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Derivation of sediment resuspension rates from acoustic backscatter time-series in tidal waters

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Abstract

Extending oceanographic forecasting models beyond dynamics to ecological parameters involves simulation of concentrations of suspended particulate matter (SPM). The latter will require assimilation of both in situ and remote sensing observations. Assimilation will need to reconcile both types of observations with modelling responses for a variety of both resuspension and settling velocity parameters. This study develops a systematic approach to this problem.

Time series of suspended sediment particulate matter (SPM) concentration are routinely obtained via indirect optical, acoustic and satellite instrumentation. However, translating such measurements into components contributed by localised sediment resuspension and horizontal advection is severely complicated by uncertainties concerning the specific SPM characteristics which cannot easily be measured in situ. Since some estimate of synoptic tidal currents is generally available, resuspension-transport-deposition models can be used to interpret these SPM concentration time series. Here, a novel methodology, incorporating an optimisation procedure and a 1-D Lagrangian particle tracking model, is developed to automate this interpretation and indicate the nature of the associated SPM.

Utilising calibrated acoustic backscatter measurements from Acoustic Doppler Current Profilers, a downhill simplex optimisation method minimises the least squares coefficient of determination (R^2) between model and observed SPM concentration time series. Advection of a linear "background" concentration gradient is incorporated into the SPM model, and the optimisation procedure decouples observed SPM concentration time series into background and resuspension components.

The model has been validated in three independent ways and good agreement between derived model parameters and independent observations has been found for settling velocity, background concentration gradients and erosion rates. Using data from two contrasting sites in the Mersey estuary and Dover Straits, agreement for concentrations involved $0.61 < R^2 < 0.83$. A modular design provides scope for more complex formulations and improvements of 20% in R^2 occurred when a time varying eddy diffusivity was employed.

1. Introduction

Models simulating transport of both cohesive and non-cohesive sediment involve a range of parameters describing the physical characteristics

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of the sediment, such as their erosion rates under varying bed shear stress and their suspended settling velocities (Gerritsen et al., 2000). Parameters can be prescribed for a particular sediment population from controlled laboratory studies or in situ measurements (Amos et al., 1992). The laboratory approach works reasonably well for cohesionless, regularly shaped sands but in most shelf sea environments SPM is dominated by complex and cohesive organo-mineral aggregates, of varying size, density, shape and degree of cohesion. Direct in situ measurements have been made in a limited range of shelf sea and estuarine environments (Amos et al., 1992; Maa et al., 1993; Dyer et al., 1996) but these have employed deployment of technologically complex, expensive instrument packages which show little prospect of becoming routine in the near future. Consequently, attention has focussed on obtaining these parameters indirectly by obtaining 'best fits' between model output and observed time series of SPM concentration at a particular site (Jones et al., 1996; Aldridge, 1997).

Optical and acoustical techniques can now provide extensive and detailed combined observations of current velocity and SPM concentration. In particular acoustic backscatter profiles obtained from devices such as Acoustic Doppler Current Profilers (ADCPs) can be calibrated to provide data of the relevant quality and quantity (Holdaway et al., 1999), although such calibrations would benefit from more detailed measurements of particle size (Thorne et al., 1991).

A procedure then needs to be established to obtain a "best fit" between models of sediment dynamics and these observations. This poses particular problems as more than one parameter is generally involved. Direct measurements of particle settling velocity are not always available so models need parameters to incorporate differing settling velocity classes and typically two parameters to describe erosion rates. Further parameters may be required if active particle aggregation/disaggregation processes need to be considered.

This paper starts with the relatively simple case of a model based on a single size class of particles, a single description of eddy difusivity and erosion rates, and does not consider flocculation. A novel methodology for utilising observed SPM data sets in conjunction with this model and an optimisation procedure is developed. The procedure provides some insights into the underlying sediment processes and associated sediment parameters.

A 1D (single point) Lagrangian particle tracking model has been developed to model the resuspension and vertical distribution of SPM in coastal waters. An optimisation procedure has been applied to estimate certain parameters in the particle tracking model by maximising agreement of modelled SPM concentration time series at all heights in the water column with corresponding observational time series obtained from ADCP backscatter measurements. The development is essentially generic and potentially applicable at a wide variety of sites and, in future, could be used as a tool for interpolation and extrapolation in both temporal and spatial scales.

Two contrasting sites in the Dover Straits and Mersey estuary have been used to validate the model. Care has been taken to select sites with relatively homogeneous bed sediments and the analysis has been limited to tidally dominant, storm free periods. The model has been validated in three independent ways by comparing model-derived settling velocities, erosion rates and background concentration gradients with independent observations. However, it is recognised that some difficulties remain in establishing a widely applicable methodology. The present approach is a "first step" and some expedient measures involving possible extensions of the techniques developed here are suggested.

2. Methodology

2.1. The particle tracking model

2.1.1. Background

There are two fundamental approaches to modelling the dynamics of suspended sediments. The first is the Eulerian technique, which solves the advection-diffusion equation (ADE) in a fixed reference frame, usually employing finite-difference schemes. Alternatively, techniques which follow a large number of particles in a Lagrangian reference frame as they move by random walk through the fluid may be used (Allen, 1982, 1991).

Particle tracking techniques have been applied to the spreading of oil slicks (Elliot et al., 1986) and more recently in suspended sediment dynamics (Clarke and Elliot, 1998; Black and Parry, 1999; Black and Vincent, 2001). The majority of sediment transport research has focused on 2D depth-averaged models. However, random walk in a 2D vertical plane has been studied in the context of sediment transport (Gani and Todorovic, 1987). A 1D model to investigate vertical mixing and a 3D model of fine sediment transport has also been developed to model phosphorus dynamics under wind-driven waves in the shallow Lake Okeechobee in the USA (Sheng et al., 1990).

Advantages of the Lagrangian method include the fact that the computational effort is concentrated in regions where most particles are located rather than treating all regions equally, as in the ADE method. Also, particle tracking allows additional measures of attributes of individual particles to be calculated, such as elapsed time within the water column as well as their concentration. Hunter (1987) reports that advection is handled particularly well by Lagrangian particle techniques whilst the diffusion term seems to be best served by Eulerian methods. He points out that the drawback for Lagrangian methods is that most hydrodynamic models are solved with finite differences, making it difficult to merge the two models. In this case, a particle tracking approach was adopted.

2.1.2. Governing equations

A 1D (single point) particle tracking model based on Jones et al. (1994) was produced in order to simulate observed concentration time series at varying height in the water column. This model assumes that the particles move independently. The model assumes that bed sediments and velocities are homogeneous in the direction of flow. Some account of lateral inhomogeneity is subsequently incorporated where advection of a background concentration gradient is considered.

Particles are moved by a vertical step length Δz at each timestep where

$$\Delta z = \pm\sqrt{(2E\Delta t)} - W_s\Delta t, \tag{1}$$

where E is the eddy diffusivity (m^2/s), W_s is the settling velocity (m/s), and Δt is the timestep (s)

The derivation of the diffusive step length $\sqrt{(2E\Delta t)}$ emerges from an analysis in Dimou and Adames (1989). The sign is randomly selected with equal weight attached to the two possibilities. Thus, the first term on the RHS of governing equation (1) simulates diffusion whilst the second term represents settling under gravity.

The particles are released from the bed according to the simple power law (directly analagous to the commonly assumed engineering "pick-up" erosion formulae):

$$N_e(t) = \mathrm{Int}\{B|U(t)|^P\}, \tag{2}$$

where N_e is the integer number of particles allowed to be eroded between time t and $t + \Delta t$, B, P are constants and $U(t)$ is the depth averaged flow.

This law is used by, amongst others, Lavelle et al. (1984) and comparisons have been made with the bed parameters they obtained in their studies. No threshold velocity is used although there is an implicit threshold because N_e is discrete and the lowest possible value of N_e for movement of sediment to occur is $N_e = 1$.

Concentrations are calculated at each timestep in terms of numbers of particles within a user-defined height range or "bin" of the form

$$z_i = z_0 + (I - 1)z_w, \tag{3}$$

where z_i is the height of the ith bin, z_0 is the midpoint of the bottom bin and z_w is the width of subsequent bins.

To facilitate subsequent intercomparison with ADCP data, the bottom bin was defined to be larger than subsequent bins since there was an absence of observational data close to the bed.

At the surface a simple reflection boundary condition was employed. That is, a particle is moved a distance Δz and if $z > h$, where h is the average water depth, then the particle assumes a position at a distance $z - h$ below the surface. At the bed, a so-called "bounce then settle" boundary condition was employed. That is, the particles are moved a diffusive distance $\sqrt{(2E\Delta t)}$, then reflected if the particle moves below the bed, then moved a distance $-W_s\Delta t$. If this latter move results in a negative height above the bed, then the particle is simply deposited on to the bed (i.e. z is set to zero). This is considered the boundary condition which most closely simulates reality (Csanady, 1973; Dimou and Adames, 1989; Bossis et al., 1992).

Round-off problems, whereby particles could be infinitesimally close to the bed but not actually considered to be on the bed, were resolved by introducing a small parameter, ε (0.0001 m), below which particles are assumed to be deposited onto the bed.

