The rheological behavior of mantle rocks determines the mechanical response of lithospheric plates as well as the nature of convective flow in the deeper mantle. Geodynamic models of these large-scale processes require constitutive equations that describe rheological properties of the appropriate mantle rocks. The forms of such constitutive equations (flow laws or combinations of flow laws) are motivated by micromechanical analyses of plastic deformation phenomena based on physical processes such as ionic diffusion, dislocation migration, and grain-boundary sliding. Critical parameters in flow laws appropriate for rock deformation such as the dependence of strain rate on stress, grain size, temperature dependence (activation energy), and pressure dependence (activation volume) must be determined from careful, well-designed laboratory experiments. Geophysical observations of the mechanical behavior of Earth’s mantle in response to changes in the stress environment due for example to an earthquake or the retreat of a glacier provide tests of the applicability of the resulting constitutive equations to deformation occurring under geological conditions.

Confident extrapolation of flow laws determined under laboratory conditions requires an understanding of the physical processes involved in deformation. Minerals and rocks deform plastically by a number of different mechanisms, each of which requires defects in the crystalline structure. Diffusion creep involves the movement of atoms or ions and thus of point defects; dislocation creep entails the glide and climb of line defects; grain-boundary sliding necessitates motion along planar defects. Within any deformation regime, more than one deformation process may be important. For example, diffusion creep can be divided into a regime dominated by diffusion along grain boundaries and another dominated by diffusion through grain interiors (the so-called grain matrix). Grain size, temperature, and pressure dictate which of these two processes is more important. Likewise, in dislocation creep, deformation can be divided into low-temperature and high-temperature regimes, with glide controlling the rate of deformation in the
former and climb in the latter. As temperature decreases, differential stress necessarily increases if deformation is to continue at a given rate.

Plastic deformation is a thermally activated, kinetic, irreversible process. Experimentalists frame laboratory investigations of plastic deformation in terms of strain rate as a function of differential stress, temperature, pressure, grain size, and other thermomechanical and structural parameters. In contrast, geodynamicists cast plasticity in terms of viscosity, the ratio of stress to strain rate. In both cases, flow laws or constitutive equations describe the relationships between these parameters, thus providing the basis for extrapolating from laboratory to Earth conditions. Tests of the appropriateness of laboratory-derived flow laws to processes occurring in Earth at much slower strain rates (higher viscosities) – and thus necessarily much lower stresses and/or much lower temperatures and/or much coarser grain sizes – include comparisons of microstructures observed in experimentally deformed samples with those in naturally deformed rocks and of viscosities derived from laboratory experiments with those determined from geophysical observations.

The present chapter, therefore, starts with an examination of the role of defects in plastic deformation, including a discussion of the influence of water-derived point defects on kinetic processes. Next, a section on some of the important mechanisms of deformation introduces constitutive equations that describe plastic flow by linking thermomechanical parameters such as strain rate, differential stress, temperature, pressure, and water fugacity with structural parameters such as dislocation density, grain size, and melt fraction. Finally, profiles of viscosity versus depth in the upper mantle are used to test the applicability of flow laws determined from micromechanical models and laboratory experiments to plastic flow taking place deep beneath Earth’s surface.

### 2.14.2 Role of Lattice Defects in Deformation

Defects in rocks are essential for plastic deformation to proceed. Zero-dimensional or point defects (specifically vacancies and interstitials) allow flow by diffusive transport of ions, one-dimensional (1-D) or line defects (dislocations) permit deformation by glide and climb, and 2-D or planar defect (grain–grain interfaces) facilitate deformation by grain-boundary sliding and migration. More than one type of defect may be simultaneously involved in deformation. For example, diffusion creep involves not only diffusion but also grain-boundary sliding (Raj and Ashby, 1971), dislocation processes also frequently couple with grain-boundary sliding (Langdon, 1994), and dislocation creep necessitates glide on several slip systems often combined with climb (von Mises, 1928; Groves and Kelly, 1969; Kelly and Groves, 1969; Paterson, 1969). In addition, dynamic recrystallization frequently acts as a recovery mechanism, producing a new generation of dislocation-free grains that are more easily deformed than the parent grains (e.g., Tullis and Yund, 1985; de Bresser et al., 2001; Drury, 2005). In this section, some fundamental aspects of point defects, dislocations, and grain boundaries are introduced as background for discussing the rheological behavior of rocks and the associated constitutive equations.

#### 2.14.2.1 Point Defects – Thermodynamics and Kinetics

##### 2.14.2.1.1 Thermodynamics

At temperatures above absolute zero, in thermodynamic equilibrium crystalline grains contain finite populations of vacancies and self-interstitials. Although the enthalpy of a crystal increases linearly with the addition of vacancies or self-interstitials, the entropy decreases nonlinearly. Hence, the Gibbs free energy of a crystal is minimized not when the crystal is perfect (i.e., defect free) but rather when the crystal contains a finite concentration of vacancies and self-interstitials. The charge neutrality condition is then approximated by
equating the concentration of the positively charged to the concentration of the negatively charged majority point defects. An example serves to illustrate the salient elements of point defect thermodynamics.

Consider the case of a simple metal oxide, MeO, where Me is an ion such as Mg, Ni, Co, Mn, or Fe. Structural elements for a system include MeMe\(^{x-}\), Me\(^{+}\), Me, \(V_0\), \(V_{Me}\), and \(O_2^\cdot\). The first two entries are termed 'regular structural elements' and the last five are called 'irregular structural elements' or point defects in the ideal crystal lattice. The Kro\"ger–Vink (1956) notation is used to indicate the atomic species, A, occupying a specific crystallographic site, S, and having an effective charge, C, relative to the ideal crystal lattice: \(A^C_S\). In this nomenclature, the symbols \(\times\), \(\cdot\), and \(\dag\) indicate neutral, one positive, and one negative effective charges. Vacant sites are denoted by a V, and ions located at interstitial sites are indicated by a subscript i. The defect Me\(^{+}\) corresponds to a 3+ ion sitting at a site normally occupied by a 2+ ion, such as a ferric iron in a ferrous iron site. In a strict sense, charge neutrality is given by

\[
[\text{Me}^{+}_\text{Me}] + 2[V^0_\text{O}] + 2[\text{Me}^\cdot] = 2[V^0_\text{Me}] + 2[O^\cdot] \tag{1a}
\]

where the square brackets, [ ], indicate molar concentration. In a Mg-rich system, one possible disorder type is the combination of Me\(^{+}\) and \(V^0_\text{Me}\) (Catlow et al., 1994), such that the charge neutrality condition is approximated as

\[
[V^0_\text{Me}] = [\text{Me}^{+}_\text{Me}] \tag{1b}
\]

In contrast, in an Fe-rich quasi-binary material such as (Mg, Fe)O, likely majority point defects are \(F^{0}_\text{Me}\) and \(V^0_\text{Me}\) (Hilbrandt and Martin, 1998), yielding the charge neutrality condition

\[
[\text{Fe}^{0}_\text{Me}] = 2[V^0_\text{Me}] \tag{1c}
\]

where \(\text{Fe}^{0}_\text{Me}\) is often written as \(h^+\), indicating a highly mobile electron hole localized at a Fe\(^{3+}\) site.

To obtain expressions that give the dependences of point defect concentrations on thermodynamic variables such as temperature, pressure, and component activities, reaction equations are needed. The first reaction equation should involve the majority point defects. In the case of the disorder type given by eqn [1b], the appropriate reaction starts with a regular structural element resulting in

\[
\text{Me}^{x-}_\text{Me} \rightleftharpoons \text{Me}^{+}_\text{Me} + V^0_\text{Me} \tag{2}
\]

for which the law of mass action yields

\[
a_{\text{Me}^{+}Me} a_{\text{Me}^{0}Me} = a_{\text{Me}^{x-}Me} K_2 \tag{3a}
\]

where \(K_2\) is the reaction constant for the reaction in eqn [2]. Since the activity of \(\text{Me}^{x-}_\text{Me}\) is little affected by the presence of a small concentration of point defects (typically \(<10^{-5}\)), \(a_{\text{Me}^{x-}Me} \approx [\text{Me}^{x-}_\text{Me}] \approx 1.\) Likewise, for ideally dilute solutions of point defects, the activities of the point defects can be replaced by their concentrations such that eqn [3a] becomes

\[
[\text{Me}^{+}_\text{Me}] [V^0_\text{Me}] = K_2 = \exp \frac{-G_F}{2RT} \tag{3b}
\]

where \(G_F\) is the Gibbs energy for the reaction in eqn [2] and RT has the usual meaning. This reaction describes the formation of the so-called Frenkel point defects (a vacancy–interstitial pair). If the charge neutrality condition in eqn [1b] and the mass action equation in eqn [3b] are combined, the dependence on temperature and pressure of the concentrations of the Frenkel defects is

\[
[\text{Me}^{+}_\text{Me}] = [V^0_\text{Me}] = \sqrt{K_2} = \exp \frac{-G_F}{2RT} \tag{4}
\]

Frenkel majority point defects are an example of a thermal disorder type, that is, the concentrations of these point defects depend on temperature and pressure but not on the activities of the components. A second example of thermal disorder occurs if Schottky point defects, \(V^0_\text{Me}\) and \(V^0_\text{O}\), are the majority point defects.

For the disorder type given by eqn [1c], the appropriate reaction involves a regular structural element plus a neutral crystal component to yield

\[
\frac{1}{2} O_2(\text{osp}) + 2\text{Fe}^{0}_\text{Me} + \text{Me}^{x-}_\text{Me} \rightleftharpoons 2\text{Fe}^{+}_\text{Me} + V^0_\text{Me} + \text{MeO}(\text{osp}) \tag{5}
\]

where the oxygen is provided from a site of repeatable growth (osp) such as a dislocation, grain boundary, or crystal–gas interface (surface). The law of mass action yields

\[
[\text{Fe}^{0}_\text{Me}]^{2} [V^0_\text{Me}] a_{\text{MeO}} = [\text{Fe}^{0}_\text{Me}]^{2} [\text{Me}^{x-}_\text{Me}] f^{1/2} \tag{6}
\]

where \(f^{0}_O\) is the oxygen fugacity. If the charge neutrality condition in eqn [1c] is now substituted into eqn [6] and the approximation \([\text{Me}^{x-}_\text{Me}] \approx a_{\text{MeO}} = 1\) is made, then

\[
[\text{Fe}^{0}_\text{Me}] = 2[V^0_\text{Me}]
\]

\[
= \sqrt{2} [\text{Fe}^{0}_\text{Me}]^{2} f^{1/6} K_3^{1/2}
\]

\[
= \sqrt{2} \left[\text{Fe}^{0}_\text{Me}\right]^{2} f^{1/6} \exp \frac{-G_5}{3RT} \tag{7}
\]

Thus, if a neutral crystal component is added from a site of repeatable growth, the concentrations of the
majority point defects will depend not only on pressure and temperature but also on component activities, often referred to as an activity-dependent disorder type. In this case, it should be noted that the material will be non-stoichiometric with typically $[V_{Me}^\nu] \gg [V_O^-]$. Also, a comparison of eqn [4] with eqn [7] demonstrates that the form as well as the magnitude of the concentration of $V_{Me}^\nu$ depends on the charge neutrality condition.

