Results from Prior NSF Support, NSF OCE 02-21035; W. Menke, PI. Integrating Geophysical Data into New Axial Volcano Magma Chamber Model, 10/01/02-09/30/05; $163,658; 2 years with additional 1 year extension.

This study was directed at understanding the effects of tidal and tectonic loading on stresses within an undersea volcano, such as Axial Volcano (Juan de Fuca Ridge). Three-dimensional, time-dependent simulations of deformation were created using J. Deng’s viscoelastic finite-element code, “FEVER”. The simulation shown below, for example, examines the time-evolution of shear stress, which is taken as a proxy for seismicity, in the days following a dike intrusion. The westward (positive x) migration of the region of strong shear stress, is controlled by the interaction of magma chamber shape (an overhanging lid, in this case), and viscous relaxation of the magma. It explains the westward migration of shallow microseismicity observed after the 1998 eruption of Axial volcano. Other, comparable, simulations (not shown) explain the amplification of tidally-induced deformation observed by ocean bottom tiltmeters deployed above the Axial magma chamber.

Figure. Cross-section through the earth showing shear stress, $\tau_{xz}$, in MPa induced by a one meter opening of a vertical dike that crosses the center of the magma chamber. (Left) Initially, stress is concentrated in walls of magma chamber. (Right) One week after viscous relaxation of $\nu=10^{11}$ Pa/s magma, stress is concentrated in overhanging lid of magma chamber.

Publications arising from this Research


Note: Postdocs Hjorleifsdottir and Nettles have never served as PI’s on NSF proposals.
1. Introduction

1.1 Thesis: The earth’s crust and upper mantle contain tectonically-important regions that are strongly heterogeneous on the length scales of a few hundred kilometers. Surface wave tomography, even the newer versions that include finite-frequency effects, probably images these areas only poorly, because they are based on weak scattering theory. Methodologies are needed that can rapidly identify these regions, assess the accuracy of the imaging in their vicinity, and offer insight into aspects of the structure that the tomography might overlook.

1.2 An Example. The hotspot-derived island of Iceland is underlain by a mantle plume that is associated with a large low velocity zone in the uppermost mantle (Tryggvason et al., 1983; Wolfe et al., 1997; Allen, 2001). While the existence of the plume is well-established, its structure and depth-extent are not yet well-understood. Surface wave tomography is starting to have a significant impact on these issues (Li and Detrick, 2003) (Figure 1). The plume is shown to be a cylindrical anomaly with shear velocity fully 10% below the global average. Images like Fig. 1 are a remarkable achievement. Still, many of the geochemical and geodynamical implications of the tomography depends upon details of structure that are at the 1-2% level, not the 10% that one sees in these images. Accessing the accuracy of the tomography is not a trivial exercise, because many of the assumptions that have gone into its formulation ignore the effects of strong lateral heterogeneity.

Consider that Iceland is only 200-300 km across, and yet has very large crustal thickness variations (Menke, 1999; Allen, 2001) (Figure 2). In places, the slope of Moho exceeds 15 degrees. A Rayleigh wave that crosses Iceland (red line in Fig. 2) experiences the effect of these very strong lateral gradients in velocity. And yet the tomography assumes: 1) that a dispersion curve determined from stations at either side of the island (red squares in Fig. 2) is equivalent to the curve for the average structure between the stations; and 2) the shear wave velocity profile determined from that dispersion curve matches the average profile between the two stations. No doubt these approximations are more-or-less right. But it makes a big difference, for example to estimates of melt concentration, whether it is ±1% or ±50% right! Might some sort of resonance between the 100 km quarter-wavelength of a Rayleigh wave and the 100 km wide depression in the Moho have a biasing effect (Yoshida, 2001)? Surface wave tomography is entering into an arena where knowing the answers to this and related questions is extremely important.

Figure 1. Shear velocity in the shallow mantle beneath Iceland determined from Rayleigh waves by Li and Detrick (2003).
1.3 The Importance of Surface Wave Tomography. Observations of seismic surface waves (Rayleigh and Love waves) have had a profound impact on our understanding of the geophysics of earth’s upper mantle. Surface wave velocities are extremely sensitive to shear modulus, which itself depends on tectonically-significant parameters such as temperature, melt fraction and crystal fabric. Surface wave measurements have been at the forefront of many fundamental discoveries, such as the differences between continent and oceans, the cooling of the ocean lithosphere, the thickness of the asthenosphere, etc.