2.1.3. Extensions

Initially, E was taken as a constant. Temporal variation in E was then incorporated as:

$$E(t) = \beta|U_b|D \tag{4}$$

where U_b is the depth averaged flow velocity, D is the average water depth and β is some constant.

This relation is valid for vertically mixed, shallow tidal water (Prandle, 1982; Lees, 1981).

Under certain flow assumptions, simple well known relations exist (Dyer, 1986; Elliot et al., 1986) which express E as an analytic function of depth but in this work, E was considered to be taken as spatially uniform. Extensions to include vertical variation would necessitate the addition of an effective drift velocity (Boughton and Delaurentis, 1987; Hunter, 1987).

The settling velocity, W_s, was also considered to be constant throughout this study which means that the effects of aggregation, where particles can coalesce on collision, were ignored. It is recognised that for the Mersey this is an over-simplification but in the low concentrations (<10 mg/l) of the Dover Straits it is probably valid. Empirical relations exist which express W_s as a function of local concentration and work is underway to utilise such an approach (Hill, 1999 (unpubl Ph.D. thesis)). Fig. 11 of Jones et al. (1994) shows a comparison over a spring-neap cycle of SPM concentrations calculated by the above model versus ADCP observations at 5 m above the bed in the Dover Straits. Prandle (1997) shows how vertical profiles of SPM vary over a spring-neap cycle for a wide range of particle sizes.

2.2. Optimisation procedure

It has been noted (e.g. Jones et al., 1994) that observed concentration time series can often be explained in terms of the superposition of advection of a semi-diurnal background concentration gradient and quarter-diurnal resuspension. It is therefore important to allow for horizontal advection even when dealing with 1D models and a means of separating these two components is required in addition to a method for determining the model parameter values which best simulate the observations. The latter involves optimisation (Bryson and Chi Ho, 1975; Boudarel et al., 1971) of a least squares fitting procedure (Weisberg, 1985; Smillie, 1966; Draper and Smith, 1966). A variety of optimisation methods were considered (Wolfe, 1978; Press et al., 1992) including Powell's method, the downhill simplex method and simulated annealing methods (Spagnol, 1994).

It can be seen from the governing equations (1) and (2) that the unknown parameters in the particle tracking model which describes the resuspension process are E the eddy diffusivity, W_s the settling velocity and B, P the bed parameters.

Generally, some external knowledge of E, such as from a hydrodynamical model of the region of interest, allows it to be estimated a priori. In the Mersey it was taken as $E = 0.01\,\text{m}^2/\text{s}$ and in the Dover Straits, $E = 0.1\,\text{m}^2/\text{s}$ (Prandle, 1982). However, in this study, as in general, no a priori knowledge regarding the three parameters B, P and W_s can be assumed (note that it is possible to include more than one representative value of settling velocity or an optimisation on eddy diffusivity can be carried out).

Consider the concentration at some height z above the bed. This comprises a resuspension component $C_r(z,t)$ and an advective component $C_a(z,t)$. The total concentration is thus,

$$C(z,t) = C_r(z,t) + C_a(z,t).$$

Here, it is assumed that the horizontal background concentration gradient being advected is both temporally and vertically constant within a Lagrangian framework. This implies that the SPM forming this concentration gradient exhibites negligible settlement over the simulation period. This is consistent with a source tens of kilometres from the site, perhaps due to a river outflow or resuspension in a regime differing from that of the localised resuspension. This advective component is proportional to the tidal excursion (Jones et al., 1994) and so

$$C(z,t) = a_1 + b_1 C_{\mathrm{mr}}(z,t) - a_2 t_{\mathrm{e}}(z,t), \tag{5a}$$

where C_{mr} is the locally resuspended concentration in units of numbers of particles produced by the particle tracking model and

$$t_{\mathrm{e}}(z,t) = \int_0^t u(z,\tau)d\tau$$

is the tidal excursion.

In the work presented here, it is assumed that the depth averaged velocity can be used in the integration and so t_{e} is depth independent. The term—$a_2 t_{\mathrm{e}}$ is the advective component described above, the minus sign being included so that a_2 is simply the background concentration gradient. The background component is thus $a_1 - a_2 t_{\mathrm{e}}$ and the resuspended component is thus $b_1 C_{\mathrm{mr}}(z,t)$. These quantities have dimensions of concentration in mg/l and must all be positive. These conditions are given the label 2.1 for future reference.

Note that for the case of two settling velocity classes Eq. (5) becomes

$$C(z,t) = a_1 + b_1 C_{\mathrm{mr1}}(z,t) + b_2 C_{\mathrm{mr2}}(z,t) - a_2 t_{\mathrm{e}}(z,t), \tag{5b}$$

where C_{mr1} and C_{mr2} are the resuspension components corresponding to the two different settling velocity classes. The model has been extended to two resuspending settling velocity classes and could be extended further to accommodate a spectrum of settling velocities, although this is constrained by the need to minimise the number of adjustable parameters.

For a given particle tracking time series $C_{\mathrm{mr}}(z,t)$ it can be seen that a multiple least squares fit can be carried out between the model output $C_{\mathrm{mr}}(z,t)$, the observed SPM concentrations and the observed tidal excursion. The problem is that $C_{\mathrm{mr}}(z,t)$ depends on W_{s}, B, P and that these parameters can take very wide ranges. It was decided that some method of optimising this fit was required. Once the optimisation is established, the solutions for the parameters a_1, a_2, b_1 and b_2 allow the signal to be decoupled into resuspension and advection components.

All optimisation procedures require a so-called cost function which they can minimise to find the optimal solution. A measure of the fit described above could be the standard least squares coefficient of determination, R^2, which can be thought of as a function of W_{s}, B, and P which maximise R^2 (or minimise the cost function, $-R^2$, since, by convention, all optimisation methods work to minimise the cost function).

It should be noted that the "global" minimum of $f(W_{\mathrm{s}}, B, P) = -R^2$ must be sought. That is, the overall minimum of f in the space of all possible values of W_{s}, B, P and not just a local minimum where $\partial f/\partial x_{\mathrm{i}} = 0$ for $i = 1,2,3$ where $x_1 = W_{\mathrm{s}}$, $x_2 = B$, $x_3 = P$. It should also be noted that the physical constraints 2.1 impose restrictions on the permitted combinations of values of the parameters W_{s}, B and P. For a concentration to have physical meaning it must clearly be positive. The problem is therefore a so called constrained optimisation problem.

There are two classes of constrained optimisation methods:

(a) those which explicitly treat the constraints, e.g. the simplex method of linear programming
(b) essentially unconstrained optimisation methods which are converted to a constrained method by adding a penalty function to the cost function when the constraints are violated.

The simplex method of linear programming is only applicable to scenarios where the constraints are linear functions of the independent parameters, which is not the case here. The unconstrained methods themselves fall into two further subcategories, namely ones which exploit knowledge of the gradient of the function to be minimised and those which do not. Now, the cost function in this case is $f(W_{\mathrm{s}}, B, P) = -R^2$ which is calculated by running the particle tracking model for a given set of parameters W_s, B, P and calculating the fit to the observed SPM concentrations. There is therefore no analytical expression for the derivative of this function. Whilst it is possible to calculate the derivative of the function, f, numerically using, say, finite differences, this is computationally expensive and so it is better to employ an optimisation method which does not require knowledge of the cost function gradient.

If there was only one independent variable to optimise, fast methods such as Brent's method (Wolfe, 1978) could be employed but we have at least a three-dimensional problem and so need to consider multi-dimensional methods. There are still a number of options which could feasibly be implemented. In recent years, new annealing methods (so called because they mimic the process in molecular crystallization where a crystal achieves its minimum energy state) have been created which address directly the problem of finding global extrema in the presence of large numbers of undesired local extrema. These methods have proved to be quite successful for some problems previously thought to be intractable, but are still under development and also carry a heavy computational cost. This could be important for future work where it is envisaged that many point models could be nested in a 2D tide model to simulate sediment transport over a wider area.

It was therefore decided to employ more established and robust techniques, such as Powell's method and the downhill simplex method, not be to be confused with the simplex method of linear programming. Initial trials with Powell's method revealed that results were found to be sensitive to the choice of the initial guess for the minima. The more robust downhill simplex method was therefore employed and minima were obtained for various initial values which converged to the same solution.

The downhill simplex method is essentially a geometric method in $N+1$ dimensional space where N is the number of parameters in the cost function. The cost function is evaluated at the $N+1$ vertices of some hyper polyhedron, called a simplex. These function values are assessed and some geometric manipulations are applied to the simplex until the method converges to the overall minimum. The method is described in some detail in Press et al. (1992). Whilst it is not as fast as Powell's method, it is very robust and in this problem proved to be more successful.

The cost function used in the optimisation process was taken as

$$f = R^2 + \lambda,$$

where λ was assigned to be $\lambda = 1$ when the conditions 2.1 were violated. Since $R^2 \varepsilon\ [-1,1]$ this ensured that a realistic minimum was always achieved.

Because water depth can vary significantly with the tide in coastal waters sigma coordinates were used which follow the surface elevation, i.e.

$$\sigma = z/D,$$

where z is the height above the bed and D the overall depth.