Since $[V_O^-]$ is generally several orders of magnitude smaller than $[V_{Me}^\nu]$, O diffuses much more slowly than Me, at least through the interiors of grains. Consequently, as discussed below, the rate of diffusion creep (specifically, Nabarro–Herring creep) and of dislocation climb are limited by the rate of O diffusion. Hence, it is important to examine the dependence of the concentration of $V_O^-$ on temperature, pressure, and oxygen fugacity. Since the dependence of the concentration of $V_{Me}^\nu$ on these parameters has been determined above, formulation of a reaction involving $V_O^-$ and $V_{Me}^\nu$ provides a good starting point. The point defects $V_O^-$ and $V_{Me}^\nu$ are related via the Schottky formation reaction

$$\text{Me}_{Me}^\nu + O_O^+ \rightleftharpoons V_{Me}^\nu + V_O^- + \text{MeO} \quad [8]$$

It should be emphasized that it is essential in reaction equations such as eqn [8] that the number and identity of the atomic species, the crystallographic sites, and the effective charges (recall $A^\nu$) be the same on the two sides of the reaction equation. The law of mass action for eqn [8] reads

$$[V_{Me}^\nu][V_O^-] = k_s = \exp \left( -\frac{G_8}{RT} \right) \quad [9]$$

again using the approximations $\alpha_{Me_{Me}}^\nu \approx \alpha_{O_O}^\nu \approx 1$ with $\alpha_{MeO}$ equal to unity since MeO is present. For the Frenkel thermal disorder type given by eqn [1b], $[\text{Me}_{Me}^\nu] = [V_{Me}^\nu]$, if eqn [9] is now combined with eqn [4], then

$$[V_O^-] = [V_{Me}^\nu]^{-1} k_s = \frac{k_s}{\sqrt{k_5}} = \exp \left( -\frac{(G_8 - G_6/2)}{RT} \right) \quad [10]$$

For the activity-dependent disorder type given by eqn [1c], $[\text{Fe}_{Me}^\nu] = 2[V_{Me}^\nu]$, if eqn [9] is combined with eqn [7], then

$$[V_O^-] = [V_{Me}^\nu]^{-1} k_s = \left( \sqrt{4}[\text{Fe}_{Me}^\nu]^{2/3} f_{O_2}^{-1/6} \frac{K_8}{K_5^{1/3}} \right)^{-1/3} = \left( \sqrt{4}[\text{Fe}_{Me}^\nu]^{2/3} f_{O_2}^{-1/6} \exp \left( -\frac{(G_5 - G_6/3)}{RT} \right) \right) \quad [11]$$

Under hydrous conditions, additional point defects must be considered. Hydrogen ions, that is, protons, p occupy interstitial sites, $H_i$, near oxygen ions thus corresponding to the point defects $(OH)O$. This water-derived point defect can be introduced through dissociation of a water molecule by the reaction

$$H_2O(srg) + 2O_O^+ + Me_{Me}^\nu \rightleftharpoons 2(OH)O^+ + V_{Me}^\nu + MeO(srg) \quad [12]$$

In addition, point defect associates such as those produced between $(OH)O$ and $V_{Me}^\nu$ can form

$$(OH)O^+ + V_{Me}^\nu \rightleftharpoons \{(OH)O^+ - V_{Me}^\nu \} \quad [13]$$

where the curly brackets $\{} \$ indicate a defect associate. This and other defect associates as well as $(OH)O$ must now be included in the charge neutrality equation, eqn [1]. New disorder types such as

$$[(OH)O] = \left( \{(OH)O - V_{Me}^\nu \} \right) \quad [14]$$

and

$$[\text{Fe}_{Me}^\nu] = \left( \{(OH)O - V_{Me}^\nu \} \right) \quad [15]$$

must also be considered. A summary of the dependencies of the concentrations of various point defects on oxygen fugacity and water fugacity for the system $\text{MeO}$ is provided in Table 1 for a range of charge neutrality conditions.

### 2.14.2.1.2 Kinetics

Diffusion of atoms and ions through crystalline solids takes place by movement of point defects, namely, vacancies and self-interstitials. Here the discussion focuses on vacancies; a parallel analysis applies for self-interstitials. Since the fraction of vacant sites, $X_V$, on any specific sublattice is small, typically $< 10^{-3}$ to $< 10^{-5}$, vacancies diffuse much more rapidly than the ions on that sublattice. From the point of view of a vacancy, all of the neighboring sites are available, while from the perspective of an ion, possibly one but more likely no sites are available at any particular moment. This connection between the movement of ions and that of vacancies leads to the following relationship between the diffusion coefficient for ions, $D_{ion}$, and that for vacancies, $D_V$:

$$X_{ion} D_{ion} = X_V D_V \quad [16a]$$

where $X_{ion}$ is the fraction of sites on a given sublattice occupied by the ions associated with that sublattice.
Constitutive Equations, Rheological Behavior, and Viscosity of Rocks

Table 1

<table>
<thead>
<tr>
<th>Charge neutrality</th>
<th>$Me_{Me}^q$</th>
<th>$V_{Me}$</th>
<th>$V_O$</th>
<th>$(Me_{Me} - V_{Me})^q$</th>
<th>$(OH)_O$</th>
<th>$(OH)<em>O - V</em>{Me}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Me_{Me}^q = 2V_{Me}$</td>
<td>1/6, 0</td>
<td>1/6, 0</td>
<td>1/6, 0</td>
<td>1/3, 0</td>
<td>1/12, 1/2</td>
<td>1/12, 1/2</td>
</tr>
<tr>
<td>$[(OH)<em>O = 2V</em>{Me}]$</td>
<td>1/4, –1/6</td>
<td>0, 1/3</td>
<td>0, –1/3</td>
<td>1/4, 1/6</td>
<td>0, 1/3</td>
<td>0, 2/3</td>
</tr>
<tr>
<td>$Me_{Me}^q = [(OH)<em>O - V</em>{Me}]$</td>
<td>1/8, 1/4</td>
<td>1/4, –1/2</td>
<td>1/4, 1/2</td>
<td>3/8, –1/4</td>
<td>1/8, 3/4</td>
<td>1/8, 1/4</td>
</tr>
<tr>
<td>$[(OH)_O = (OH)<em>O - V</em>{Me}]$</td>
<td>1/4, 0</td>
<td>0, 0</td>
<td>0, 0</td>
<td>1/4, 0</td>
<td>0, 1/2</td>
<td>0, 1/2</td>
</tr>
</tbody>
</table>

Since $X_{ion} = (1 - X_V) \approx 1$, eqn 16a is usually approximated as

$$D_{ion} \approx X_V D_V \quad [16b]$$

such that

$$D_{ion} \ll D_V \quad [16c]$$

that is, vacancies diffuse orders of magnitude faster than ions because $X_V \ll 1$.

One consequence of the rapid diffusion of vacancies (as well as of electron holes) is that equilibrium vacancy concentrations are attained relatively rapidly in response to changes in oxygen partial pressure or oxide activity, typically in much less than an hour under laboratory conditions for which diffusion distances are on the scale of the sample size (Mackwell et al., 1988; Wanamaker, 1994). Point defect concentrations respond even more quickly to changes in temperature and pressure for which the appropriate diffusion length is the distance between sites of repeatable growth, which serve as sources and sinks for point defects.

The concentration of vacancies in a mineral will differ from one sublattice to the next. In a system composed of $Me = (Mg, Fe)$, Si, and O, the concentration of vacancies on the Me sublattice is generally several orders of magnitude larger than the concentrations on the Si and O sublattices, that is, $[V_{Me}^p] \gg [V_{Si}^p]$, $[V_{Si}^p]$ (e.g., Nakamura and Schmalzried, 1983, 1984; Tsai and Dieckmann, 2002). As a result, $D_{Me} >> D_{Si}$.

For deformation by diffusion creep or dislocation climb processes, the flux of ions directly enters expressions for the strain rate. The deformation rate is calculated from analyses of the flux of ions between regions experiencing different stress states, such as between grain boundaries in a rock that are oriented normal to and those that are oriented parallel to the maximum principal stress. In an ionic solid, the fluxes of all of the constituent ions must be coupled in order to maintain stoichiometry (Rouff, 1965; Readey, 1966; Gordon, 1973; Dimos et al., 1988; Jaoul, 1990; Schmalzried, 1995, pp. 345–346). As a consequence, the creep rate of a monomineralic rock is controlled by the rate of diffusion of the slowest diffusing ion.

In the case of olivine, $Me_2SiO_4$, for diffusion along parallel paths (i.e., 1-D diffusion), the fluxes of $Me^{2+}$, $Si^{4+}$, and $O^{2-}$ are given by (e.g., Schmalzried, 1981, p. 63; Schmalzried, 1995, pp. 78–82)

$$\dot{J}_{Me^{2+}} = - \frac{D_{Me} C_{Me}}{RT} \frac{\partial \mu_{Me^{2+}}}{\partial \xi} + \frac{2F \partial \phi}{\partial \xi} \quad [17a]$$

$$\dot{J}_{Si^{4+}} = - \frac{D_{Si} C_{Si}}{RT} \frac{\partial \mu_{Si^{4+}}}{\partial \xi} + \frac{4F \partial \phi}{\partial \xi} \quad [17b]$$

and

$$\dot{J}_{O^{2-}} = - \frac{D_{O} C_{O}}{RT} \frac{\partial \mu_{O^{2-}}}{\partial \xi} - \frac{2F \partial \phi}{\partial \xi} \quad [17c]$$

where $j$ is the flux, $D$ the self-diffusivity, $C$ the concentration, and $\eta$ the electrochemical potential of the appropriate ionic species. The electrochemical potentials for the various ions are written in terms of the chemical potential, $\mu$, and the electrical potential, $\varphi$, as (e.g., Schmalzried, 1981, p. 63)

$$\eta = \mu + zF \varphi \quad [18]$$

where $z$ is the charge on the ion and $F$ is the Faraday constant. To maintain stoichiometry during diffusion-controlled creep in which transport of entire lattice molecules must occur between sites of repeatable growth such as dislocations and grain boundaries, the flux coupling condition is

$$\frac{\dot{J}_{Me^{2+}}}{C_{Me}} = \frac{\dot{J}_{Si^{4+}}}{C_{Si}} = \frac{\dot{J}_{O^{2-}}}{C_{O}} \quad [19a]$$
Equation [19a] applies specifically to the case in which the all of the ions diffuse along a single path or parallel paths; the more general case in which diffusion occurs along multiple paths (e.g., along grain boundaries and through grain interiors) is treated in Dimos et al. (1988; see also Kohlstedt, 2006).