In the fifty years since surface wave velocity measurements have started to become routine, there has been a gradual evolution of the technique towards finer-and-finer spatial resolution. Efforts to image the Pacific Ocean illustrate this progression. The earliest studies (e.g. Kuo et al., 1962) treated the Pacific ocean as a single unit, and were concerned with the broad features of Rayleigh wave dispersion (that is, the variation of velocity with period), and inferences that could be made about the variation of shear modulus with depth, assuming that the whole ocean was laterally homogeneous. As high-quality seismic data became more plentiful, lateral heterogeneity was investigated either by comparing waves whose paths were confined to distinct parts of the oceans (e.g. Adams, 1964; Savage and White, 1968) or by performing regionalized inversions (e.g. Forsyth, 1972; Yu and Mitchell, 1979; Nishimura and Forsyth, 1988). In the later method, prior information from tectonics and geology was used to divide the Pacific into multiple regions, and the structure in each region was varied to fit the dispersion data. These early inversions incorporated the “path-average” approximation, in which the overall dispersion curve for an earthquake-receiver pair is assumed to be the average, weighted by path-length, of the dispersions curves of each of the regions crossed by the great circle ray path connecting earthquake and receiver. Once the period-dependent dispersion curve for a particular region is determined, it is then used to infer the variation of shear modulus with depth. Hence the procedure is a “two-step approach”, first determining laterally-varying dispersion curves, and then inverting these curves for
depth-dependent modulus. More sophisticated tomographic inversions for either the Pacific (e.g. Suetsugu and Nakanishi, 1985; Zhang and Tanimoto, 1989) or the earth as a whole (e.g. Trampert and Woodhouse, 1995) followed. They used a very large number of small regions (or their spline equivalent), thus allowing finer spatial resolution and less dependence on prior information. This early tomography used both the path-average approximation and the two-step approach. Most authors assumed great circle ray paths, but the effect of ray deflection by lateral heterogeneity was first implemented in surface wave inversion by Pollitz (1994) and is now more-or-less routine, as is the inclusion of the azimuthal variation of phase velocity due to anisotropy (e.g. Montagner and Tanimoto, 1990) and multipathing (e.g. Forsyth and Li, 2005). The most-detailed Pacific tomography, such as Forsyth et al.’s (1988) work in the MELT region, Forsyth et al.’s (2005) work in California, and Schutt and Dueker’s (2004) work in Yellowstone resolves features as small as 100 km in horizontal dimension.

The quarter wavelength of a 100-second period Rayleigh wave – a period that is sensitive to lithospheric structure – is about 100 km. In focusing on features that are of this length scale or smaller, surface wave tomography has entered the realm in which the diffraction effects are important.

1.4 The Development of Finite Frequency Tomography. Finite-frequency tomography was first developed in the context of body waves. The literature of the subject is huge, and we make no attempt to review it here. It relies on the idea that all points in the earth scatter seismic waves, so that the wavefield observed at a receiver depends on earth properties everywhere, and not just on the properties of the earth along the ray path connecting source to receiver. The earliest approaches to finite-frequency tomography used observations of the wavefield itself to directly image heterogeneity (e.g. Tarantola 1988). A perturbation method, such Born or Rytov approximation, is used to linearize the wave equation about a wave in a reference medium, and thus to relate a weak velocity heterogeneity to a weak perturbation in the wavefield. The tomography inverts this process, determining heterogeneity from the wavefield. Unfortunately, the complicated wavefields often encountered in real-world seismology have limited the application of this technique.