The concentrations in the least squares fitting procedure are therefore defined in terms of σ coordinate and time. The optimisation was carried out over the entire depth. Optimisation of three parameters over a tidal cycle required approximately two hours run time with a machine with 3×190 MHz processors producing 293MFLOPS. No attempt was made to maximise computational efficiency. Given the likelihood of continuing pausity of observational data, this aspect of the study was not regarded as a priority issue.

3. Sites under study

The model is forced by depth averaged current velocity and requires time series of SPM concentration profiles for the optimisation procedure. Suitable data were available from two contrasting sites in the Dover Straits and Mersey estuary.

3.1. The dover straits

From May 1990 to June 1991 an experiment was conducted (Prandle, 1990) to examine the physical processes involved in sediment resuspension and advection of contaminants in the English Channel. The study sites lie between Dungeness and Cap Griz-Nez (see Fig. 1) in a region of strong

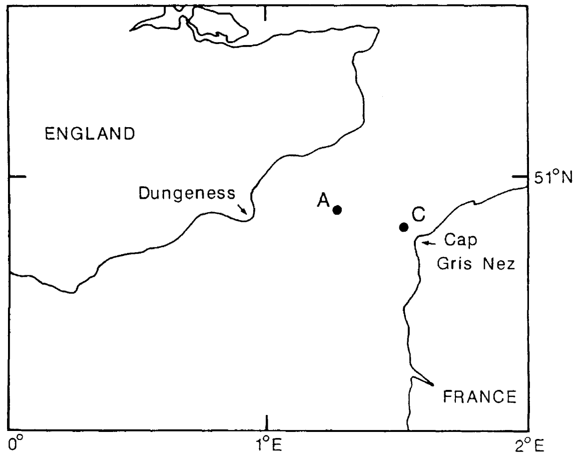


Fig. 1. Mooring positions in the Dover Straits.

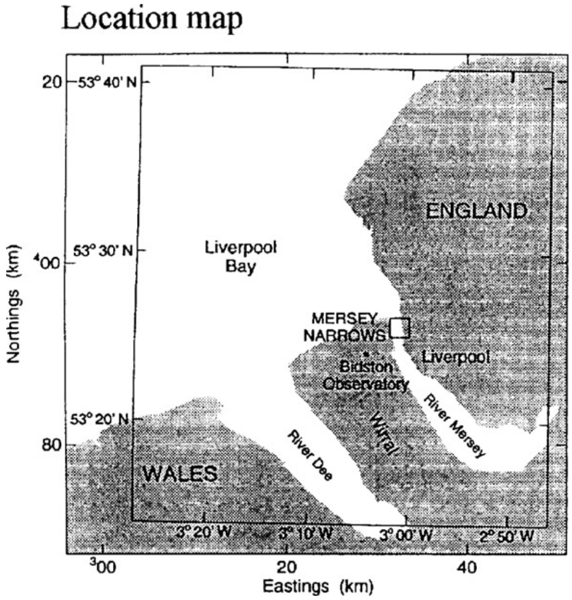


Fig. 2. Location map of the River Mersey.

(> 1 m/s) rectilinear tidal currents (the M_2 amplitude at site A was 0.7 m/s). The water depth is approximately 30 m and the water column is well mixed throughout the year. Moorings were deployed at site A (50° 56′N, 01° 16′E) for one year and at site C (50° 53′N, 01° 32′E) for one month (22 May–14 June 1990) (Prandle, 1990).

The moorings comprised a 1 MHz sea-bed mounted ADCP built at the Proudman Oceanographic Laboratory (Griffiths and Flatt, 1987).The ADCP measures vector averaged North and East components of current at levels centred on bins with mid points at $3.9\,\text{m} + (n-1) \times 1.418\,\text{m}$, for $n = 1, 2, \ldots 24$ throughout the water column. The backscatter intensity is also recorded, averaged over the same time interval of 10 min.

A U-shaped mooring was also deployed at each site with an Anderaa RCM7 current meter together with an optical beam transmissometer positioned at 5 m above the bed. This latter instrument was designed and built at the University of Wales, Bangor and was calibrated against SPM concentrations determined from water bottle samples taken at the site (Jones et al., 1994).

3.2. The mersey

The tides in the Mersey estuary are dominated by the M_2 constituent whose vertical amplitude is approximately 3 m (Lane et al., 1997). The water depth at the site under consideration was on average about 10 m. Vertical variation in salinity peaked at 1.5 psu during the study but was generally much smaller (Jones and Jago, 1993). With tidal velocities upwards of 1 ms^{-1} in a water depth of 10 m, the assumption of being well mixed vertically is reasonable.

In a similar manner to the Dover Straits experiment, ADCPs were deployed in June 1992 on the seabed at 4 sites in the Mersey Narrows (see Figs. 2 and 3). A transmissometer was also mounted on the ADCP frame at 1.5 m above the bed. Whilst this instrument lies in the so-called "near field" of the ADCP, the lobe effects are allowed for in the calibration procedure described in the next section.

4. Determination of SPM concentration from ADCP

The use of the ADCP to measure SPM concentrations is still developing and there is no universal agreement on how to calibrate the backscatter signal into meaningful concentrations.

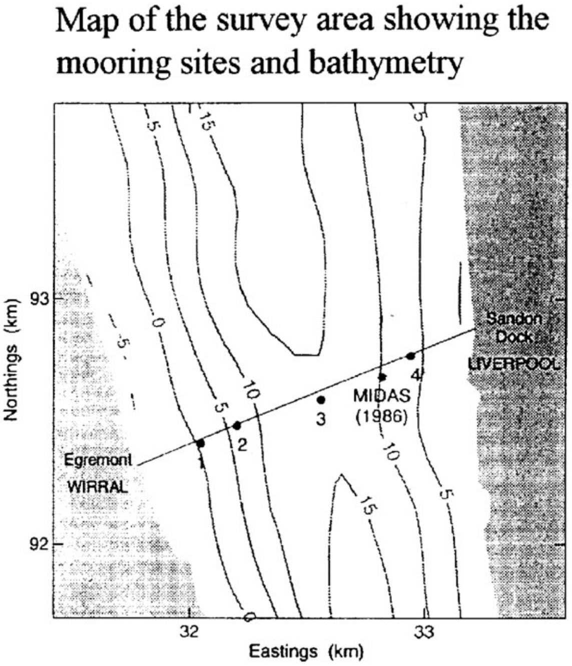


Fig. 3. Measurement positions and bathymetry in the River Mersey.

One of the earliest uses of acoustic backscatter instruments was in the high energy boundary layer experiment (HEBBLE) on the Nova Scotia continental rise in deep waters (Lynch et al., 1991; Libicki et al., 1989) and a comparison of acoustical and optical instruments is detailed in Thevenot and Kraus (1993).

To extract information about concentration at a fixed depth, corrections must be made for spherical spreading of energy in the beam, absorption by water and absorption by intervening SPM. The general theory of backscattering of sound by particles in water is given by Coates (1990), for example. For the frequencies commonly employed in ADCP's this process is described by Rayleigh scattering with the amplitude of the backscattered signal approximately proportional to the third power of the particle "diameter", where it is assumed that sediment particles can be represented by perfect spheres of non-cohesive material. Since SPM often comprises a wide range of particle sizes, practical application of the above theory is both complicated and effectively limited. Shorter range Acoustic Backscatter Sensors circumvent this difficulty by employing multi-frequency acoustic signals enabling both sediment concentration and particle size spectra to be determined (Thorne et al., 1994, 1996). Lynch et al. assumed a uniform size distribution which is also assumed in this study.

In this work, a method following the theoretical work done by Thorne et al. (1991, 1993) and Thorne and Hanes (2002) developed for SPM within 1m of the bed, is pursued. They performed calibration experiments to verify the theory with considerable success. An integral expression based upon Rayleigh scattering is devised.

Following Thorne, the observed ADCP signal and desired concentration is given by

$$\langle P_b \rangle^2 = \frac{k_0^2 \tau c M(r)}{r^2 \psi^2} e^{-4r(\alpha_w + \alpha_s)}, \tag{6}$$

where $\langle P_b \rangle$ is the ensemble averaged rms pressure signal (Pa), τ is the pulse length (m), c is the sound speed in m/s, ψ is the near field function (see below), k_0 is a constant that depends on the instrument and the scattering function, r is the radial distance along the beam, M is the mass concentration of SPM, α_{w} is the water absorption coefficient (Neper/m) and α_{s} is the SPM absorption coefficient.

Owing to a complex beam pattern close to the ADCP, a near field function is required which shows how the acoustic backscatter is modified in this region.

$$\psi = 1 \quad \text{for } r > \varepsilon r_n,$$

$$\psi = \frac{1}{3}\left(2 + \varepsilon \frac{r_n}{r}\right) \quad \text{for } r < \varepsilon r_n,$$

where $r_{\mathrm{n}} = \pi a_t^2 / \lambda$ with a_{t} being the radius of the transceiver and $\lambda = c/f$ is the acoustic wavelength.

Following Thorne, ε is set as $\varepsilon = 2$, somewhat arbitrarily.

For the ADCP, $a_t = 0.038\,\mathrm{m}$, $f = 10^6\,\mathrm{Hz}$ and $c = 1500\,\mathrm{m/s}$ so $r_{\mathrm{n}} = 3.024\,\mathrm{m}$.

Thus, the near field function is used for $r < 6.049\,\mathrm{m}$ which affects the first two bins.