The flux coupling equation provides a means of eliminating the $\frac{\partial \mu_{Me^+}}{\partial \xi} + 2F \frac{\partial \phi}{\partial \xi}$ term from the flux equations. The flux coupling condition for olivine in eqn [19a] can be rewritten as

$$2j_{Me^+} = 4j_{O^-} = j_{O^+}$$

Then, since $D_{Me} \gg D_{O} > D_{Si}$, the approximation

$$\frac{\partial j_{Me^+}}{\partial \xi} = \frac{\partial \mu_{Me^+}}{\partial \xi} + 2F \frac{\partial \phi}{\partial \xi} \approx 0$$

must hold in order to satisfy the flux coupling condition given by eqn [19b]. Using the implicit assumption of local thermodynamic equilibrium, the chemical potentials of the ions can be written in terms of the chemical potentials of the oxides as

$$\mu_{Me^+} + \mu_{O^+} = \mu_{MeO}$$

and

$$\mu_{Si^+} + 2\mu_{O^-} = \mu_{SiO_2}$$

Thus, the flux coupling condition in eqn [19b] leads to the relation

$$D_{Si} \frac{\partial \mu_{SiO_2}}{\partial \xi} = (D_{O} + 2D_{Si}) \frac{\partial \mu_{MeO}}{\partial \xi}$$

The flux of Si$^{4+}$, the slowest ionic species, then becomes

$$j_{Si^{4+}} = -\frac{C_{Si}D_{Si}}{RT} \left( \frac{\partial \mu_{SiO_2}}{\partial \xi} + 2 \frac{\partial \mu_{MeO}}{\partial \xi} \right)$$

Since the chemical potential of olivine is

$$\mu_{Me_{2}SiO_4} = 2\mu_{MeO} + \mu_{SiO_2}$$

the flux of Si$^{4+}$ is

$$j_{Si^{4+}} = -\frac{C_{Si}D_{Si}}{RT} \frac{D_{O}}{D_{Si}} \frac{\partial \mu_{Me_{2}SiO_4}}{\partial \xi}$$

The chemical potential of olivine contains an activity, $a$, and a stress, $\sigma$, term such that

$$\frac{\partial \mu_{Me_{2}SiO_4}}{\partial \xi} = RT \frac{\partial \ln a_{Me_{2}SiO_4}}{\partial \xi} - V_{Me_{2}SiO_4} \frac{\partial \sigma}{\partial \xi}$$

where $V_{Me_{2}SiO_4}$ is the molar volume of olivine (e.g., Ready, 1966; Gordon, 1973; Dimos et al., 1988; Schmalzried, 1995, p. 334). The first term on the right-hand side of eqn [24] can be expressed in terms of the gradient in the concentration of vacancies on the Me sublattice (Dimos et al., 1988; Jaoul, 1990) as

$$RT \frac{\partial \ln a_{Me_{2}SiO_4}}{\partial \xi} \approx -2RT \frac{\partial \mu_{Me}}{\partial \xi} \approx -2(V_{Me}^{\prime})_0 F_{Me_{2}SiO_4} \frac{\partial \sigma}{\partial \xi}$$

where $[(V_{Me}^{\prime})_0]$ is the concentration of Me vacancies under hydrostatic stress conditions. The gradient in the concentration in metal vacancies results from the gradient in normal stress between sources for vacancies (regions of minimum compressive stress) and sinks for vacancies (regions of maximum compressive stress). Since $[(V_{Me}^{\prime})_0]$ is small, $< 10^{-3}$, the second term in eqn [24] dominates such that

$$\frac{\partial \mu_{Me_{2}SiO_4}}{\partial \xi} \approx -V_{Me_{2}SiO_4} \frac{\partial \sigma}{\partial \xi}$$

Therefore, if $D_{O} \gg D_{Si}$

$$j_{Si^{4+}} = \frac{C_{Si}D_{Si}}{RT} \frac{D_{O}}{D_{Si}} \frac{\partial \mu_{Me_{2}SiO_4}}{\partial \xi}$$

### 2.14.2.2 Line Defects – Structure and Dynamics

Dislocations are line defects that separate regions of a crystal that have slipped from those that have not slipped. Their motion by glide and climb is central to deformation of most crystalline solids, much like point defects are essential for diffusion. Unlike point defects, dislocations are not equilibrium defects. While the change in enthalpy associated with the formation of a dislocation is quite large, the change in entropy is relatively small. In a simplistic analogy, the former might be considered equal to the change in enthalpy due to the formation of of $N$ vacancies, where the vacancies form a row of the same length as the dislocation line. In contrast, the change in entropy resulting from the formation of $N$ vacancies constrained to lie along a line will be significantly less than that associated with the formation of $N$ vacancies randomly distributed in a crystal. The result is that, at equilibrium, a crystal is expected to be dislocation free (Devereux, 1983, pp. 368–372).

A dislocation is characterized by two vector quantities, its line direction, $\ell$, which defines the direction of the dislocation at each point along its length, and a displacement or Burgers vector, $b$, which defines the displacement of the lattice produced as the dislocation moves through a crystal. For a dislocation loop such as illustrated in Figure 1, the tangential line
Direction changes from one point to the next along the loop, while the Burgers vector is the same at every point. Regions along the dislocation loop for which \( \mathbf{b} \) is perpendicular to \( \ell \) are termed edge segments, while regions for which \( \mathbf{b} \) is parallel to \( \ell \) are termed screw segments. Regions of a dislocation for which \( \mathbf{b} \) and \( \ell \) are neither perpendicular nor parallel are referred to as mixed segments.

Movement of a dislocation can take place by glide and climb. An edge dislocation can move in one of two ways. If an edge dislocation moves in the glide plane defined by \( \mathbf{b} \times \ell \), its motion is conservative. In contrast, if an edge dislocation moves normal to its glide plane, the process is nonconservative in that lattice molecules are added to or taken away from the dislocation line by diffusion such that the dislocation climbs out of its glide plane. A screw dislocation does not have a specific glide plane since \( \mathbf{b} \) and \( \ell \) are parallel to each other. Hence, a screw dislocation generally glides on the plane that offers least resistance to its motion. If it encounters an obstacle to its movement, a screw dislocation can cross slip off of its original glide plane onto a parallel glide plane in order to continue moving. A dislocation must always move in a direction perpendicular to its line direction. The displacement resulting from glide of an edge dislocation is thus parallel to the direction of dislocation glide, while the displacement resulting from movement of a screw dislocation is perpendicular to its direction of motion defined by the velocity vector, \( \mathbf{v} \), as illustrated in Figure 2. Finally, a slip system is defined for a dislocation by the combination of a unit vector normal to the slip plane, \( \mathbf{n} \), and \( \mathbf{b} \).

Slip generally occurs on the closest packed plane that contains \( \mathbf{b} \), that is, the most widely separated

\[ E_{\text{disl}} = \frac{G\mathbf{b}^2}{4\pi(1-v)} \ln \frac{r}{r_c} \]  

where \( G \) is the shear modulus, \( r \) is the mean spacing between dislocations, and \( r_c \) is the radius of the dislocation core. In eqn [28], the contribution of the core of the dislocation to the elastic strain energy has been neglected. For elastically anisotropic materials, the full matrix of elastic constants must be considered, as discussed in detail by Hirth and Lothe (1968, pp. 398–440).

Slip generally occurs on the closest packed plane that contains \( \mathbf{b} \), that is, the most widely separated.

**Figure 1** Sketch of dislocation loop indicating the direction of the Burgers vector, \( \mathbf{b} \), and of the dislocation line, \( \ell \), at each point along the loop. The edge and screw segments of the dislocation loop are denoted by \( e \) and \( s \), respectively.

**Figure 2** Sketch illustrating that (a) the movement of an edge dislocation, \( e \), and (b) the movement of a screw dislocation, \( s \), with the same Burgers vector (c) produce the same displacement of the upper half of a crystal relative to the lower half. Adapted from Kelly A and Groves GW (1970) *Crystallography and Crystal Defects*, 428 pp. Reading, MA: Addison-Wesley Publishing Company.
planes and thus, in general, the most weakly bonded planes. In olivine, \([100] < [001] < [010]\) suggesting that \(\mathbf{b} = [100]\) should dominate. However, plastic deformation of a polycrystalline material cannot proceed without opening up void space if only one or two independent slip systems operate. Homogeneous plastic deformation of a rock requires dislocation glide on five independent slip systems (von Mises, 1928). This constraint is relaxed to glide on four independent slip systems if deformation is allowed to be heterogeneous (Hutchinson, 1976). If dislocations both glide and climb, homogeneous deformation necessitates the operation of just three independent slip systems (Kelly and Groves, 1969). Processes such as grain-boundary sliding, ionic diffusion, and twinning can also relax the von Mises criterion.

Glide and climb of dislocations involve steps along the dislocation line. Steps that lie in the glide plane, kinks, facilitate glide. Steps out of the glide plane, jogs, facilitate climb.

Dislocations interact with one another and with an applied stress field through the stress field that arises due to the elastic distortion caused by the presence of each dislocation. The stress fields associated with edge and screw dislocations in an elastically isotropic and elastically anisotropic media are derived in Hirth and Lothe (1968, pp. 398–440) and in Weertman and Weertman (1992, pp. 22–41). The force on a dislocation is then calculated using the Peach–Koehler equation (Peach and Koehler, 1950). In its simplest form, the force per unit length, \(f\), on a dislocation is

\[
f = \sigma \mathbf{b}
\]

where the stress \(\sigma\) arises, for example, from a neighboring dislocation or an external force.

### 2.14.2.3 Planar Defects – Structure and Energy

Planar defects include twin boundaries, stacking faults, grain boundaries, and interphase boundaries. The emphasis in this chapter is on grain boundaries and interphase boundaries, because of the important role of these two types of interfaces in plastic deformation.

If the misorientation across a grain boundary is not too large, say, \(<15^\circ\) (i.e., a low-angle boundary), the interface can be constructed from a periodic array of dislocations. A low-angle tilt boundary consists of a series of parallel edge dislocations, as illustrated schematically in Figure 3 and imaged in Figure 4. In the case of a low-angle tilt boundary, the spacing between dislocations, \(h\), can be expressed in terms of the misorientation angle, \(\vartheta\), across the boundary as

\[
h = \frac{\mathbf{b}}{2\sin(\vartheta/2)}
\]

For a small misorientation, the misorientation angle can be approximated by

\[
\vartheta \approx \frac{b}{b}
\]

Figure 3 Sketch of a low-angle tilt boundary formed by a series of periodically spaced edge dislocations, which are denoted by the inverted “T” symbols. Adapted from Read WT Jr (1953) Dislocation in Crystals, 175pp. New York: McGraw-Hill Book Company.