More recent research has moved away from a focus on wavefields and toward a focus on travel times of specific band-limited “arrivals” (e.g. P-wave, S-waves) observed in a seismogram. The key development was a method of calculating the relationship between a weak velocity heterogeneity and a perturbation in the travel time of a band-limited arrival (the so-called “Frechet derivative or kernel”) (Marquering et al, 1999; Dahlen et al., 2000; Hung et al. 2000; Tromp et al., 2005). Not every scatterer in the earth contributes equally to the perturbation. The most important scatterers lie within first Fresnel zone, that is, the ellipsoidal volume in which scattered waves arrive within a half wavelength of the wave that travels the direct path, and thus constructively interfere with it. Scatterers in higher order Fresnel zones make a smaller contribution, but as the sign of the kernel reverses between zones, they can nevertheless have important effects in some
cases. The ellipsoidal shape has led to the Frechet derivatives being nicknamed “banana-doughnut” kernels. There has been enough application of this technique to abstract two general differences of this kind of tomography, compared to standard ray-based tomography (Hung et al., 2004; Montelli et al., 2004). First, the amplitude of the recovered heterogeneity is larger, by as much as a factor or two or three; and second, the formal resolution kernels, which characterize the ability of the method to resolve small length-scale features, are wider (poorer). The later factor does not argue against the use of the method, but rather indicates that ray methods are overestimating the true ability of the data to resolve small length-scale structure.

Surface wave dispersion curves are just a way of summarizing period-dependent surface wave traveltimes. Applying the ideas of finite-frequency traveltime tomography to seismic surface waves is therefore relatively straightforward. The earliest efforts used a very simple kernel, one which was constant within the first Fresnel zone (or some fraction of it) and zero outside (e.g. Ritzwoller et al. 2002; Yoshizawa and Kennett, 2005). Subsequent efforts used more complicated kernels calculated using perturbation theory (Zhou et al., 2005). Resulting tomographic inversions are generally similar to those produced by older, ray-based tomography, but with some differences (=improvements). In particular, as noted above, the amplitude of the heterogeneity tends to be higher.

1.5 The Impact of Strong Heterogeneity. Finite-frequency tomography, at least as it is implemented today, is fundamentally based on perturbation theory, and thus assumes that weak heterogeneity. Nevertheless, a brief survey of the literature (Figs. 3-7) identifies many regions where tomography has detected rather strong lateral heterogeneities (though of course we cannot be certain that the tomography has fully-imaged these “worst-case” parts of the earth).

Figure 3. Surface wave tomography in the Barrent Sea by Levshin et al., 2006. Note strong lateral heterogeneity of 7% over horizontal distance of about 500 km.
Figure 4. North America surface wave tomography by Van der Lee and Nolet (1997). Note strong lateral heterogeneity of 10% over distances of 300-500 km across both the passive eastern and active western margins of the North American craton (green and blue region).

Figure 5. Surface wave tomography across MOMA array, eastern North America by Li et al. (2003). Profile is about 1500 km across. Top: Shear wave velocity; Bottom: shear wave anomaly. Note strong lateral gradient of 6% near center of profile.

Figure 6. Rayleigh wave phase velocities at 25 second period across the MELT array, East Pacific Rise, by Forsyth et al. (1998). Note strong lateral gradient of 8% over horizontal distance of ~200 km.

The point here is that Iceland is by no means unique in having tectonically and geodynamically important regions which are characterized by strong (>5%) lateral
gradients in mantle velocities over length scales of a few hundred kilometers. Ocean-continent boundaries, ridges and extensional zones like the US Basin and Range do, too.

Figure 7. Compressional velocity beneath Iceland based on finite-frequency body wave imaging by Hung et al. (2004). Note strong lateral gradients of about 4% over distances of 100 km.

1.6 One-theta variation of phase velocity as a detector of strong, small-scale heterogeneity. One aspect of the tomography assessment problem is to identify areas where the data suggest that strong, small-length scale heterogeneity might be present. One-theta azimuthal variation of phase velocity, estimated using a small-aperture triad array, would be one such example. Such variation has been detected along the continental margin of northeastern US (Menke and Levin, 2002) (Figure 8). It occurs when there is a lateral gradient in earth properties near the array.

Figure 8. Azimuthal variation of 40s Rayleigh phase velocity in the range in northeastern US (Menke and Levin, 2002). Symbol represents phase velocity for a single earthquake by fitting a
planar wavefront to surface waves arriving at a triad of seismometers. Different shaped symbols are for different (but geographically nearby) triads, all with an aperture of about 200 km. Note strong “one theta” variation of phase velocity of about 15-20%.