The problem is that α_s depends on $M(r)$ via

$$\alpha_s = \frac{1}{r}\int_0^r \zeta M(r)\,\mathrm{d}r \qquad (7)$$

so that it is not possible analytically to invert the equation to find a general solution $M(r)$ as a function of $\langle P_b \rangle$. Instead, a numerical technique has to be applied, though a direct inversion method for a specific case is reported by Lee and Hanes (1995).

Linear regression of backscatter from the appropiate bin of the ADCP against SPM concentrations obtained from a transmissometer moored in the near-bed region is used to calculate the parameter k_0 (Eq. (6)). It is this parameter that depends on particle size but here it is assumed to be constant. The moored transmissometer was first intercalibrated with a profiling transmissometer, which had been independently calibrated by gravimetric analysis of water samples obtained over tidal cycles at varying heights above the bed. A recursive relation can then be derived for successive bins and a numerical Newton–Raphson algorithm used to solve for the root of the ensuing equation. Although calibration constants depend on particle size and compostion for both optical and acoustic instrumentation, any error incurred due to variation in these characteristics is assumed to be less than other errors in the technique (e.g. in gravimetric sampling or least squares fitting). Despite such reservations about sensitivities to particle size the technique appears to be reliable when compared with other methods.

5. Results

The results from the optimisation procedure described in Section 2.2 are presented for observations extending over typical semi-diurnal tidal cycles during springs and neaps at Site A in the Dover Straits and Site 4 in the Mersey.

Measured current velocities were used and the stated constant values for E for the Dover Straits and the Mersey were taken. They are consistent with Eq. (4). For time varying E, $\beta = 0.002$ was taken (Prandle, 1982). In all runs, 5000 particles were initially at rest on the bed and the model was spun-up over three tidal cycles before an equilibrium was reached and the optimisation performed. The "bin" size z_0 (Eq. (3)) was taken as $z_0 = 3.9$ m and $z_w = 2.83$ m, so that every second model bin corresponded with an ADCP bin. The average depth for the site in the Dover Straits was taken as $D = 33$ m, whereas for Site 4 in the Mersey $D = 10$ m.

The optimised parameters are presented together with the corresponding measure, R^2, of the fit between model and observed SPM concentration time series in Table 1. The fits were significant at the 95% confidence level. The table also indicates the constants a_1, a_2, b_1 (and b_2 for

Table 1
Summary of optimal parameter values

	DS (One W_s)		Mersey (One W_s)		DS (Two W_s)		Mersey $E = E(t)$	
	Springs	Neaps	Springs	Neaps	Springs	Neaps	Springs	Neaps
R^2	0.61	0.74	0.69	0.69	0.64	F	0.71	0.83
W_{s1} (m s^{-1})	0.0068	0.0103	0.0014	0.0037	0.0093	A	0.0035	0.008
W_{s2} (m s^{-1})	NA	NA	NA	NA	0.0004	I	NA	NA
B (m$^{-(P+2)}$s^{P-1})	59.2	69.3	47.2	30.6	190.6	L	46.7	78.3
P	7.10	9.73	2.98	4.57	6.92	E	4.47	8.4
a_1 (kg m^{-3})	0.864	0.761	82.86	26.96	0.393	D	0.166	1.522
b_1 (kg m^{-1})	0.687	1.997	0.838	2.056	0.216		0.210	1.227
a_2 (kg m^{-4})	4.49×10^{-5}	1.29×10^{-6}	-4.89×10^{-3}	-2.02×10^{-3}	0.0241		9.54×10^{-5}	3.84×10^{-5}
b_2 (kg m^{-1})	NA	NA	NA	NA	0.0341		NA	NA
U_0 (m s^{-1})	0.56	0.65	0.27	0.47	0.47		0.42	0.60

NB: All results shown are significant at the 95% confidence level.

the two settling velocity case) obtained during the procedure (see Eq. (5)). Results from four separate optimisation procedures are presented: (i) constant E, one value of W_s, Dover Straits (DS), (ii) constant E, one value of W_s, Mersey, (ii) constant E, two values of W_s (DS), and (iv) time varying E, one value of W_s, Mersey. Two values of W_s were included for the Dover Straits site because measured in situ settling velocity distributions indicated the presence of two modes in addition to a slow settling background population (Jones et al., 1994).

NB: Note that the background concentration gradient is a_2. The sign is such that more SPM is present to the South West in the Dover Straits than in the North Sea, whereas in the Mersey, more SPM is present in the estuary than offshore. For the neap tide case, no results were obtained since the optimisation procedure did not find an optimum value after 500 iterations. The table also includes calculation of the critical threshold velocity U_0, obtained from Eq. (2) with $N_e = 1$ as being $U_0 = (1/B)^{1/P}$. These values can be used to estimate threshold shear stress by converting to velocity at 1m above the bed and assuming a standard drag coefficient (0.002). They indicate threshold bed shear stress of around 0.4 Pa for the Dover Straits and 0.15–0.3 Pa for the Mersey estuary. These are within the range (0.11–0.5 Pa) measured in situ in fine sediments by Amos et al. (1997).

Fig. 4a shows, for spring tides, the results for the Dover Straits study (all results are for site A) at the height of $\sigma = 0.4$ (i.e. 40% up the water column from the bed). Fig. 4a shows the model concentration (i.e. the combined background and resuspension components) compared with the corresponding SPM concentration obtained from the ADCP. Fig. 4b shows a breakdown of this modelled concentration into the constituent background (i.e. $a_1 + a_2(-\int u\,dt)$) and resuspension component ($b_1 C_s$).

Fig. 5a and b show, for spring tides, the results at a sample height of $\sigma = 0.4$ for the Mersey. Fig. 6a and b shows that the model and data agreement is better in the Mersey higher up in the water column at $\sigma = 0.7$. It is thought that the comparison lower down in the water column may be adversely influenced by suspect data in the first bin of the ADCP. The backscatter data from the ADCP, calibrated in the manner described in Section 4, has been plotted together with the CTD mounted profiling transmissometer in the Mersey. Fig. 7a and b show the results from the ADCP during spring and neap tides respectively. The corresponding graphs for the transmissometer are presented, drawn on the same scales as the ADCP, in Fig. 8a and b.

6. Validation

The model validation did not merely consist of comparing modelled and observed concentration time series. The model was validated in 3 more, independent ways.

6.1. Settling velocities

The W_s values in the simulations with a single settling velocity class fall within realistic ranges for the types of SPM found at the two sites (Dyer, 1986).

A small yet significant improvement in the model results is achieved by incorporating two settling velocities in the Dover Straits. The improvement in R^2 was 4.2%. The settling velocity achieved from this two settling velocity class model agree well with the measurements from quasi in-situ settling tube observations in the Dover Straits where a largely bi-modal distribution was found with settling velocity values of 0.006 and 0.0001 m/s (Jones et al., 1994).

6.2. Erosion rates

A means of calculating an erosion rate is now described. Recall that the number of particles eroded per metre squared within a timestep Δt is given by $N_e = B|U(t)|^P$. The number of particles is fixed at 5000 so the mass of sediment, m, represented by each particle varies for each optimised run according to the concentration of sediment in the water. A calculation of m follows:

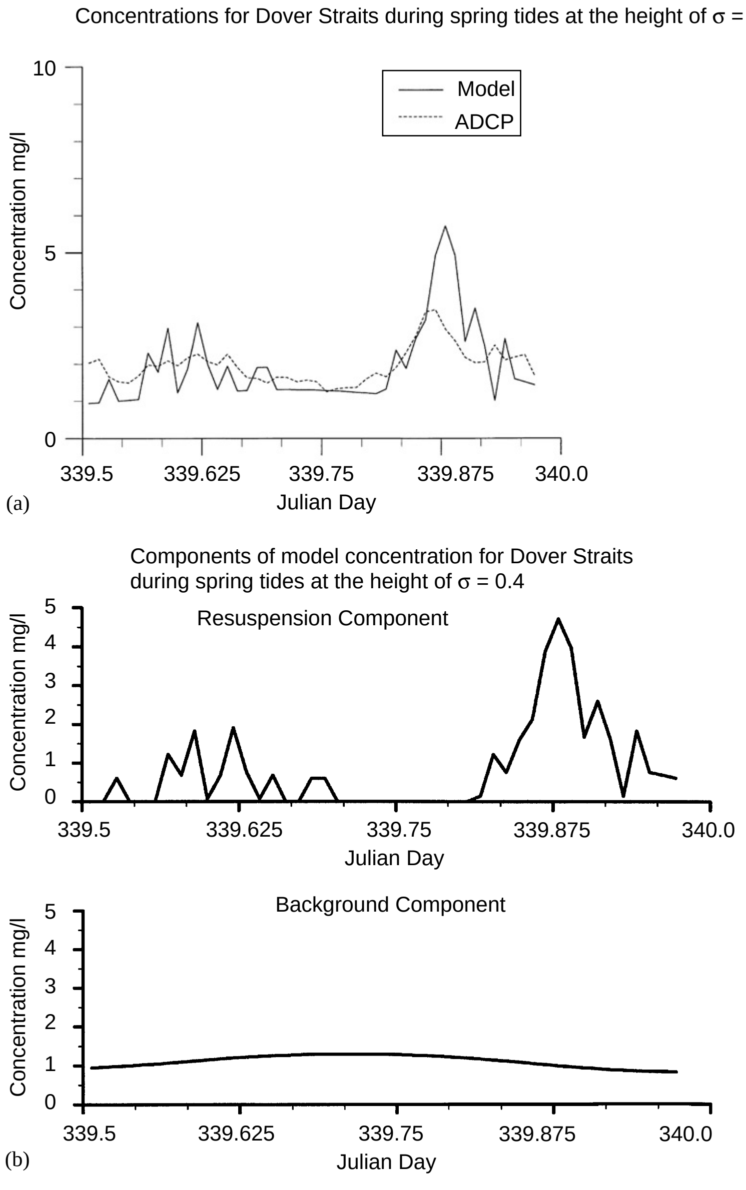


Fig. 4. (a) Concentrations for Dover Straits during spring tides at the height of $\sigma = 0.4$, (b) Components of model concentration for Dover Straits during spring tides at the height of $\sigma = 0.4$.