Figure 4 Bright-field transmission electron micrograph of a low-angle tilt boundary in olivine. Two sets of dislocations, one with \(\mathbf{b} = [100]\) and the other with \(\mathbf{b} = [101]\), are present in this boundary. The darker \(\mathbf{b} = [101]\) dislocations are more widely spaced than the lighter, periodically spaced \(\mathbf{b} = [100]\) dislocations. This image was taken with the diffraction condition \(\mathbf{g} = (002)\), hence the \(\mathbf{b} = [100]\) dislocations exhibit only residual contrast. Adapted from Goetze C and Kohlstedt DL (1973) Laboratory study of dislocation climb and diffusion in olivine. Journal of Geophysical Research 78: 5961–5971.
Starting with eqn [28], the elastic strain energy per unit area of a low-angle tilt boundary can then be derived (Read and Shockley, 1950)

\[
E_{\text{tilt}} = \frac{Gb^2}{4\pi(1-v)} \left( A - \frac{1}{2} \ln \frac{\theta}{\lambda} \right) \tag{31}
\]

where the parameter \( A \) accounts for the fraction of the elastic strain energy associated with the cores of the dislocations. Once the cores of the dislocations in a tilt boundary begin to overlap, this simple description of a grain boundary breaks down.

A low-angle twist boundary requires two, generally orthogonal, sets of screw dislocations. The elastic energy per unit area for a low-angle twist boundary is similar to that for a low-angle tilt boundary. A sketch of the atomic structure for a low-angle twist boundary formed by rotating the lattices of two grains relative to each other through an angle \( \theta \), thus forming orthogonal sets of dislocations, is presented in Figure 5. A transmission electron micrograph of a twist boundary is shown in Figure 6.

As illustrated in Figure 7, grain-boundary energy increases rapidly with increasing \( \theta \), as described by eqn [31], up to \( \theta \approx 20^\circ \). At larger values of \( \theta \), grain-boundary energy is approximately constant except for a few minima that occur at specific orientations. Other than twin boundaries, these minima occur at orientations for which a significant fraction of the lattice points in the two neighboring grains are nearly coincident (Chan and Balluffi, 1985, 1986). A simple example of this coincidence structure illustrating the type of periodicity that can be present in specific grain boundaries is shown in Figure 8 for a twist boundary.

Grain and interphase boundaries influence the physical properties of rocks in several important ways. First, grain-boundary sliding provides a mechanism for producing strain in response to an applied differential stress. Grain-boundary sliding takes place by the movement of grain-boundary dislocations. As discussed in the next section, grain-boundary sliding is a critical part of diffusion creep (Raj and Ashby, 1971). Grain-boundary sliding can also be important during deformation in which dislocation glide and climb operate (e.g., Langdon, 1994). In this case, lattice dislocations can dissociate and enter grain boundaries resulting in enhanced or stimulated grain-boundary sliding (Pshenichnyuk et al., 1998). As discussed below, grain-boundary sliding often provides an additional mechanism of deformation and can be particularly important in rocks composed of minerals with fewer than five independent slip systems.

Second, grain boundaries migrate in response to a difference in dislocation density between neighboring grains. As expressed in eqn [28], each dislocation introduces elastic strain energy into a crystalline grain. If during deformation the dislocation density becomes higher in one grain than in the next grain, a thermodynamic driving force exists that can cause the grain boundary to migrate toward the region of higher dislocation density, absorbing lattice dislocations into the grain boundary thus reducing the
stored strain energy. A dislocation-free region is left in the wake of the migrating boundary.

Third, grain boundaries are short-circuit paths for diffusion. The rate of diffusion along a grain boundary is in general several orders of magnitude faster than through a grain interior (Shemon, 1983, pp. 164–175). Thus, even though the cross-sectional area of a grain boundary is small relative to that of the grain itself, the flux of atoms passing through a grain boundary can be significant. The result is that, in the diffusion creep regime, grain-boundary diffusion is often the primary mechanism for deformation. It should also be noted that grain boundaries provide storage sites for elements that, due to size or charge, are incompatible in the grain interior (Hiraga et al., 2003, 2004, 2007; Hiraga and Kohlstedt, 2007). The presence of incompatible elements at grain and interphase boundaries perturbs the structure and the local charge state of the boundary, which in turn can affect diffusion kinetics (Mishin and Herzi, 1999).

2.14.3 Mechanisms of Deformation and Constitutive Equations

The mechanism of deformation that dominates the plastic deformation of a rock depends both on the properties of the rock such as grain size, d, and phase content, Φ, and on thermodynamic parameters such as temperature, pressure, P, water fugacity, and component (oxide) activity, a_{ox}. The dependence of strain rate, \dot{\varepsilon}, on these quantities can be expressed in a general constitutive equation or flow law as

\[ \dot{\varepsilon} = \dot{\varepsilon}(d, \Phi, T, P, f_{H_2O}, a_{ox}, \ldots) \]  

Diffusion-controlled creep tends to be important at low differential stresses in fine-grained rocks. In contrast, dislocation-dominated processes govern flow at higher stresses in coarser-grained rocks. In this section, some of the constitutive equations used to describe flow in the diffusion and dislocation
deformation regimes are first introduced. Then, the influence on rheological behavior of two important fluids, water and melt, are discussed.

2.14.3.1 Constitutive Equations

2.14.3.1.1 Diffusion creep

Diffusion creep can be divided into two regimes, one in which diffusion through the interiors of grains controls the rate of deformation (Nabarro, 1948; Herring, 1950) and the other in which diffusion along grain boundaries governs the rate of deformation (Coble, 1963). The relative importance of these two mechanisms depends primarily on grain size and temperature.

\[ \dot{\varepsilon}_{NH} = \alpha_{NH} \frac{\sigma V_m D_{gm}}{RT} \frac{1}{d^3} \]  

where \( \alpha_{NH} \) is a geometrical term and \( D_{gm} \) is the diffusion coefficient for the slowest species diffusing through the grain matrix (gm), that is, the grain interior. The important points to note are that strain rate is linearly proportional to the differential stress, inversely proportional to the square of the grain size, and exponentially dependent on temperature and pressure through the diffusion coefficient since

\[ D_{gm} = D_{gm}^0 \exp \left( -\frac{\Delta E_{gm} + P \Delta V_{gm}}{RT} \right) \]

where \( D_{gm}^0 \) is a material-dependent parameter and \( \Delta E_{gm}, \Delta V_{gm} \) and \( \Delta H_{gm} \) are the activation energy, activation volume, and activation enthalpy for grain-matrix diffusion, respectively.

\[ \dot{\varepsilon}_C = \alpha_C \frac{\sigma V_m \delta D_{gb}}{RT} \frac{1}{d^3} \]  

where \( \alpha_C \) is a geometrical term, \( \delta \) is the diffusion width of the grain boundary, which is approximately equal to the structural width of \( \sim 1 \) nm (e.g., Atkinson, 1983a, 1983b), and \( D_{gb} \) is the diffusion coefficient for the slowest species diffusing along the boundaries. For grain boundaries

\[ D_{gb} = D_{gb}^0 \exp \left( -\frac{\Delta E_{gb} + P \Delta V_{gb}}{RT} \right) = D_{gb}^0 \exp \left( -\frac{\Delta H_{gb}}{RT} \right) \]

where \( D_{gb}^0 \) is a material-dependent parameter and \( \Delta E_{gb}, \Delta V_{gb} \) and \( \Delta H_{gb} \) are the activation energy, activation volume, and activation enthalpy for grain-boundary diffusion, respectively. Again, the strain rate increases linearly with increasing differential stress and increases exponentially with inverse temperature while decreasing exponentially with increasing pressure. However, in contrast to the Nabarro–Herring case, strain rate for Coble creep varies inversely as the cube, rather than the square, of the grain size.

Comparison of eqn [33] with eqn [35] reveals the following points: (1) Both creep mechanisms give rise to Newtonian viscous behavior (\( \dot{\varepsilon} \propto \sigma \)) with viscosity, \( \eta \equiv \sigma / \dot{\varepsilon} \), independent of stress. (2) As long as the grain size is small enough that diffusion creep dominates over dislocation creep, Nabarro–Herring creep (\( \dot{\varepsilon} \propto 1/d^2 \)) is more important than Coble creep (\( \dot{\varepsilon} \propto 1/d^1 \)) at larger grain sizes. (3) Nabarro–Herring creep dominates at higher temperatures and Coble creep at lower temperatures because \( \Delta H_{gb} < \Delta H_{gm} \).

The analyses of Nabarro (1948) and Herring (1950) and of Coble (1963) strictly apply to deformation of a single spherical grain. Subsequent analyses pointed out the necessity of grain-boundary sliding in diffusion creep of polycrystalline materials (Lifshitz, 1963; Raj and Ashby, 1971). The fact that grains tend to be equiaxed in samples deformed in the diffusion creep field indicates that grain-boundary sliding and grain rotation are crucial. The creep process is essentially that of grain-boundary sliding accommodated by diffusion both through grain interiors and along grain boundaries, if grain boundaries are too weak to support shear stress. For this case, \( \alpha_{NH} = 14 \) and \( \alpha_C = 14\pi \). Since grain-boundary and grain-matrix diffusion are independent/parallel processes, eqn [33] and [35] can be combined to give

\[ \dot{\varepsilon}_{\text{diff}} = 14 \left( \frac{\sigma V_m}{RT} \right) D_{gm} \left( \frac{\pi \delta D_{gb}}{d^2} \right) \left( \frac{d^3}{d^2} \right) \]

In general, this deformation mechanism is referred to simply as diffusion creep.
2.14.3.1.2 Deformation involving dislocations

As differential stress and/or grain size increases, a transition occurs from diffusion creep to dislocation-dominated deformation. One estimate of the stress required to move from the diffusion creep regime to the dislocation creep regime is based on a calculation of the stress required to operate a Frank–Read dislocation source, $\sigma_{FR}$ (Frank and Read, 1950) and thus generate the dislocations necessary to sustain deformation:

$$\sigma_{FR} \approx \frac{2Gb}{L}$$

where $L$ is the length of the dislocation segment operating as the source of new dislocations. If, the length of a dislocation line is limited by the grain size, then for $G = 70$ GPa, $b = 0.5$ nm, and $L = d = 10 \mu$m, $\sigma_{FR} \approx 7$ MPa.

