The Rocket in a Barn Paradox

retold by Bill Menke, February 18, 2014

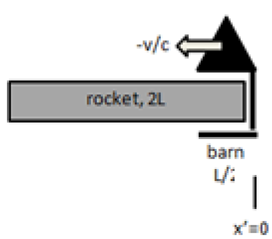
Statement of the paradox: A long rocket can fit entirely inside a short barn if it is going fast enough to be Lorentz contracted to the length of the barn. At the moment it is inside, the doors of the barn can be flicked shut, so the barn completely contains the rocket. Well and good. But wait! From the perspective of the rocket, the barn is Lorentz contracted, making it even shorter than if it had been at rest. How can the rocket fit inside?

Resolution of the paradox: From the point of view of the rocket, the barn doors do not flick shut simultaneously, rather the rear door flicks closed later than the front door. The nose of the rocket is already out of the front door when the rear door closes. The simultaneity of flicking the doors of the barn shut is not an absolute; whether they shut simultaneously or not depends upon one's perspective. And if the doors are not shut simultaneously, the rocket is not fully inside the barn. There is no paradox!

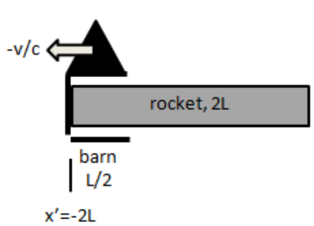
Analysis:

From the rocket's point of view (the primed frame):

Event 1. The rocket's length is 2L and it is stationary, with its nose pointing to the right. The barn is rushing towards it, right to left, at velocity, say, $-v/c = -\sqrt{3}/2$ which corresponds to $\gamma = 2$. At time $ct' = 0$ the nose of the rocket is even with the right side of the barn, at $x' = 0$. The right barn door flicks closed (and reopens):



Event 2. The barn moves a distance of $3L/2$ so that its left door is at $x' = -2$ at time $ct' = (3L/2)/(v/c)$. The left side of the barn is now even with the tail of the rocket and the left barn door flicks closed. Note that the rocket was never fully inside the barn.



We calculate the times and places of events using the Lorentz Transformations. From the barn's point of view (the unprimed frame):

The Lorentz Transformation for Event 1 is:

$$x = \gamma\left(x' + \left(\frac{v}{c}\right)ct'\right) = 0$$

$$ct = \gamma\left(ct' + \left(\frac{v}{c}\right)x\right) = 0$$

And the Lorentz Transformation for Event 2 is:

$$x = \gamma\left(x' + \left(\frac{v}{c}\right)ct'\right) = 2(-2L + (3L/2)) = -L$$

$$ct = \gamma\left(ct' + \left(\frac{v}{c}\right)x\right) = 2\left(\frac{\frac{3L}{2}}{\frac{\sqrt{3}}{2}} - \frac{\sqrt{3}}{2}2L\right) = 2\left(\sqrt{3}L - \sqrt{3}L\right) = 0$$

Thus, from the point of the view of the barn, the two events are simultaneous at time $ct = 0$ and separated by a distance L, the width of the barn. The rocket, which is of rest length 2L is Lorentz contracted to a length L in the barn's frame, so it just fits inside.