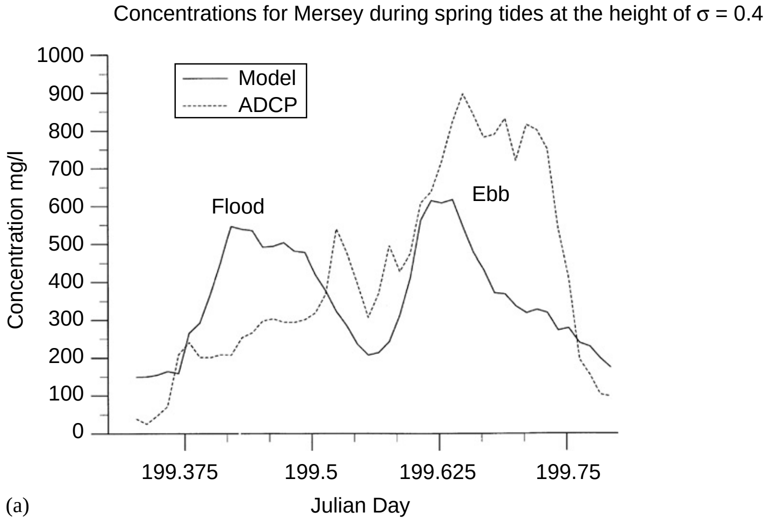
The erosion rate is

$$\mu = N_e m/\Delta t \quad (\mathrm{kg\,m^{-2}s^{-1}}). \tag{8}$$

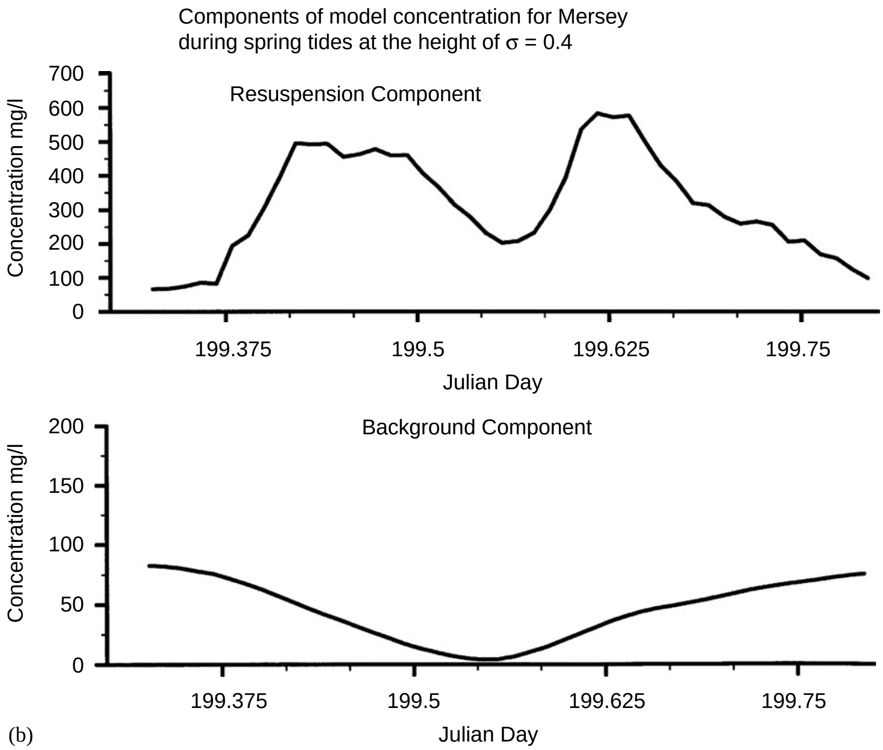
The model computes N particles per square metre in each depth bin of height h_b metres so the mass in each depth bin is Nm kg m^{-2} and thus the resuspended component of the concentration in each depth bin is

$$Nm/h_b(\mathrm{kg\,m^{-3}}).$$

But b_1N is the resuspension component of the model concentration in mg/l. Equating the



(a)



(b)

Fig. 5. (a) Concentrations for Mersey during spring tides at the height of $\sigma = 0.4$, (b) Components of model concentration for Mersey during spring tides at the height of $\sigma = 0.4$.

two concentrations, allowing for units yields a value of

$$m = b_1 h_b / 1000.$$

Substituting for m and N_e in the erosion rate expression, Eq. (8) gives

$$\mu = BU^P b_1 h_b / (1000 \Delta t)\,(\mathrm{kg\,m^{-2}\,s^{-1}}). \tag{9}$$

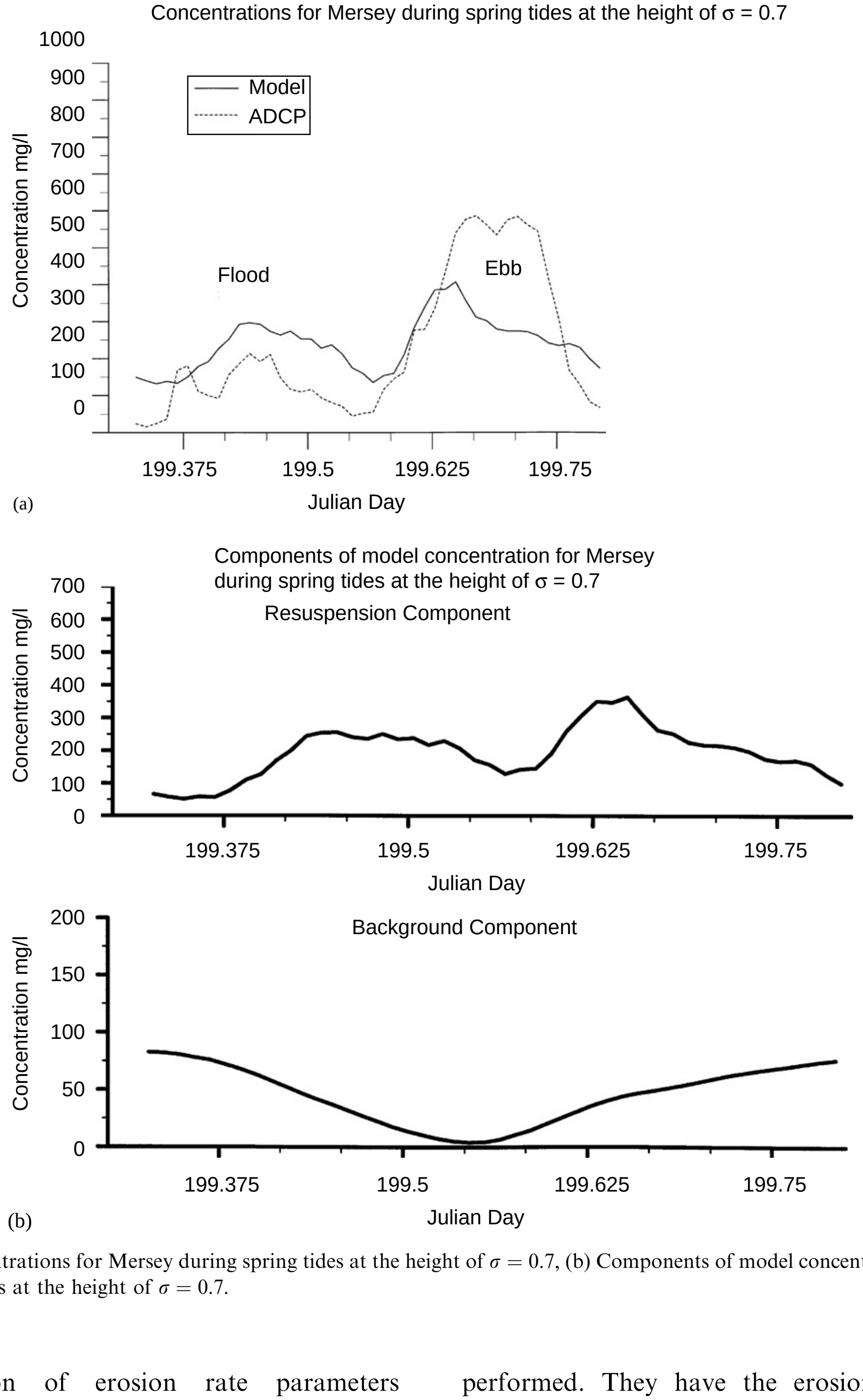
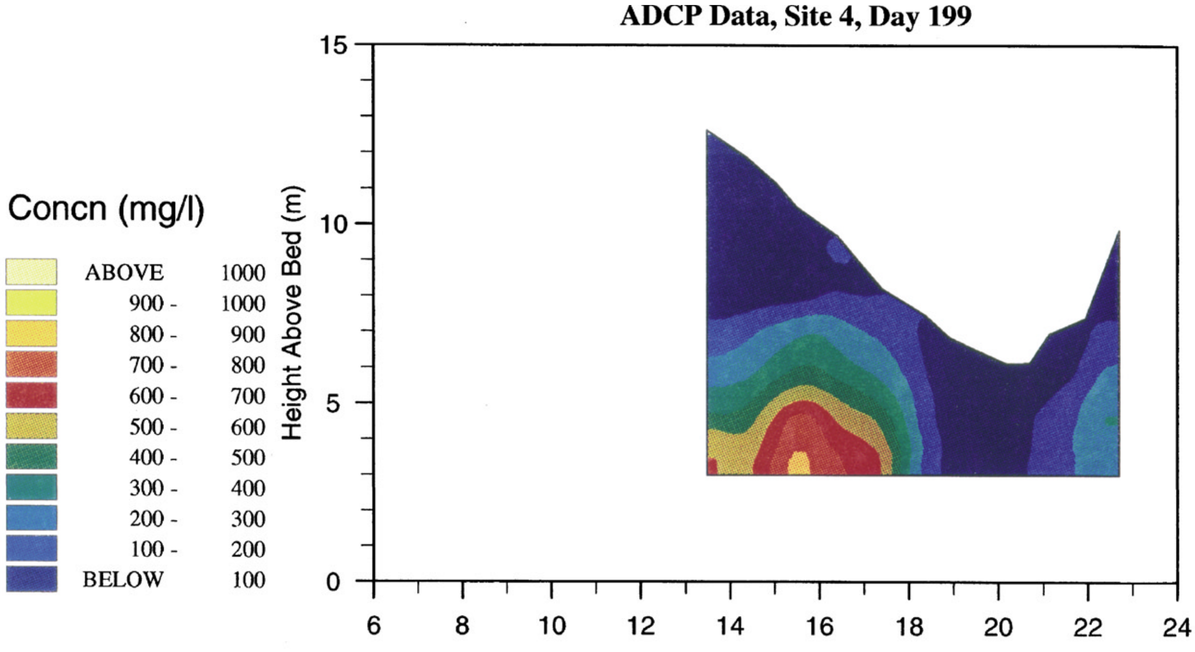