The strain produced by a single dislocation moving across a grain is very small, thus many dislocations must be generated and move to accomplish a significant amount of deformation. The strain, $\varepsilon$, produced in a cubic grain of dimension $d$ by one dislocation moving through the grain is

$$\varepsilon = \frac{b}{d}$$

Thus, if $d = 1$ mm, $\varepsilon \approx 5 \times 10^{-7}$. Greater strain is achieved by moving a larger number of dislocations through the grain. The density of dislocations, $\rho$, scales with applied differential stress as

$$\rho \approx \left(\frac{\sigma}{Gb}\right)^2$$

such that $\rho \approx 10^9$ m$^{-2}$ for a differential stress of 1 MPa. The strain produced by this density of dislocation moving a distance $x$ is

$$\varepsilon = \rho bx$$

For $x = d$, $\varepsilon \approx 5 \times 10^{-4}$ (still a small value). To produce geologically significant strains, dislocations must be generated, move through crystalline grains, and then be removed so that new dislocations can be generated to maintain an approximately constant density of dislocations and keep the deformation process going. If steady-state deformation is attained, then a balance must be achieved, and one step in this series of steps will limit the rate of deformation.

A number of models have been proposed to describe the rate of deformation in terms of applied stress, temperature, pressure, and other thermodynamic and structural variables (e.g., Poirier, 1985, pp. 94–144; Evans and Kohlstedt, 1995). In the present chapter, characteristic elements of these models are reviewed in order to provide a framework for examining laboratory-determined deformation data and extrapolating to geological conditions.

The dependence of the rate of deformation accomplished by dislocation processes on differential stress can be examined through the Orowan equation, which is obtained by differentiating eqn [41] with respect to time and noting that in steady state the dislocation density is independent of time, $\partial \rho/\partial t = 0$ (Orowan, 1940; Poirier, 1985, pp. 62–63):

$$\dot{\varepsilon} = \rho \bar{v}$$

where $\bar{v}$ is the average dislocation velocity. It should be noted, however, that the $\partial \rho/\partial t$ term is critical when discussing transient deformation (e.g., deformation immediately following a change in stress). As discussed in reference to eqn [40], the dislocation density is proportional to approximately the square of the stress, such that $\dot{\varepsilon} \propto \rho \propto \sigma^2$. Additional dependence of strain rate on differential stress enters through the dislocation velocity term.

In this discussion, dislocation creep is divided into three regimes. As with diffusion creep, the main parameters determining the relative importance of the various dislocation creep mechanisms are differential stress, temperature, pressure, and grain size. First, at high temperature (low stress) and relatively large grain size, all of the strain can be accomplished by glide and climb of dislocations. These conditions define the dislocation creep regime. Second, at high temperature but smaller grain size, grain-boundary sliding operates in conjunction with dislocation processes to produce strain. These conditions define a regime in which dislocation processes are accommodated by grain-boundary sliding, simply referred to here as the grain-boundary sliding regime. Third, at low temperatures (high stresses), deformation takes place by dislocation glide limited by the intrinsic resistance of the lattice. These conditions delineate the low-temperature plasticity regime.

2.14.3.1.2(i) Dislocation creep To date, by far the majority of analyses of plastic deformation in geological materials have used a power-law equation
to describe experimental results. A general form of this flow law is

\[ \dot{\varepsilon} = A \frac{\sigma^n}{R^m} \exp \left( -\frac{Q}{RT} \right) \]

where \( A \) is a material-dependent parameter and \( Q \) is the activation energy (strictly, enthalpy) for creep. The power-law form of the creep equation arises by considering the role of dislocation climb and grain-boundary sliding in the deformation process.

The potential importance of dislocation climb as the rate-controlling step of high-temperature, steady-state creep was suggested in the early 1950s by Mott (1951, 1953, 1956). Subsequently, several climb-controlled creep models were developed that relate strain rate to differential stress and temperature, the latter though the self-diffusivity (for reviews see, e.g., Weertman (1978), Poirier (1985), Cannon and Langdon (1988), Evans and Kohlstedt (1995), and Weertman (1999)). The rate of climb of edge dislocations depends directly on diffusive fluxes of ions (e.g., Hirth and Lothe, 1968, pp. 506–519; Poirier, 1985, pp. 58–62). The justification for emphasizing the importance of climb in high-temperature \((T > 2/3)T_m\), where \( T_m \) is the melting temperature) deformation of crystalline materials is the observed one-to-one correlation between the activation energy for creep and the activation energy for self-diffusion of the slowest ionic species. This correlation has been observed for a large number of metallic and ceramic materials (e.g., Dorn, 1956; Sherby and Burke, 1967; Mukerjee et al., 1969; Takeuchi and Argon, 1976; Evans and Knowles, 1978) as well as for olivine (Dohmen et al., 2002; Chakraborty and Costa, 2004; Kohlstedt, 2006).

Two climb-based models of dislocation creep merit particular attention because of their pioneering contributions in this area and because they incorporate most of the elements found in subsequent models. Weertman (1955, 1957a) developed a flow law for steady-state dislocation creep in which dislocation glide produces the strain but dislocation climb controls the strain rate. In this model, a dislocation source generates a dislocation loop that expands by gliding until it encounters an obstacle such as a dislocation loop that was generated on a parallel glide plane. The two dislocations interact to form a dipole that prevents further glide until the two dislocations comprising the dipole climb to annihilate one another. Once the dipole is annihilated, the dislocation sources produce new dislocation loops (i.e., dislocation multiplication) to continue the deformation process. If glide is rapid and climb is slow then the average dislocation velocity is

\[ \dot{\varepsilon} = \frac{\sigma}{G} \frac{D}{b^2 \ln \left( \frac{R}{r_c} \right)} \]

where \( \dot{\varepsilon} \) is the dislocation glide, \( \varepsilon_c \) is the climb distance, and \( v_c \) is the climb velocity (Poirier, 1985, p. 110; Weertman, 1999). The climb velocity, which is determined by diffusion of the slowest atomic/ionic species, is given by the expression (Hirth and Lothe, 1968, pp. 506–519)

\[ v_c = 2\pi \frac{\sigma V_m D}{R T^2 b} \frac{1}{\ln \left( R_o / r_c \right)} \]

where the average spacing between dislocations, \( R_o \), is usually written in terms of the dislocation density as \( R_o \approx 1/\sqrt{\rho} \), and the inner cutoff (core) radius, \( r_c \), is generally set at \( r_c \approx b \). A flow law is then obtained by inserting eqn [40] and [45] into the Orowan equation, eqn [42], to yield (Weertman, 1999)

\[ \dot{\varepsilon} = 2\pi \frac{GV_m}{RT} \left( \frac{\sigma}{G} \right)^{1/3} \frac{D}{b^2 \ln \left( \frac{4G}{4\pi\sigma} \right)} \]

Nabarro (1967) formulated a model for steady-state dislocation creep based solely on dislocation climb. Bardeen–Herring sources (Bardeen and Herring, 1952) generate dislocations that form a network and continuously climb. Dislocation multiplication occurs by operation of dislocation sources thus increasing their density, while climb of dislocations of opposite sign toward one another results in annihilation thus decreasing their density. A balance between multiplication and annihilation results in steady-state creep described by the flow law (Nabarro, 1967; Nix et al., 1971)

\[ \dot{\varepsilon} = 2\pi \frac{GV_m}{RT} \left( \frac{\sigma}{G} \right)^{1/3} \frac{D}{b^2 \ln \left( \frac{4G}{4\pi\sigma} \right)} \]

Although the steady-state strain rates obtained from the models developed by Weertman and Nabarro, eqns [47] and [46], respectively, differ in magnitude, these flow laws share fundamental elements. Both yield a cubic dependence of strain rate on differential stress and a linear dependence of strain rate on diffusivity. That is, both result in power-law equations similar to that given in eqn [43] with \( m = 0 \). The exponential dependence of strain rate on temperature enters through \( D \) as does at least part of the dependence of strain rate on
oxygen and/or water fugacity. In detail, the dependence of diffusivity on fugacity enters through the concentration of the point defects (e.g., vacancies) that enable diffusion of ions, as expressed in eqn [16].

As discussed by Hirth and Lothe (1968, pp. 506–529) and Poirier (1985, pp. 58–62), the climb velocity is directly proportional to the concentration of jogs, \( \epsilon_j \) times the migration velocity of jogs, \( v_j \):

\[
\frac{\partial \epsilon_j}{\partial t} = \epsilon_j v_j
\]

where the value of the parameter \( A_{gbs} \) depends on the details of the grain-boundary sliding process and the dislocation recovery mechanism (see Langdon, 1994), has a value of order 10. This type of analysis is applied to the situation in which a subgrain structure does not build up within the individual grains so that dislocations move relatively freely from one grain boundary, across the grain, to the opposite grain boundary.

If grains contain subgrains, then dislocations that are generated by sliding along a grain-boundary glide across the adjoining grain until they encounter a
subgrain boundary. The dislocations then climb into the subgrain boundary, interact with dislocations in the subgrain wall, and are annihilated (Langdon, 1994). This type of analysis leads to a flow law of the form

\[ \dot{\varepsilon} = B_{\text{gbs}} \frac{D_{\text{gm}} G V_m}{d^3} \frac{\sigma^3}{RT} \]

with \( B_{\text{gbs}} \approx 1000 \). Note that eqns [49] and [50] differ in their dependence on stress (\( \sigma^2 \) vs \( \sigma^3 \)), on grain size (\( 1/d^2 \) vs \( 1/d \)), and on temperature (\( Q_{\text{gb}} \) vs \( Q_{\text{gm}} \)).

In silicates, evidence for creep involving grain-boundary sliding and dislocations motion has been identified in deformation data for dry olivine (Kohlstedt and Wang, 2001; Hirth and Kohlstedt, 2003) and is apparent in the results for dry clinopyroxene (Bystricky and Mackwell, 2001) for which a coarse-grain rock is a factor of \( \sim 10 \) stronger than a fine-grained sample fabricated from powders prepared from the coarse-grain rock. Interestingly, this grain-boundary sliding regime appears to be absent in olivine deformed under hydrous conditions, possibly because an enhancement of dislocation climb provides the extra deformation mechanism required to fulfill the von Mises criterion (Mei et al., 2002; Hirth and Kohlstedt, 2003).

In addition to their contribution to deformation by producing strain as they slide, grain boundaries facilitate deformation by migrating in response to the strain energy associated with high densities of dislocations within the grains. This dynamic recrystallization process does not itself produce strain; however, it does remove dislocations within grains, resulting in a population of small grains that are, at least initially, free of dislocations. Dislocations generated at grain boundaries can more easily glide across these dislocation-free grains, unimpeded by interactions with other dislocations, than through larger grains containing high densities of dislocations. In addition, since the contribution of grain-boundary sliding to deformation increases with decreasing grain size as indicated in eqns [49] and [50], grain-boundary sliding can contribute to deformation in regions of small grains, thus also enhancing the rate of deformation. Hence, dynamic recrystallization often results in ’strain softening’, a reduction in creep strength with progressive deformation (i.e., increasing strain) of a rock (e.g., de Bresser et al., 2001; Drury, 2005). Dynamic recrystallization can also occur by subgrain rotation by which dislocations are removed from the interiors of grains by addition to low-angle boundaries (e.g., Poirier, 1985, pp. 169–177). As dislocations are added to these (low-angle) subgrain boundaries, the misorientation angle increases until distinct new grains (i.e., high-angle grain boundaries) form. An example of dynamic recrystallization involving grain-boundary migration and simultaneous subgrain rotation is presented in Figure 9. Again, the removal of dislocations from within the grains reduces temporarily the number of obstacles to dislocation glide, and the formation of smaller grains makes the operation of grain-size-sensitive deformation processes (diffusion creep and grain-boundary sliding) more favorable.

