Example of Rolling Gaussian Process Regression

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In this example, a 30×30 image is reconstructed, using data that is acquires at the rate of 10 observrvations per time step. The true image abruptly changes from a four-lobed to a three-lobed pattern at time step 25. Two variants of the rolling process are examined. In the first (Figure 1A), the size of the rolling set of observrvations increases with time up to an upper bound of 90. The first patterm is correctly reconstructed, and then, after a period of elevated error, the second patterm is correctly reconstructed. In the second variant (Figure 2B), the posterior variance, σ_d^2, is monitored, and when it increases past a threshold, the size of the rolling set of observations is reduced, but not below a lower bound of 50. Once the error has declined below the threshold, the set size is allowed to increase, up to the upper-bound. The duration of the interval of elevated error is reducted compared to the first variant.

Image and GTR parameters. The goal of this numerical experiment is to reconstruct a two-dimensional field, $m(x,y)$, on the interval $0 \le x \le 1$, $0 \le y \le 1$, evaluated on a 30×30 grid of uniformly-spaced target points. At each time step, a total of 10 data are collected, drawn at randomly chosen points from the true function:

$$m(x,y) = \begin{cases} \sin(2\pi x)\sin(2\pi y) & (t < 25) \\ \sin(\pi x)\sin(3\pi y) & (t \ge 25) \end{cases}$$

and with variance $\sigma_d^2 = 0.01$ The field is assumed to have the Gaussian autocovariace, $C(x,x',y,y') = \exp(-½r^2/s^2)$, with $r^2 = (x-x')^2 + (y-y')^2$ and scale length $s = 0.22$.

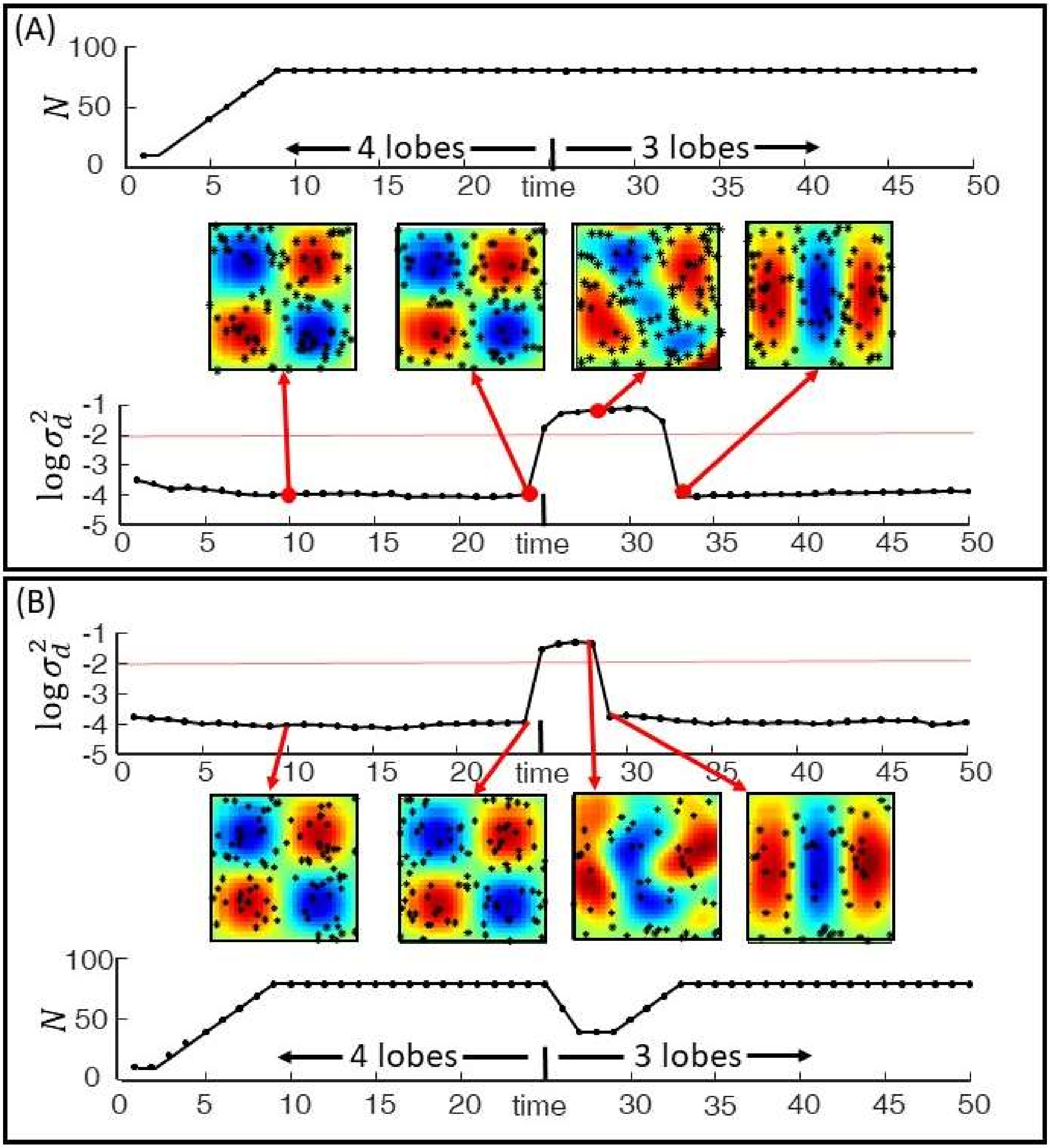

Figure 1. Example of rolling GPR. The two-dimensional field, $m(x, y)$ (colors), that is being imaged abruptly changes from a four-lobed patter to a three-lobed pattern, at time step 25. (A) Variant 1. The number, N, of data increase with time at the rate of 10 per time step, up to an upper bound of 90. The two patterns are correctly reconstructed, except during a 15-time-step interval beginning at the transition. The posterior variance, σ_d^2, is high during the transition. (B) Variant 2. When the posterior variance exceeds the threshold, $\log \sigma_d^2 > -2$ (red line), the number of data, N, is decreased at the rate of 10 per time step, to a lower bound of 50, and is allowed to increase again when the posterior error drops below the threshold. The length of the high-error interval is decreased to 9 time-steps.