This is the surface wave analog of the well-known effect in body wave propagation, that phase velocities are biased toward high values when measurements are made “updip”, and toward low values when observed “downdip”. The effect does not require strong heterogeneity, but rather can be understood in the context of perturbation theory and banana-doughnut kernels. It results from the kernel for differential traveltime between two neighboring stations being bi-symmetrical in shape (Figure 9). The kernel interacts differently with structure local to the array as the azimuth to the earthquake changes.

![Figure 9](image)

**Figure 9.** The banana shape of the kernel implies that phase velocity, when estimated using two neighboring stations in a region of laterally-varying structure, will produce a one-theta variation in phase velocity. A) Kernel for an earthquake located due south of Station 1. B) Kernel at Station 2 for same earthquake, located 100 km south of Station 1. C) Kernel for differential traveltime. D) Velocity structure containing a lateral gradient. E) Phase velocity computed by convolving kernels for a suite of earthquakes at different azimuths with the structure in D. Note one-theta variation in estimated velocity.

1.7 Rayleigh-Love mode conversions as a detector of strong, small-scale heterogeneity (including gradients in anisotropy). The presence of quasi-Love waves -
Love waves generated by strong Rayleigh wave scattering – is also indicative in strong lateral gradients in earth properties, and especially of gradients in upper mantle anisotropy (Levin and Park, 1998; Kobayashi and Nakanishi, 1998). As shown in Figure 10, these waves sometimes have strong regional variability, which suggests that the underlying structure that causes their excitation has a short length scale.

Figure 10. Rayleigh waves from an earthquake in Indonesia, observed on an array in Italy (V. Levin, personal communication, 2006). A strong quasi-Love waves arrives just prior to the Rayleigh wave on the southern stations, but is absent on the northern stations. Levin ascribes this behavior to a strong anisotropy gradient across the 200 km wide array.

2. Proposed Research

2.1 Application of Spectral Element Synthetics. Tomography shows that strong, small-length scale heterogeneity is present in many tectonically and geodynamically important areas. Specific examples can be found where the surface wave seismograms have clear indications of interactions with small-length scale heterogeneities. But, in general, we do not have a full understanding of the overall impact of these structure on the accuracy of the current flavors of surface wave tomography (i.e. with either ray-based or banana-doughnut kernels). Existing appraisals (e.g. Kennett and Yoshizawa, 2002; Sieminski et
al. 2004), while raising interesting points, lack the “ground-truth” of credible three-dimensional synthetics.

Until recently, such as assessment was difficult because there were no good methods of calculating finite-frequency surface wave synthetics in a three-dimensional earth model that came anywhere close to having a realistic velocity structure. Approximate techniques (e.g. Meier and Malischewsky’s (2000) mode-matching) gave intriguing hints, but suffered from unassessable error. This goal was been an elusive one during the 1980’s and 1990’s, even as finite-element and finite-difference synthetics began to be usefully applied to body waves. Part of the problem was the lack of adequate computing power, which limited calculations to two-dimensions or to just Love waves, or both (e.g. Bullen and Bolt, Section 12.3.3, 1985; Yoshida, 2001. This dismal situation has completely changed in the last five years, with the development of fully three-dimensional spectral element techniques (Tromp et al., 2005; Komatitsch et al., 2005). This method has been proven useful studying surface waves in both structural (Chen et al., 2005) and earthquake source (Hjorleifsdottir et al. 2006) contexts. For example, Chen et al. (2005) successfully use the spectral element method to model 20-second period Rayleigh wave scattering from mountain ranges (Figure 11), and to quantitatively compare predictions with observations.

These same spectral element techniques can also be used to compute banana-doughnut kernels, through use of a mathematical methodology that employs adjunct operators (Tromp et al. 2005).

Figure 11. Synthetic seismogram, calculated using the spectral element method by Chen et al. 2005, of Rayleigh wave (arc crossing lower part of figure) from an Alaska earthquake crossing North America. Note the strong scattering from a region in west-central US, that leads to a series of circular wavefronts radiating outward from that region. These secondary waves were observed by the authors on arrays in western US (rectangles).
2.2 Goal of the Proposal. Our goal is to use spectral element simulations to systematically examine the ways in which surface waves interact with strong, short length scale heterogeneities and to develop a rigorous assessment of the accuracy of both ray-based and banana-doughnut based tomographic images.