2.14.3.1.2.(iii) Low-temperature plasticity

At lower temperatures and hence higher stresses, power-law formulations underpredict the observed increase in strain rate with increasing stress. As temperature decreases, diffusion becomes too slow to permit a significant contribution of dislocation climb to the deformation process. The rate of deformation then becomes limited by the ability of dislocations to glide past obstacles. In silicate minerals with a significant component of covalent bonding, dislocations must overcome the resistance imposed by the lattice itself, often referred to as the Peierls barrier (e.g., Frost and Ashby, 1982, pp. 6–9). Motion of dislocations over the Peierls barrier requires the nucleation and migration of kinks (steps along dislocation lines that lie in the glide plane), with the
The nucleation rate of pairs of kinks generally assumed to limit the dislocation velocity (Frost and Ashby, 1982, p. 8). The dislocation velocity is determined by the glide velocity, \( v_g \), and consequently by the kink velocity, \( v_k \):

\[
v \approx v_g = \alpha_k v_k
\]

where \( \alpha_k \) is the concentration of kinks. The activation enthalpy for the nucleation and migration of double kinks, \( \Delta H_k(\sigma) \), is a function of differential stress. A detailed analysis for \( \Delta H_k(\sigma) \) yields (Kocks et al., 1975, p. 243; Frost and Ashby, 1982, p. 8)

\[
\Delta H_k(\sigma) = \Delta H_k^0 \left[ 1 - \left( \frac{\sigma}{\sigma_p} \right)^r \right]^{1/\gamma}
\]

where \( \Delta H_k^0 \) is the Helmholtz free energy of an isolated pair of kinks, \( \sigma_p \) is the Peierls stress, and \( r \) and \( s \) are model-dependent parameters (Frost and Ashby, 1982, p. 9). From Orowan’s equation, eqn [32], the flow law then becomes

\[
\dot{\varepsilon} = \dot{\varepsilon}_p \left( \frac{\dot{\varepsilon}_0}{\sigma} \right) \exp \left\{ \frac{\Delta H_k^0}{RT} \left[ 1 - \left( \frac{\sigma}{\sigma_p} \right)^{r/\gamma} \right] \right\}
\]

where \( \dot{\varepsilon}_p \) is a material-dependent parameter.

An illustrative method for displaying the conditions under which a given deformation mechanism dominates flow is the deformation mechanism map (Frost and Ashby, 1982). At present, olivine is the only mineral for which data exist in the diffusion, dislocation, grain-boundary sliding, and low-temperature plasticity regimes. Four parameters are usually considered when constructing a deformation mechanism map: strain rate, stress, grain size, and temperature. Examination of eqn [32] indicates that other parameters could be included (e.g., pressure, water fugacity, and melt fraction) depending on the application. Here, the classical form is first used in which stress is plotted as a function of temperature for a fixed grain size (and fixed pressure, water fugacity, etc.) with strain rate shown parametrically. In Figures 10(a) and 10(b), differential stress is plotted as a function of temperature for dunite rocks composed of 10-\( \mu \)m grains and 1-mm grains, respectively. Flow parameters for the diffusion, dislocation, and grain-boundary sliding regimes are taken from Hirth and Kohlstedt (2003), while those for the low-temperature plasticity regime are from Goetze (1978) and Evans and Goetze (1979). For a fine-grained rock with \( d = 10 \mu m \) such as might be found in a shear zone (Figure 10(a)), diffusion creep dominates at geological strain rates, \( 10^{-14} - 10^{-10} \) s\(^{-1} \) for stresses below \( \sim 300 \) MPa. At higher stresses, grain-boundary sliding becomes important, and at still higher stresses, low-temperature plasticity dominates. For a coarser-grained rock with \( d = 1 \) mm such as might be expected in the upper mantle (Figure 10(b)), deformation at geological strain rates and stresses appropriate for the asthenosphere occurs near the boundary between the diffusion and dislocation creep regimes, while deformation in the lithosphere requires dislocation creep or low-temperature plasticity.

**Figure 10** Deformation mechanism maps plotted as differential stress as a function of temperature for dunite with grain sizes of (a) 10 \( \mu \)m and (b) 1 mm. (a) At the smaller grain size for a strain rate of \( 10^{-15} \) s\(^{-1} \), deformation occurs by diffusion creep for stresses up to \( \sim 300 \) MPa above which deformation is dominated by grain-boundary sliding and then by low-temperature plasticity. (b) At the larger grain size for a strain rate of \( 10^{-15} \) s\(^{-1} \), with increasing stress deformation moves from the diffusion creep regime to the dislocation creep regime to the low-temperature plasticity regime.
plasticity. Strong seismic anisotropy in the mantle at depths less than \( \sim 250 \) km indicates that deformation occurs by dislocation processes (e.g., Montagner, 1985; Nishimura and Forsyth, 1989; Gaherty and Jordan, 1995). In contrast, the apparent lack of seismic anisotropy at greater depths, below the Lehmann discontinuity (Lehmann, 1959, 1961), has been interpreted as an indicator of diffusion creep (Karato, 1992), although more recent observations indicate that a weak anisotropy exists in this deeper region, characteristic of dislocation activity on the \((b\overline{0}0)[001]\) slip system (Mainprice et al., 2005).

An alternative form of the deformation mechanism map that provides insight into mantle deformation process is obtained by plotting stress as a function of strain rate again shown parametrically, as illustrated in Figure 11. The greater importance of grain-boundary sliding at the lower temperature (800°C vs 1300°C) is clear in this comparison. This behavior reflects the fact that the activation energy for grain-boundary sliding below \( \sim 1250°C \) appears to be smaller than that for dislocation creep, \( \sim 400 \text{kJ mol}^{-1} \) versus \( \sim 500 \text{kJ mol}^{-1} \) (Hirth and Kohlstedt, 2003) although this point remains to be confirmed experimentally (e.g., Mei and Kohlstedt, 2000b). Also, the contribution of low-temperature plasticity is, as expected, greater at 800°C. Finally, at 1300°C for geological stresses, strain rates, and asthenospheric grain sizes, deformation occurs close to the boundary between diffusion and dislocation creep.

2.14.3.2 Role of Fluids in Rock Deformation

Two important, often-present components that can result in a substantial weakening of rocks are melt and water. If either is present, even in small concentration, it can profoundly reduce the viscosity of a rock.

2.14.3.2.1 Role of melt in rock deformation

The influence of melt on rock viscosity enters constitutive equations in two ways (Kohlstedt, 2002; Xu et al., 2004). First, melt provides a path along which ions can diffuse more rapidly than they can either through grain interiors or along grain boundaries. Second, since melt does not support shear stresses, for a given applied macroscopic stress, the stress at the grain scale will be locally increased if melt is present.

Both of these contributions to the high-temperature strength of a rock were included in the analysis by Cooper et al. (1989) of the effect of melt on diffusion creep of an aggregate composed of a fluid plus a solid phase. Such analyses depend critically on the distribution of the melt phase. In this particular model, the melt distribution was assumed to be that dictated by surface tension for an isotropic system under a hydrostatic state of stress for which the dihedral angle, \( \theta \), is determined by the relative values

![Figure 11](image-url)  
*Figure 11* Deformation mechanism maps plotted as differential stress as a function of grain size for dunite at temperatures of (a) 800°C and (b) 1300°C. (a) At the lower temperature for a strain rate of \( 10^{-15} \text{s}^{-1} \), deformation occurs by diffusion creep for grain sizes less than \( \sim 200 \mu \text{m} \) and by dislocation creep at larger grain sizes. At higher strain rates, grain-boundary sliding contributes at grain sizes smaller than \( \sim 50 \mu \text{m} \) such as might be present in shear zones. (b) At the higher temperature, deformation is dominated by diffusion creep at finer grain sizes and by dislocation creep at coarser grain sizes.
of the solid–liquid and solid–solid interfacial energies, $\gamma_{sl}$ and $\gamma_{ss}$, respectively:

$$\cos \left( \frac{\theta}{2} \right) = \frac{\gamma_{ss}}{2\gamma_{sl}}$$ \hspace{1cm} [54]$$

For the olivine + mid-ocean ridge basalt (MORB) case, the average dihedral angle is $\sim 40^\circ$ (i.e., $0 < \theta < 60^\circ$) such that, ideally, melt forms an interconnected network along three-grain junctions and through four-grain corners, such as illustrated in Figure 12. Average values for dihedral angle of $< 60^\circ$ are observed for many rocks composed of silicate minerals plus a silicate melt. However, as illustrated in Figure 13, for melt fractions greater than $\sim 0.05$ the model of Cooper and Kohlstedt (Cooper et al., 1989) significantly underpredicts the decrease in viscosity measured in experiments on partially molten rocks in the diffusion creep regime (Hirth and Kohlstedt, 1995a; Zimmerman and Kohlstedt, 2004).

One important aspect not taken into account in the above analysis is the fact that the wetting behavior of most partially molten systems is not isotropic (e.g., Cooper and Kohlstedt, 1982). As illustrated in Figure 14, microstructural observations demonstrate that melt wets a fraction of the grain boundaries, behavior not predicted for an isotropic system with $\theta > 0^\circ$. Furthermore, the percentage of grain boundaries wetted by melt increases with increasing melt fraction, $\phi$ (Hirth and Kohlstedt, 1995b). At the time of writing of this chapter, a model that appears to deal adequately with the effect of this microstructural distribution of melt on diffusion creep behavior is being developed (Takei and Holtzman, 2006). This model expresses the viscosity of a partially molten rock in terms of grain-boundary contiguity (the ratio of grain-boundary area to grain-boundary area plus grain-melt interfacial area). This model extends that of Cooper and Kohlstedt (Cooper et al., 1989) from two to three dimensions and includes the observed anisotropic wetting of grains. One important result of

![Figure 12](image-url) **Figure 12** Sketch of melt distribution in a partially molten rock with isotropic interfacial energies. In (a) with $0^\circ < \theta \leq 60^\circ$, melt wets all of the triple junctions and four-grain junctions; (b) cross-section through a triple junction midway along a grain edge illustrates the presence of melt. In (c) with $\theta > 60^\circ$, melt is confined to four-grain junctions; (d) cross-section through a triple junction midway along a grain edge illustrates the absence of melt. Adapted from Riley GN, Jr and Kohlstedt DL (1990) An experimental study of melt migration in an olivine-melt system. In: Ryan MP (ed.) Magma Transport and Storage, pp. 77–86. New York: John Wiley & Sons.
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Figure 13  Plot of normalized strain rate, $\dot{\varepsilon}(\phi)/\dot{\varepsilon}(0)$, vs melt fraction. Circles are the experimentally determined values. Dashed curves are based on model of Cooper et al. (1989) for $1^\circ \leq \theta \leq 59^\circ$. Solid line is a fit of the experimental data to the empirical relationship $\dot{\varepsilon}(\phi)/\dot{\varepsilon}(0) = \exp(\alpha\phi)$, yielding $\alpha = 22$. For $\phi < 0.15$, data are from Hirth and Kohlstedt (1995a). For $\phi > 0.15$, data are from Scott and Kohlstedt (2006).

