Throat and pore of a tetrahedral arrangement of spheres

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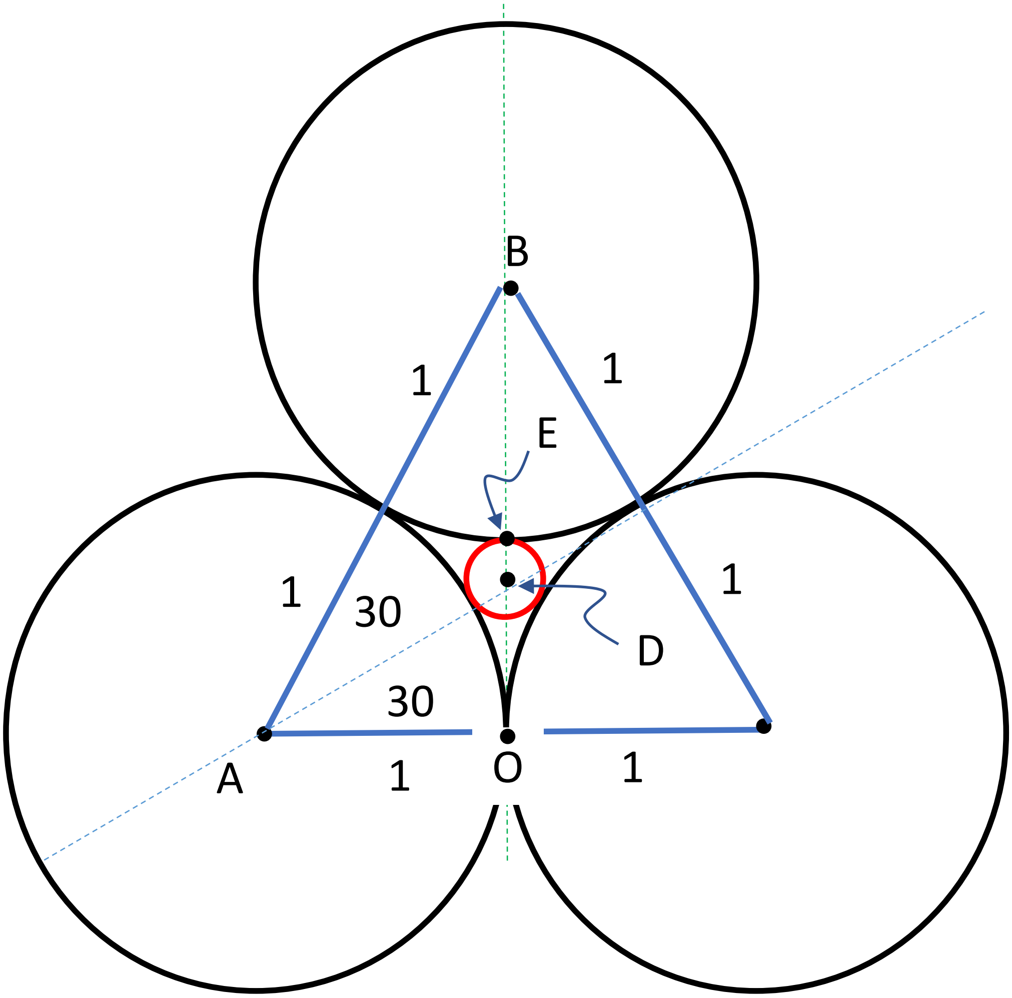
Part A: The "throat" is the largest sphere that can fit through the center of three unit spheres arranged on a plane, i.e. the radius $\overline{\mathrm{DE}}$.



$$\overline{\mathrm{OB}} = 2 \ \sin 60 = 2 \frac{\sqrt{3}}{2} = \sqrt{3}$$

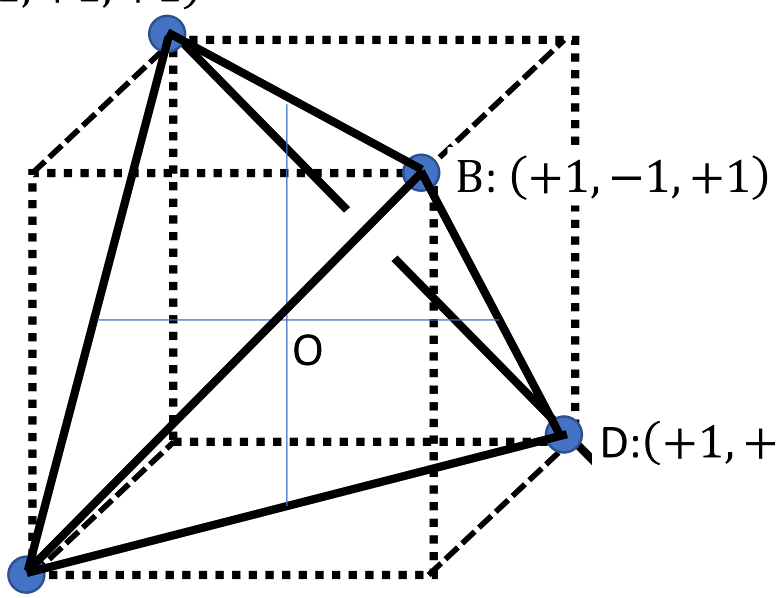
$$\overline{\mathrm{O}D} = 1 \ \tan 30 = \frac{1}{\sqrt{3}}$$

$$\overline{\mathrm{BE}} = 1$$

$$\overline{\mathrm{DE}} = \overline{\mathrm{O}B} - \overline{\mathrm{BE}} - \overline{\mathrm{O}D} = \sqrt{3} - 1 - \frac{1}{\sqrt{3}} \approx 0.155$$

Part B: The "pore" is the largest sphere that can fit inside a tetrahedral arrangement of unit spheres.

We start by constructing a tetrahedron using a cube centered on the origin and with edges of length 2



$$\text{length of edge} = \text{length } \overline{\text{CD}} = 2\sqrt{2}$$

$$\text{distance of vertex from center} = \text{length } \overline{\text{CO}} =$$
$$= \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

Now consider the fours spheres that make up the tetrahedron. The sphere centered at C and the sphere centered at D touch halfway along $\overline{\text{CD}}$,and thus have radii $R = ½ \text{ length } \overline{\text{CO}} = \sqrt{2}$.

Now consider largest sphere that fits inside the tetrahedron. It touches one of the tetrahedron spheres along $\overline{\text{CO}}$. Therefore, its radius is $r = \overline{\text{CO}} - R = \sqrt{3} - \sqrt{2}$. Rescaling for tetrahedron spheres of unit radius yields $r' = \sqrt{\frac{3}{2}} - 1 \approx 0.225$.