430	9.4536	5.1396	-3.787E-04	-4.211E-04	0.89924
440	9.4819	5.1574	-3.787E-04	-4.218E-04	0.89782
450	9.5014	5.1697	-3.766E-04	-4.156E-04	0.90601
460	9.528	5.1864	-3.678E-04	-4.063E-04	0.90516
470	9.5628	5.2083	-3.562E-04	-3.968E-04	0.89759
480	9.5966	5.2295	-3.481E-04	-3.893E-04	0.89398
490	9.6298	5.2504	-3.529E-04	-3.896E-04	0.90589
500	9.6629	5.2712	-3.773E-04	-3.904E-04	0.96665
510	9.6962	5.2922	-4.002E-04	-3.915E-04	1.0222
520	9.7298	5.3135	-4.171E-04	-3.926E-04	1.0624
530	9.7633	5.3349	-4.314E-04	-3.938E-04	1.0954
540	9.7969	5.3562	-4.372E-04	-3.951E-04	1.1066
550	9.8304	5.3776	-4.382E-04	-3.964E-04	1.1053
560	9.864	5.3989	-4.383E-04	-3.976E-04	1.1024

570	9.8976	5.4201	-4.380E-04	-3.989E-04	1.098
580	9.9312	5.4413	-4.374E-04	-4.001E-04	1.0932
590	9.9648	5.4624	-4.358E-04	-4.010E-04	1.0869
600	9.9984	5.4836	-4.328E-04	-3.960E-04	1.0929
610	10.032	5.5047	-4.295E-04	-3.926E-04	1.094
620	10.058	5.5213	-4.268E-04	-3.917E-04	1.0897
630	10.077	5.5333	-4.243E-04	-3.906E-04	1.0863
640	10.1	5.5476	-4.219E-04	-3.895E-04	1.0831
650	10.137	5.571	-4.202E-04	-3.887E-04	1.081
660	10.2	5.6104	-4.180E-04	-3.731E-04	1.1203
670	10.791	5.9607	-3.420E-04	-3.211E-04	1.0652
680	10.848	6.0156	-2.958E-04	-3.239E-04	0.91332
690	10.88	6.0468	-2.942E-04	-3.256E-04	0.90355
700	10.899	6.0673	-2.934E-04	-3.267E-04	0.89806
710	10.922	6.0898	-2.929E-04	-3.289E-04	0.89055
720	10.952	6.1184	-2.920E-04	-3.328E-04	0.87743
730	10.981	6.1471	-2.899E-04	-3.358E-04	0.86324
740	11.008	6.1736	-2.870E-04	-3.369E-04	0.85208
750	11.033	6.1954	-2.839E-04	-3.367E-04	0.84325
760	11.055	6.21	-2.809E-04	-3.352E-04	0.83814
770	11.075	6.22	-2.779E-04	-3.323E-04	0.83609
780	11.093	6.2289	-2.748E-04	-3.284E-04	0.83655
790	11.109	6.2361	-2.716E-04	-3.234E-04	0.83971
800	11.124	6.2408	-2.683E-04	-3.172E-04	0.84585