Figure 14  Backscattered scanning electron micrograph of sample of olivine +5% basalt. Both melt-filled triple junctions and wetted grain boundaries are identified. The presence of wetted grain boundaries demonstrates that the interfacial energies are anisotropic in this system, since melt would be confined to triple junctions in an isotropic system with a similar dihedral angle of $\theta \approx 40^\circ$.

this model for the diffusion creep regime is a factor-of-five decrease in viscosity caused by increasing $\phi$ from 0 to 0.001 (i.e., 0.1% melt). This model yields a good fit to the experimental data for $\phi > 0$. The predicted factor-of-five decrease in viscosity in going from $\phi = 0$ to $\phi = 0.001$ has not been observed experimentally, possibly because most of the samples fabricated for these experiments contain a trace amount of melt due to impurities introduced during the crushing and grinding process or produced from inclusions within the olivine crystals used to prepare fine-grained powders from which samples were synthesized (Faul and Jackson, 2006).

Experimental determinations of strain rate as a function of melt fraction in both the diffusion creep regime and the dislocation creep regime have been empirically fit to the relation

$$
\dot{\varepsilon}(\phi) = \dot{\varepsilon}(0)\exp(\alpha\phi')
$$

[55]

for melt fractions from near zero to the rheologically critical melt fraction (RCMF) with $r=1$ for the olivine + basalt system (Kelemen et al., 1997; Mei et al., 2002; Scott and Kohlstedt, 2006) as well as for an organic crystal plus melt system (Takei, 2005) and $r=3$ for a partially molten granitic rock (Rutter and Neuman, 1995, Rutter et al., 2006). The RCMF is the melt fraction at which, with increasing melt fraction, viscosity of a melt-solid system moves from that of a solid framework with interstitial melt to that of a melt containing grains in suspension.

Melt has a greater effect on viscosity in the dislocation creep regime than in the diffusion creep regime, at least in the olivine + basalt and partially molten lherzolite systems (Mei et al., 2002; Zimmerman and Kohlstedt, 2004; Scott and Kohlstedt, 2006). This point is illustrated in Figure 15 with $\alpha = 21$ in the diffusion creep regime and $\alpha = 32$ in the dislocation creep regime. At melt fractions greater than the RCMF, a partially molten rock behaves as a fluid (melt) containing suspended particles. The Einstein–Roscoe equation (Roscoe, 1952),

$$
\eta_{ER}(\phi) = \frac{\mu}{(1.35\phi - 0.35)^2}
$$

[56]

which reasonably well describes the viscosity of a system with $\phi > 0.25–0.30$, is also shown in Figure 15. In eqn [56], $\mu$ is melt viscosity. Finally, it should be noted that the onset of melting will affect grain-boundary chemistry, which will indirectly affect rock viscosity through its influence on the rates of ionic diffusion along grain boundaries (Hiraga et al., 2007).

2.14.3.2.2 Role of water in rock deformation

Water weakening of nominally anhydrous silicate minerals was first observed in the mid-1960s in an experimental study of the strength of quartz (Griggs and Blacic, 1965). In these solid-medium deformation
experiments, samples deformed with a hydrous
confining medium (talc) were weaker than samples
deformed with an anhydrous confining medium.
Subsequently, water weakening has been reported
for other nominally anhydrous minerals (NAMs)
including olivine (e.g., Avé Lallemant and Carter,
1970), pyroxene (Avé Lallemant, 1978), and feld-
spar (e.g., Tullis and Yund, 1980). These first
studies of water weakening emphasized the phen-
omenon. A second generation of experiments
focused on the mechanism of water weakening by
studying the dependence of viscosity on water
fugacity, that is, on the concentration of water-
derived point defects (Kronenberg and Tullis,
1984; Kohlstedt et al., 1995; Post et al., 1996; Mei
and Kohlstedt, 2000a, 2000b; Karato and Jung,
2003; Chen et al., 2006).

The first model of water or hydrolytic weakening
was built on a mechanism in which water hydrolyzes
strong Si–O bonds forming weaker Si–OH · · · OH–Si
bonds (Griggs, 1967). Thus, dislocation glide is easier
under hydrous conditions than under anhydrous
conditions; effectively, the Peierls stress/barrier
decreases as Si–O–Si bridges become hydrolyzed.

In this analysis, the rate of dislocation glide is limited
by the propagation of kinks along dislocations, a
process facilitated by diffusion of water along dislo-
cation cores. The dislocation velocity is then taken to
be proportional to the concentration of water
(Griggs, 1974).

More recent models of water weakening have
emphasized the role of water-derived point defects,
particularly hydrogen ions (protons) (Hobbs, 1981,
1983, 1984; Poumellec and Jaoul, 1984; Mackwell
et al., 1985). Protons diffuse rapidly in NAMs (for a
review, see Ingrin and Skogby, 2000) such as
quartz (Kronenberg et al., 1986), olivine (Mackwell
and Kohlstedt, 1990; Kohlstedt and Mackwell,
1998, 1999; Demouchy and Mackwell, 2003), and
pyroxene (Ingrin et al., 1995; Hercule and Ingrin,
1999; Carpenter Woods et al., 2000; Stalder
and Skogby, 2003). Thus, initially dry, millimeter-size
samples can be hydrated in high-temperature,
high-pressure experiments lasting a few hours or
less. The presence of protons in a NAM affects the
velocity of a dislocation in two possible ways. First,
since protons are charged, a change in the concen-
tration of protons will result in a change in the
concentration of all other charged point defects.
Hence, the introduction of protons into a NAM
will directly affect the concentrations of vacancies
and self-interstitials on each ionic sublattice and
consequently the rates of diffusion of the constit-
uent ions and the rate of dislocation climb. In
addition, extrinsic point defects, specifically
water-derived point defects, can affect the concen-
trations of kinks and jogs along dislocation lines
and thus dislocation velocity as expressed in eqns
[48] and [51] (Hirsch, 1979, 1981; Hobbs, 1981,
1984). The effect of water-derived point defects
on the concentrations of kinks and jogs can enter
in two ways. First, for minerals containing a tran-
sition metal such as Fe (i.e., semiconducting
silicates), the presence of water-derived point
defects will affect the concentrations of kinks and jogs
by ionizing initially neutral jogs (Hirsch, 1979, 1981;
Hobbs, 1984). For the case of kinks, a positively charged kink, k′, can be
produced from a neutral kink by the reaction

\[ k^\circ + h' \rightleftharpoons k' \]  \[ 57 \]

for which the law of mass action yields

\[ [k'] = K_5 [k^\circ][h'] \]  \[ 58 \]

\[ \]
The total concentration of kinks then becomes

$$[k^{\text{tot}}] = [k^x] + [k] = [k^x](1 + k_5\gamma [p])$$  \hspace{0.5cm} [59]

where the second term can be much larger than unity and dominate the kink population. The dependence of kink concentration on water (proton) concentration or water fugacity then enters through concentration of positively charged kinks, thus enhancing the dislocation glide velocity. A similar argument can be used for increasing the concentration of jogs, and thus the dislocation climb velocity. While this approach provides one mechanism for increasing dislocation velocity in NAMs, it is not easily extended to minerals such as quartz that do not contain significant concentrations of transition metals. Therefore, a more direct influence of water-derived point defects on kink and jog concentrations needs to be considered. In point defect notation, the addition of protons, $p$, to a nominally anhydrous silicate will affect the concentration of kinks through a reaction such as

$$k^x + p\{k^x - p\}$$  \hspace{0.5cm} [60]

where the curly brackets $\{\}$ indicate the formation of a neutral kink associated with a proton. Application of the law of mass action yields

$$[\{k^x - p\}] = k_{10}[k^x][p]$$  \hspace{0.5cm} [61]

The total concentration of kinks is then

$$[k^{\text{tot}}] = [k^x] + [\{k^x - p\}] = [k^x](1 + k_{10}[p])$$  \hspace{0.5cm} [62]

Again, a similar equation applies for jogs. The dependence of kink and jog concentrations on water concentration/fugacity then enters directly through the dependence of proton concentration on water fugacity.

The high-temperature deformation behaviors of anorthite, clinopyroxene, and olivine deformed in the dislocation creep regime under anhydrous and hydrous conditions are compared in Figure 16. A similar comparison is made for the diffusion creep regime by Hier-Majumder et al. (2005a). In the dislocation creep regime, the dependence of strain rate on water fugacity has been quantified for olivine (Mei et al., 2002; Hirth and Kohlstedt, 2003; Karato and Jung, 2003) and for clinopyroxene (Chen et al., 2006). In the case of olivine, strain rate increases as water fugacity to the $\sim 1^{\text{st}}$ power, while in the case of clinopyroxene, strain rate increases as water fugacity to the $\sim 3^{\text{rd}}$ power. In the diffusion creep regime, creep rate increases as water fugacity to the $\sim 1^{\text{st}}$ power for olivine (Mei and Kohlstedt, 2000a) and to the $\sim 1.4$ power for clinopyroxene (Hier-Majumder et al., 2005a). To assess possible explanations for these observed dependences of strain rate on water fugacity, Table 2 summarizes the relationships between the concentration of various point defects and water fugacity for several charge neutrality conditions for a transition-metal silicate. (1) For olivine under both anhydrous and hydrous conditions, the activation energies for dislocation creep (Hirth and Kohlstedt, 2003) and those for Si self-diffusion (Dohmen et al., 2002; Chakraborty and Costa, 2004) agree within experimental error (Kohlstedt, 2006). This observation suggests that the high-temperature creep of olivine is controlled by dislocation climb limited by diffusion of the slowest ionic species, Si, under both dry and wet conditions. In this case, the jog concentration is unity, that is, the dislocations are fully saturated with jogs (see eqn [48] and the discussion that follows). Under hydrous conditions, the water fugacity exponent, $q$, in eqn [43] of $\sim 1$ is consistent with Si diffusing by a vacancy mechanism with the major Si vacancy being part of the point defect
associate \(2(\text{OH})_O - V^{\text{me}}_{\text{Si}}\) \(\equiv (2H)_{\text{Si}}^+\) and charge neutrality given by \([\text{(OH)}]_O = [\text{H}_{\text{Me}}]\) (see Table 2). Diffusion of Me ions also varies as water fugacity to the 1st power (Hier-Majumder et al., 2005b), indicating that \((2H)_{\text{Me}}\) is the primary point defect on the Me sublattice responsible for diffusion (see Table 2). (2) The situation appears to be more complex for clinopyroxene. The stress exponent is \(n = 2.7 \pm 0.3\), consistent with the climb-controlled creep models discussed above. Unfortunately, diffusion data are lacking for comparison with the creep data. The relatively strong dependence of strain rate on water fugacity, \(q = 3.0 \pm 0.6\), in the dislocation creep regime suggests that a simple climb-controlled mechanism may not apply for high-temperature deformation of clinopyroxene (see Table 2). One possible way to reconcile this large value for the water fugacity exponent is as follows: (1) Creep is climb controlled; (2) charge neutrality condition is given by \([h] = [\text{H}_{\text{Me}}]\) or by \([p] = [\text{H}_{\text{Me}}]\); (3) strain rate is limited by Si diffusion by a vacancy mechanism involving the defect associate \(4(\text{OH})_O - V^{\text{me}}_{\text{Si}}\) \(\equiv (4H)_{\text{Si}}^+\); (4) dislocations are undersaturated with jogs with the jog concentration dominated by neutral jogs associated with protons, \(k^+ - p\), as in eqns [61] and [62]. These scenarios yield a water fugacity exponent for strain rate of 2/4 and 2/4, both within the experimental uncertainty of the measured value. Clearly, diffusion data are critical in order to develop this type of argument more completely.