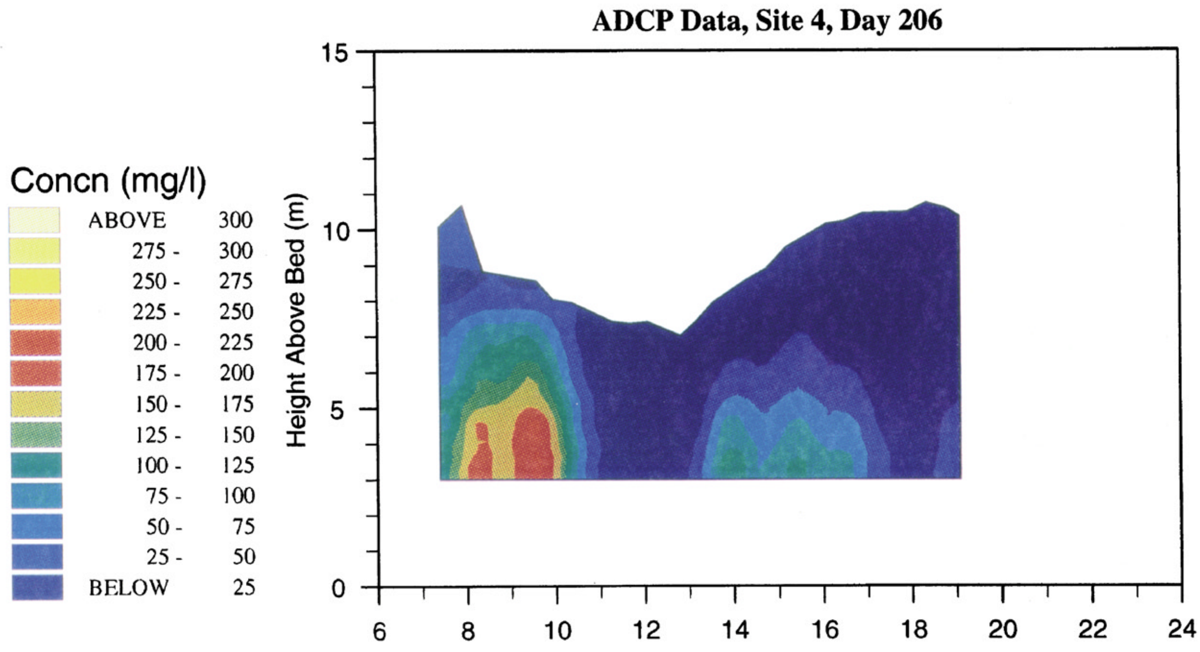
Fig. 6. (a) Concentrations for Mersey during spring tides at the height of $\sigma = 0.7$, (b) Components of model concentration for Mersey during spring tides at the height of $\sigma = 0.7$.

A comparison of erosion rate parameters with previously published values for Puget Sound, Washington and a number of other sites in Lavelle and Mofjeld (1984) can now be performed. They have the erosion rate in the form

$$\mu_{\mathrm{LM}} = \alpha|\tau|^{\eta}\ (\mathrm{g\,cm^{-2}s^{-1}}),$$



(a)



(b)

Fig. 7. (a) Profile from ADCP in the Mersey during spring tides, (b) Profile from ADCP in the Mersey during neap tides.

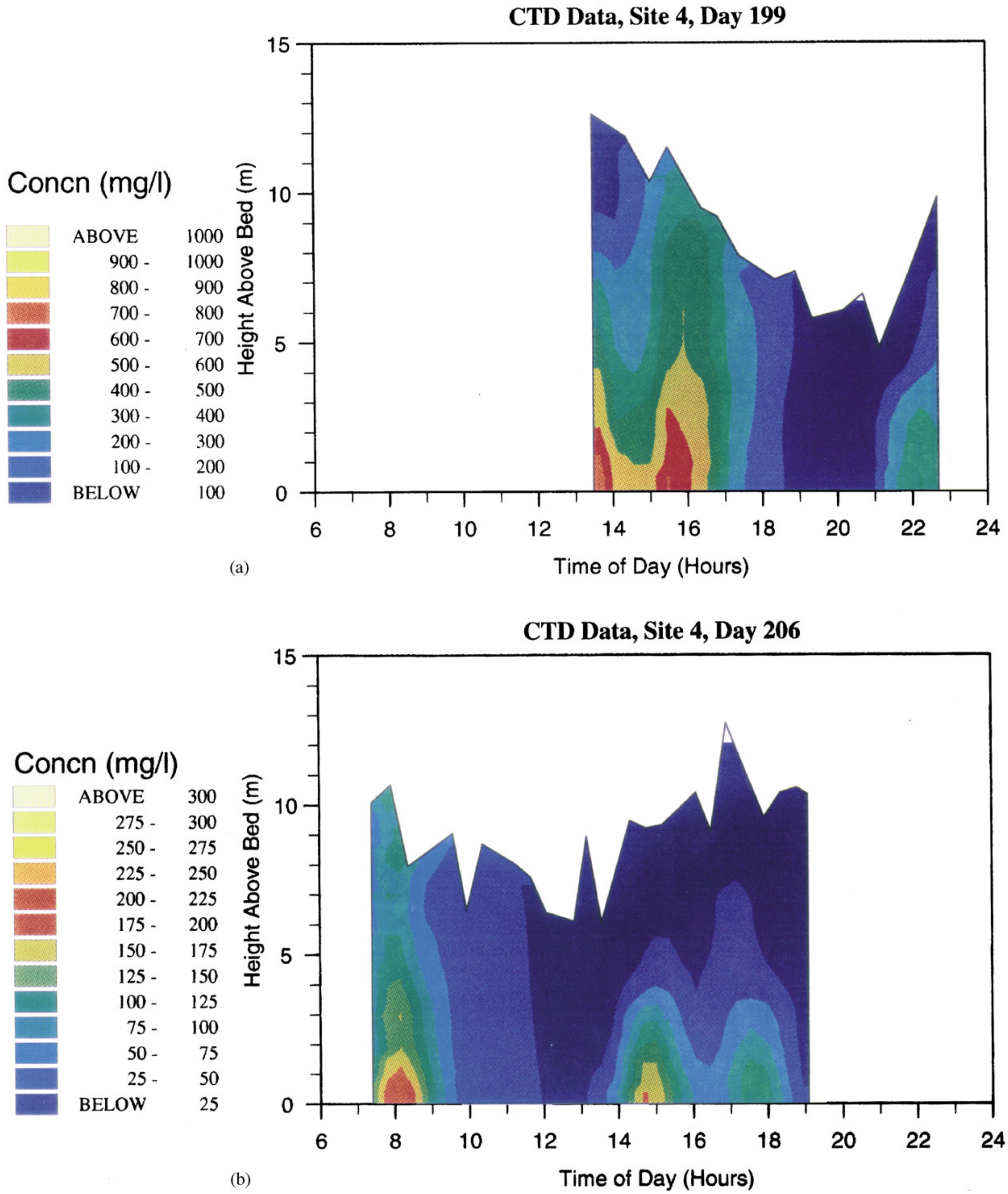


Fig. 8. (a) Profile from CTD-mounted transmissometer in the Mersey during spring tides, (b) Profile from CTD-mounted transmissometer in the Mersey during neap tides.

where τ is the bottom stress in dynes/cm^2. Now,

$$\tau = \rho C_D |U| U,$$

where U is the free stream velocity (cms^{-1}), ρ is the fluid density (g cm^{-3}) and C_D is a drag coefficient.