2.3 Research Team. The team consists of 3 people, who collective have expertise in all necessary aspects of the problem (spectral element synthetics, surface wave interpretation, and tomography):

Bill Menke, a senior seismologist with broad experience in seismic data processing methodology, including seismic tomography, anisotropic wave propagation and time series analysis.

Vala Hjorleifsdotir, a postdoc just completing a PhD thesis in which she modified and applied Tromp’s implementation of the spectral element method to the problem of using surface waves to perform earthquake fault slip inversions (Hjorleifsdotir, Tromp and Kanamori, 2006); and

Meredith Nettles, a postdoc who recently completed a PhD thesis in which she created a new, three-dimensional shear velocity model for North America determined via surface wave tomography (Nettles, M. and A. Dziewonski, 2004; Nettles, 2005). She is broadly knowledgeable about surface wave analysis and imaging techniques.

2.4 Work Plan

Part 1. Test Cases: We plan a thorough examination of examination of specific, idealized test earth structures; including an Iceland-like hot spot, and East-Pacific Rise-like mid-ocean ridge; a NE US-like passive continental margin; an Aleutian-like subductions zone and Rocky Mountain-like orogenic belt.

Our basic approach will include:

A) Building a suite of earth models for each test case, informed by existing tomography, but with some containing features with shorter length scales that can tomography can resolve when such features are geodynamically plausible. For example, in the North American case, where tomography already indicates a sharp cratonic boundary, we will examine cases where it is extremely sharp.

B) Computing spectral element synthetics for surface waves crossing the structures in these models. We will model a full suite of source backazimuths (the density being limited by the computer power of our Beowulf cluster), so as to be able to assess the directionality of structural interactions and also to generate a synthetic dataset that is suitable for topographic inversion.

C) Examining the resultant seismograms (both by eye and through application of array processing techniques) for interesting features linked to the heterogeneity. Of particular interest will be structures that have strong one-theta dependence of phase
velocities (or two-theta patterns not related to anisotropy), that generate strong Rayleigh-Love coupling, or that otherwise are not easily interpretable under the weak-scattering assumptions inherent in tomographic inversion algorithms.

D) Performing inversions (both ray-based and banana-doughnut) on inter-station phase velocities derived from the synthetics. And

E) Assessing the accuracy of each inversion.

The main purpose of this effort is to “inform our intuition”, to understand what things can be relied upon in tomographic inversions, and what things are likely to be artifacts; and to develop specific quantitative tests (e.g. tests post-facto applied to the original seismograms) that can be used to recognize trouble spots. One byproduct will be the development of a gallery of test cases that can be used by the community to exercise tomographic imaging codes.

**Part 2. North America Assessment.** We will conduct a detailed examination of the North America continental-scale tomography, using spectral element synthetics. We will focus mainly Nettle’s tomographic model, but will also compare it to results from van dee Lee and Nolet (1997) and Li et al. (2003), in areas where these models have the most significant differences. Our general procedure will be similar to that outlined in Part 1, except that we will:

A) Examine measurement error, that is, quantify the degree to which synthetic seismograms, and synthetic phase velocity curves based on those seismograms, differ from what is observed.

B). Development geographically specific measures of error (e.g. as might be quantified using small sub-arrays of the data) and the relationship between that error and causative three dimensional structures in that region.

C). Use this knowledge to improve the North American velocity model.

**3. Management Plan**
Menke, the PI, will be responsible for the timely completion of the project. Menke, Hjorleifsdottir and Nettles, assisted by a GRA, will work on all aspects of the project.

**4. Timetable**
This research is expected to take two years, with Part 1 being completed during the first year and Part 2 during the second.
5. Dissemination of Results
We will maintain archives of data and preliminary results on our web sites, which are already quite extensive. See:

www.ldeo.columbia.edu/user/menke

www.seismology.harvard.edu/~nettles/

www.gps.caltech.edu/~vala/

We will make the test case models freely available, so that others can use them. I will present results at scientific national meetings, such as the Fall AGU, and make a best-faith effort to publish them rapidly in peer-reviewed journals.