### 2.14.4 Upper-Mantle Viscosity

In this section, viscosity profiles for the upper mantle constrained by geophysical observations and calculated from laboratory-derived flow laws are compared. Two situations are considered: first, global average values determined from analyses of glacial isostatic adjustment are examined relative to those calculated for dunite containing 1000 H/10^6 Si. Second, the viscosity of the upper-mantle region beneath the western US is compared to the viscosity predicted for dunite deforming under water-saturated conditions.

Laboratory results for dunite are used because it is the only rock for which data are available over a wide range of upper-mantle conditions. Justification for this choice is based on the observations that olivine is the major mineral in the upper mantle and that, under conditions for which data are available, the flow behavior of dunite is nearly identical to that of lherzolite (Zimmerman and Kohlstedt, 2004). Furthermore, flow laws for melt-free dunite are used because, at small melt fractions, homogeneously distributed melt has a relatively small effect on viscosity. For \(\phi = 0.02\), viscosity is a factor of <2 smaller than the viscosity of a melt-free rock (Figures 13 and 15); larger melt fractions would be difficult to trap in the mantle over long periods of time. By comparison, the viscosity of dunite containing 1000 ppm water is a factor of >100 smaller than the viscosity of dry dunite (Figure 16). Thus, the effect of water rather than melt is emphasized here. It should be noted that melt might have a larger effect on viscosity if it segregates into melt-rich bands as observed in large-strain experiments on partially molten peridotites (Holtzman et al., 2003a, 2003b, 2005); however, this area remains to be fully explored.

Finally, viscosity profiles are calculated for dislocation creep rather than diffusion creep. Three points justify this choice. First, strong seismic anisotropy at depths of <250 km (e.g., Nishimura and Forsyth, 1989) and weak but persistent seismic anisotropy at great depths (Mainprice et al., 2005) indicate that deformation in the upper mantle is dominated by dislocation processes. Second, analysis of the influence of rheological properties on the evolution of the tip of a subducting slab indicates that non-Newtonian
behavior (i.e., dislocation creep) facilitates the initiation of subduction (Billen and Hirth, 2005). Third, as illustrated in Figures 10 and 11 as well as in Hirth (2002) and Hirth and Kohlstedt (2003), deformation of dunite at temperatures above ~1000°C occurs primarily by dislocation glide and climb with contributions from grain-boundary sliding.

### 2.14.4.1 Upper-Mantle Viscosity Profile

Recent papers by Kaufmann and Lambeck (2002) and Hirth and Kohlstedt (2003) have investigated upper-mantle viscosity profiles based on analyses of glacial isostatic adjustment and laboratory-derived flow laws, respectively (Dixon et al., 2004). Kaufmann and Lambeck (2002) examined changes of Earth’s surface associated with the retreat of Late Pleistocene ice sheets to obtain global average models of viscosity profiles. They obtained a rather narrow range of values for upper-mantle viscosity of between $3 \times 10^{20}$ and $3 \times 10^{21}$ Pa s for the depths of 60–410 km with a best fitting result of $3 \times 10^{20}$ to $5 \times 10^{20}$ Pa s for 120–410 km, as illustrated in Figure 17, for which a rigid, elastic lithosphere overlies the viscoelastic upper mantle.

Hirth and Kohlstedt (2003) critically reviewed published experimental data from laboratory experiments on dunite rocks and olivine aggregates. Best-fit values for the flow-law parameters in eqn [43], summarized in their table 1, were used in the present analysis. The activation volume as a function of depth was calculated using their equation 3, which is based on a calculation of the elastic strain necessary to create and move point defects. The temperature as a function of depth was determined based on an adiabatic geotherm with a potential temperature, $T_p$, of 1350°C. A water concentration of 1000 H/10^6 Si was employed following the study of Hirth and Kohlstedt (1996). Two viscosity profiles are presented in Figure 17, one assuming constant stress ($\sigma = \text{constant}$) and the other assuming constant rate of viscous energy dissipation, that is, $\sigma \dot{\varepsilon} = \text{constant}$ (e.g., Christensen, 1989). For additional details, the reader is referred to Hirth and Kohlstedt (2003). The viscosity profile with $\sigma \dot{\varepsilon}$ constant, which changes by a factor of three in viscosity in going from 60 to 410 km, is much steeper than the viscosity profile with $\sigma$ constant, which changes by a factor of 120 in viscosity over the same depth range. Hence, it is difficult to reconcile profiles obtained with $\sigma$ constant with the constraints obtained from glacial isostatic adjustment results. In contrast, profiles produced from laboratory data with $\sigma \dot{\varepsilon}$ constant are in reasonably good agreement in terms of both the narrow width and the magnitude reported from the glacial isostatic adjustment analyses.

#### 2.14.4.2 Western US Mantle Viscosity Profile

In their analysis of the role of water on the lateral variation in upper-mantle viscosity, Dixon et al. (2004) tabulated constraints on the viscosity structure of the upper mantle beneath the western US (their table 1). Values of viscosity were based on the investigations of Bills et al. (1994), Kaufmann and Amelung (2000), Nishimura and Thatcher (2003), and Pollitz et al. (2000, 2003). These studies examined the response of Earth’s surface to smaller, more local loads (earthquakes and lake level changes) than analyzed in the glacial isostatic adjustment work. Thus, their resolution is best in the shallower portion of the upper mantle. Dixon et al. (2004) emphasized the distinctly lower values of mantle viscosity observed in the western US ($10^{18}–10^{19}$ Pa s) compared with the global average values ($10^{20}–10^{21}$ Pa s). These authors make a compelling case that this difference is due to enrichment in water of the upper mantle beneath the western US associated with subduction of the Farallon Plate, which hydrates and weakens the mantle. In Figure 18, the range of viscosity obtained for
the western US is compared with viscosity profiles calculated based on the flow laws determined by Hirth and Kohlstedt (2003). Viscosity profiles are again calculated for $\sigma$ constant and for $\dot{\varepsilon}$ constant. The geothem was taken from figure 1 of Dixon et al. (2004), which was obtained from P-wave tomodiographic inversion for the western US (Goes and van der Lee, 2002). Water concentration was taken to be saturated following the solubility law determined by Kohlstedt et al. (1996) and Zhao et al. (2004). As illustrated in Figure 18, the flow law for dislocation creep of dunite determined from laboratory experiments yields a viscosity profile that falls within the range of values obtained from measurements of Earth’s response to changes in more local surface loads. The profile with $\dot{\varepsilon}$ constant yields a relatively small decrease in viscosity with increasing depth, while the profile with $\sigma$ constant produces a much larger decrease in viscosity over the same depth range. In either case, the good agreement between the viscosity profiles determined from field observation and those calculated based on experimentally determined flow laws supports extrapolation of the laboratory-derived constitutive equations to problems of geophysical interest.

2.14.5 Concluding Remarks

While the good agreement between laboratory and field observations for the viscosity structure of the upper mantle is encouraging, even exciting, several important areas remain that require experimental investigation of the rheological properties of mantle rocks. Possibly the most obvious void is the paucity of deformation studies on transition zone and lower-mantle minerals and rocks. Progress in this field has been slow, largely due to a lack of apparatuses capable of carrying out experiments under well-controlled thermomechanical conditions at very high pressures. However, recent developments combining a new generation of deformation apparatuses such as the deformation DIA (D-DIA) and the rotational Drickamer apparatus (RDA) with high-intensity, synchrotron-produced X-rays hold promise for significant progress in this area (e.g., Durham et al., 2002; Thurel et al., 2003; Xu et al., 2003, 2005).

For the upper mantle, further work is needed to quantify the deformation behavior in the low-temperature plasticity regime, particularly under hydrous conditions, for application to studies of deformation of the lithosphere. Furthermore, laboratory (e.g., Jung and Karato, 2001) and seismic (e.g., Mainprice et al., 2005) observations indicate that a transition in slip system occurs below a depth of ~250 km in Earth’s upper mantle, related to changes in water, stress, and/or pressure conditions. At present, flow laws for the upper mantle are primarily based on experiments carried out at relatively modest pressures for which measurements of lattice-preferred orientations indicate that deformation in olivine is dominated by dislocations with [100] Burgers vectors. If dislocations with [001] Burgers vectors become important in deeper portions of the upper mantle, the flow law determined at very high pressures may deviate from that extrapolated from lower pressure conditions (Couvy et al., 2004).

Finally, to date, most studies of plastic deformation of rocks have concentrated on steady-state creep with little attention given to transient deformation. While a steady-state approach may work reasonably well for describing convective flow in the mantle, it cannot provide an adequate description of deformation that occurs in an environment for which thermomechanical conditions are changing and microstructures are simultaneously evolving (e.g., Evans, 2005). This point has been appreciated by a number of researchers, some of whom have taken
a state-variable or mechanical-equation-of-state approach (e.g., Lerner and Kohlstedt, 1981; Covey-Crump, 1994, 1998; Rutter, 1999; Stone et al., 2004). Such approaches will be critical in the development of the next generation of constitutive equations, which must be capable of describing flow in regions such as shear zones and metamorphic belts where changes in thermal and chemical environment including fluid chemistry and evolution of materials parameters including dislocation density, grain size, and lattice-preferred orientation occur continuously.

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