Thus,

$$\mu_{LM} = \alpha(\rho C_D)^\eta |U|^\eta U^\eta \ (\text{g cm}^{-2}\,\text{s}^{-1})$$

Writing

$$A = Bb_1 h_b/(1000\Delta t), \tag{10}$$

it can be seen from Eq. (9) that

$$\mu = A|U|^P \ (\text{kg m}^{-2}\,\text{s}^{-1}),$$

where U here is in m/s.

Noting that μ is in kg m^{-2}s^{-1} whereas the erosion rate in Lavelle and Mofjeld, μ_{LM}, is in cm/s, and noting that Lavelle and Mofjeld's velocity is in cm/s, it can be seen that

$$\eta = P/2 \tag{11}$$

and

$$\alpha = \frac{A}{10(100)^P(\rho C_D)^\eta}, \tag{12}$$

Now, $h_b = 2.83$ m and $\Delta t = 89.42$ s (500 timesteps per tidal cycle) so values of A can be calculated from Eq. (10) using values of B and b_1 from Table 1. Lavelle and Mojfeld parameters were then obtained taking $C_D = 1.6 \times 10^{-3}$ and $\rho = 1$ g cm^{-3}. The results are presented for the cases of just one settling velocity class for both sites in Table 2.

Lavelle and Mojfeld found values of α in the range 1.9×10^{-9} to 2.3×10^{-6} g cm^{-2} s^{-1} and η in the range 1.2–5, so these results are compatible. Note that C_D depends on sediment type but changing its value does not significantly change the values obtained for α.

6.3. Background horizontal concentration gradient

Recall from Sections 2.2 and 5 that the background horizontal concentration gradient being advected with the flow is just the tabulated value a_2. Some theory is now described which allows an estimate of this gradient to be calculated from observations of the measured time series for concentration and currents.

In depth-averaged form, the mass conservation equation is

$$\frac{\partial C}{\partial t} + (\underline{u} \cdot \nabla)C = E^2 C - \text{sinks} + \text{sources}. \tag{13}$$

The assumption is made, backed up by observations (Prandle et al., 1990) that for tidally resuspended material, the semi-diurnal cycle is dominant in generating the SPM signal.

So we can represent C and U by

$$C = C_0 + C_1 e^{i\omega t} + C_2 e^{i2\omega t}$$

and U_1 is approximately a rectilinear tidal current with

$$U = U_1 e^{i\omega t},$$

where ω is the angular frequency of the M_2 cycle. Selecting terms for the M_2 cycle, it can be seen from Eq. (13) that

$$i\omega C_1 + U_1 \frac{\partial C_0}{\partial x} = 0,$$

where the RHS is zero since the diffusion term is small compared to the other two terms (see Prandle et al., 1990). The background concentration gradient is then seen to be

$$\left|\frac{\partial C_0}{\partial x}\right| = \omega \frac{|C_1|}{|U_1|} \tag{14}$$

Table 2

Erosion rate parameter	Dover Straits (one W_s)		Mersey (one W_s)	
	Springs	Neaps	Springs	Neaps
A (kg$^{\pm 1}$m$^{-(3\pm 2)}$s^{-1})	1.29×10^{-10}	4.38×10^{-10}	1.25×10^{-10}	1.99×10^{-10}
P	7.1	9.73	2.98	4.57
(g cm^{-2} s^{-1})	6.8×10^{-9}	6.08×10^{-10}	2.01×10^{-6}	3.54×10^{-7}
	3.55	4.87	1.49	2.29

Table 3
Units are $\mathrm{mg\,l^{-1}\,m^{-1}}$

	Dover Straits	Mersey
Model	4.60×10^{-05}	0.85×10^{-02}
Estimate	5.38×10^{-05}	1.96×10^{-02}

Thus, a harmonic analysis of the measured series for concentration and currents will give values for $|C_1|$ and $|U_1|$ which can be substituted into Eq. (14) to infer background concentration gradient from observations.

In the Mersey, the presence of largely rapidly settling SPM is required for the analysis to be valid, else the RHS of Eq. (13) could not be assumed to be negligible. The above analysis is not valid in the Dover Straits where more than one particle class is to be found. Instead a value has been taken from the literature for the Dover Straits (Dyer and Moffat, 1992).

Table 3 shows the model estimates compared with the harmonically derived estimate from the observations in the Mersey and the directly observed value for the Dover Straits.

The order of magnitude agreement achieved is very good. Recall that the sign of the gradient is such that less material is offshore in the Mersey whilst in the Dover Straits it is greater to the South West.

7. Discussion

7.1. Characteristics of observed and model SPM time series

These results have to be understood in the context of the limitations of the approach. For example, the sites in the Dover Straits were in regions where bed sediments are homogeneous and where the bed itself is quite flat. These prerequisites for the model are perhaps compromised in the application to the Mersey. The flow is also assumed to be tidally dominant and data was chosen to be during storm-free periods. It is important to realise that it has been assumed that these waters are well mixed since the sediment eddy diffusivity has been equated to the eddy viscosity. This assumption is supported by observations.

Good agreement between modelled and observed SPM concentration time series has been shown for both spring and neap tidal cycles with even the most basic of the models presented, with an $R^2 = 0.61$ during springs and $R^2 = 0.74$ during neaps in the Dover Straits. The high frequency signal seen in the model results in Fig. 4a is believed to be a numerical problem with the resolution of the model, perhaps resulting in too few particles being suspended. In the Mersey, it is noticeable that the peak in concentration during ebb flow is modelled better than the flood. The flatter peak (Fig. 5a) observed during flood tide is perhaps due to the influence of relatively low SPM concentration water entering the estuary from the Irish Sea. There is an interesting pronounced dip in the observed concentrations at slack water during neap tides in the Mersey which may be due to flocculation of particles causing the settling velocity to increase (Eisma, 1993). One can see very high concentrations of around $1\,\mathrm{g\,l^{-1}}$ close (<5 m) to the bed at times of maximum flow. The characteristic "twin peaks" signal can be clearly seen.

The time series plots depicted in Fig. 9a and b for the 4 bins closest to the bed during spring and neap tides respectively show that agreement, whilst not perfect, between the two instruments is quite good. The tendency for the transmissometer to give higher values than the ADCP higher up in the water column could be due to the fact that the acoustic instrument is relatively more sensitive to larger diameter material. These differences could perhaps be exploited in future work if particle size measurements were made at the same time.

7.2. Enhanced correlation

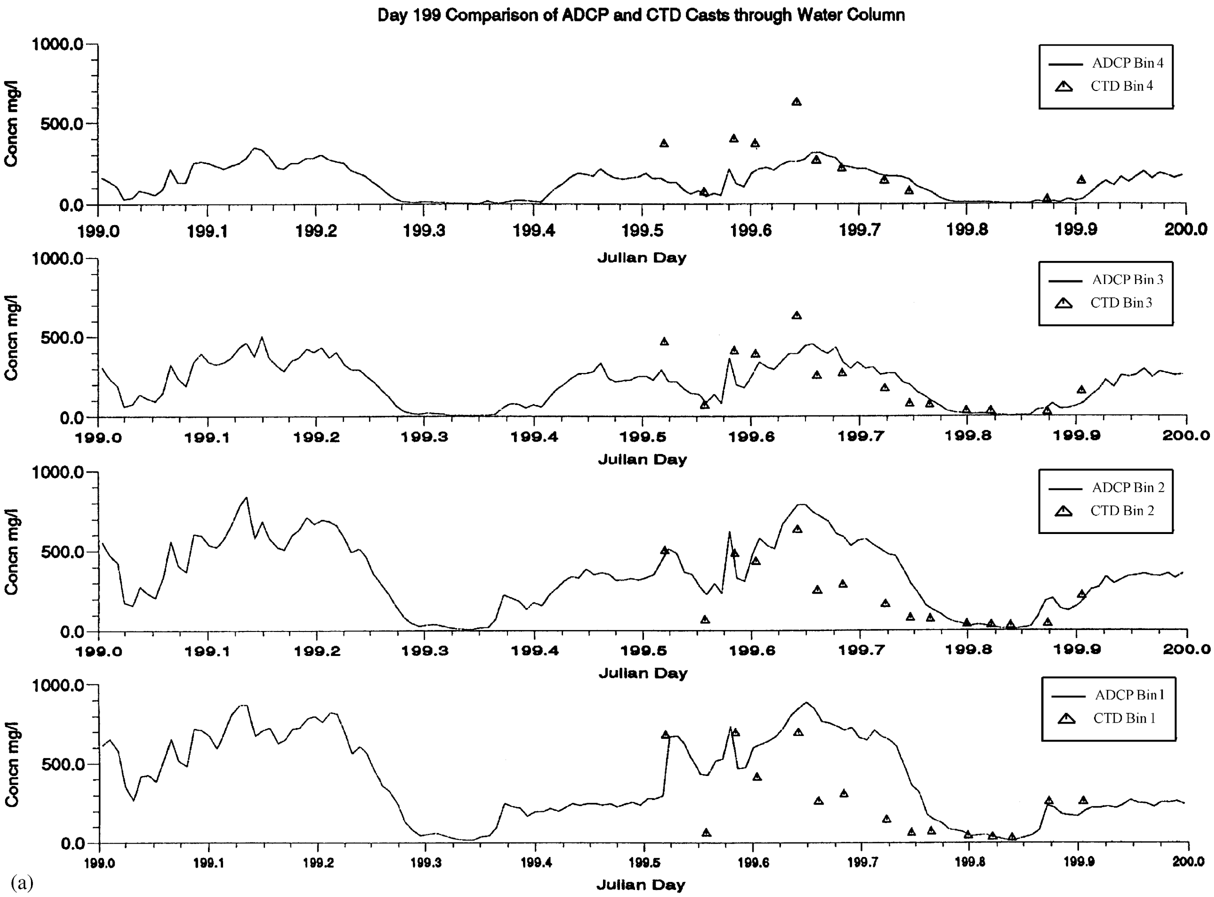
Substantial improvements in agreement were achieved when a time varying eddy diffusivity was included in the Mersey runs. The improvement in R^2 was 3.5% at springs but 19.8% at neaps.

The improvement in R^2 gained when introducing a second settling velocity class in the Dover Straits was small because imposing a background concentration can effectively simulate a low settling velocity class already in suspension. The explicit inclusion of the additional settling velocity class is only a slightly more accurate representation of reality.

7.3. Optimisation procedure

Regarding the optimisation procedure, the downhill simplex method was chosen since it is a robust method. It is robust to the choice of the $N+1$ vertices of the initial simplex that is given by the user (where N is the number of parameters being optimised) in its search in the N-dimensional space for the minimum of the cost function. Extensive tests were carried out with different initial simplex values that showed that the same solution was obtained irrespective of the starting position. This was not always the case for other optimisation methods that were explored, such as Powell's method. With hindsight, further gains in performance might be achieved by utilising simulated annealing methods which are purpose designed to find global minima. However, there is a large computational cost associated with this choice.

One reason why the optimisation procedure failed to find a solution during neap tides in the Dover Straits application with two settling velocity classes might be that the lower current speeds were only capable of eroding the smaller settling velocity class so the procedure was being asked



(a)

Fig. 9. (a) Comparison of ADCP and CTD casts through the water column during spring tides, (b) Comparison of ADCP and CTD casts through the water column during neap tides.

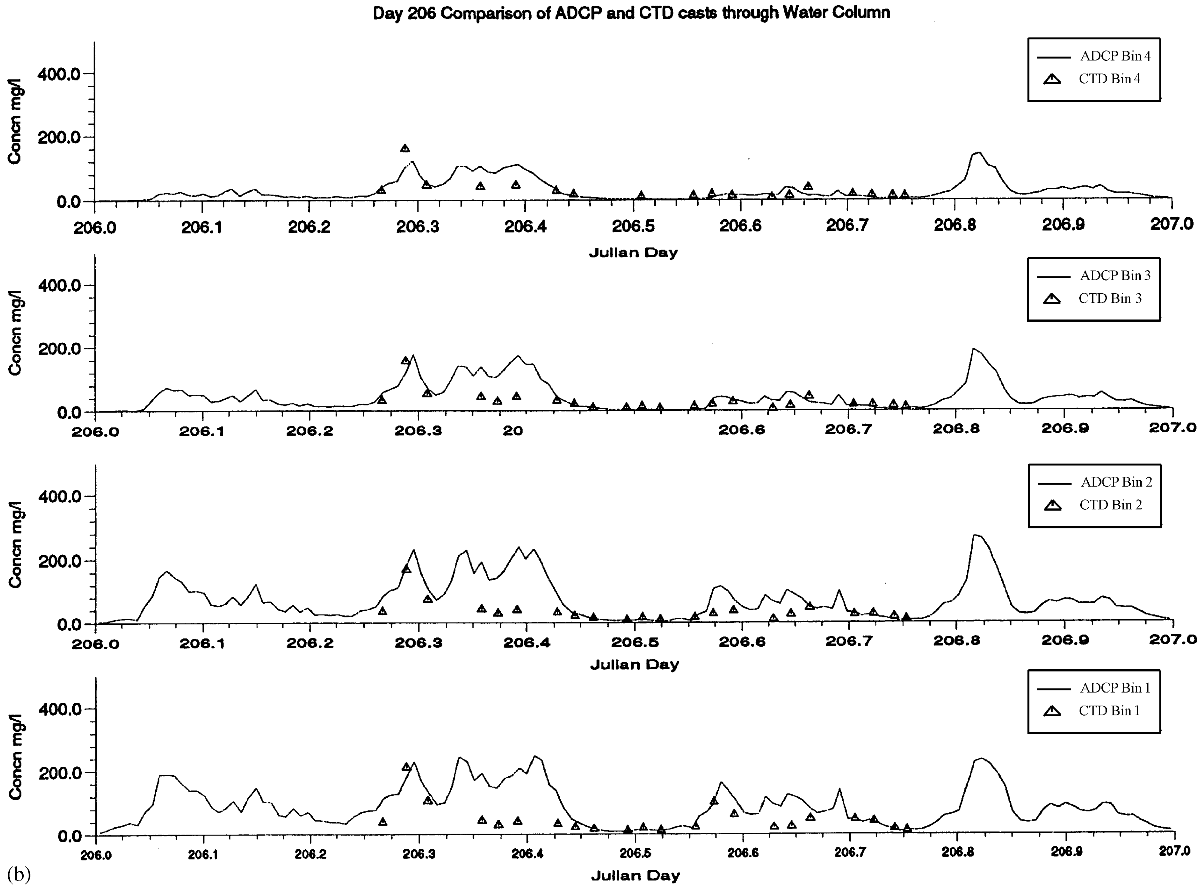


Fig. 9 (*continued*).

to solve an intractable problem since only one, not two, classes were present.

7.4. Future developments

The techniques developed here can be extended to cover complete spring-neap cycles. Whilst such an extension adds complexity, it also incorporates important additional characteristics of the sediment regime. This is likely to require incorporation of a wider range of particle sizes and time-variation in the background advective terms. Some pre-selection of appropiate time sequences (to avoid episodic events) and frequency filtering may be necessary to focus on the more tractable aspects. No particle size data were collected for these studies but one challenge for the future might be to utilise particle size measurements (now more commonly available from in-situ laser instruments) to improve the calibration of the ADCP. It is certainly to be recommended that such particle size measurements be made in conjunction with ADCP backscatter measurements.

Extending the model to include direct calculation of turbulent intensity is an obvious next step and would readily accommodate effects such as aggregation, wind-wave interactions and damping by excess density. Future work could also concentrate on better representations of the vertical structure, including parameterisations of the eddy diffusivity as a function of depth. It is to be noted that the modular structure of the optimisation methodology is such that any vertical model of SPM dynamics, not just the particle tracking model described herein, could be inserted quite easily.

8. Conclusions

A methodology has been established for obtaining estimates of physical sediment parameters, specifically background concentration gradient, erosion rates and settling velocities, by employing a simple vertical sediment dynamic model with an optimisation scheme which utilises combined current velocity and SPM concentration data obtained from Acoustic Doppler Current Profilers. Validation of the methodology has been performed by comparison with SPM concentration measurements at a range of heights above the bed during both spring and neap tides at two contrasting sites whose mean SPM concentrations differ by around three orders of magnitude. Good agreement at both sites was achieved using the simplest formulation of the model. Its modular design allowed extension to include a time varying eddy diffusivity for the Mersey estuary site, which produced a 20% improvement in R^2 during neap tides. A small improvement of 4% was also achieved for the Dover Straits site during spring tides when two settling velocities were incorporated.

The model was validated in three further, independent ways. Values for the settling velocity calculated in the optimisation process showed good agreement with settling tube measurements. The optimisation process allowed the estimation of two bed erosion parameters, which in turn yielded erosion formulae consistent with values presented in the literature. The SPM concentration signal has been decoupled into quantified resuspension and advection components. Comparisons of the model background concentration gradient showed good agreement with estimates obtained from observations at both sites. These independent validations establish the methodology as a useful tool in the modelling and analysis of SPM dynamics and show how much implicit information has been derived from the ADCP backscatter with the aid of a particle tracking model and an optimisation procedure. The present focus on optimal fitting over a semi-diurnal tidal cycle could easily be extended to complete spring-neap cycles. However, shorter discrete applications have the advantage of responding to spring-neap variations in sediment type and associated (limited) availability. A succession of applications over a spring-neap cycle can then provide a useful basis for intercomparison, together with indications of the associated cyclical variability.

Acknowledgements

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