EESC 9945
Geodesy with the Global Positioning System
Class 1: Introduction

1. Class meeting time
2. Overview of term
3. Term Project: Point positioning
4. Introduction to the Global Positioning System
Aim of Class

To introduce the principles of the use of the Global Positioning System, including:

- GPS components (segments)
- GPS observations
- Using GPS observations to estimate positions

*Emphasis is for geoscience user of GPS, not necessarily for geodesists*
Slides from Classes

- Available at [http://www.ldeo.columbia.edu/~jdavis/eesc9945.htm](http://www.ldeo.columbia.edu/~jdavis/eesc9945.htm) soon after class
Course Outline

- Introduction
- Satellite orbits
- GPS constellation
- GPS observables
- Propagation medium
- Point positioning using pseudorange
- Relative positioning using carrier-beat phase (2 classes)
- Terrestrial reference frames
- Positioning using stochastic filtering (2 classes)
- GPS errors
Term Project (for credit): *Point positioning using Coarse Acquisition Code*

- Can work in teams of 2–3
- Overall goal of project is to write code to calculate Earth-fixed positions using pseudorange (similar to navigation devices)
- Can use any computer programming language (including MATLAB)
- Project will have four sub-goals; each sub-goal has one subroutine to develop:
  1. **Due 15 Feb**: Read broadcast ephemerides in RINEX format for 24-hour block and return Cartesian coordinates for all satellites for closest epoch
  2. **Due 1 Mar**: Calculate Earth-centered Cartesian coordinate for GPS satellite given broadcast ephemeris for specific epoch
  3. **Due 29 Mar**: Read GPS observations in RINEX observation file and return list of observed satellites and ionosphere-free pseudorange observables
  4. **Due 26 Apr**: Use observables and satellite coordinates in least-squares solution for site position
- Further details next week
- **This week’s assignment**: E-mail me team members before next class
Course Topics

- Satellite orbits
  - What are the motions of artificial satellites?
  - How is their position described?
  - How do we translate between Keplerian satellite ephemerides and Earth-centered Cartesian coordinates?
  - Where can we obtain GPS ephemerides and what is their format?
Course Topics

• GPS Constellation
  – How many satellites are there and how do we label them?
  – How are they configured within this constellation?
  – How are GPS satellites controlled and how do they keep time?
  – What is the signal broadcast by the GPS satellites?
Course Topics

• GPS Observables
  – What are the phase and pseudorange observables?
  – Models for these observables
Course Topics

• Propagation medium
  – How do the ionosphere and neutral atmosphere impact propagation of the GPS observations?
  – How do we deal with these effects?
Course Topics

• Point positioning using pseudorange
  – Point positioning
  – How do we estimate the point position of the GPS receiver using least squares and the pseudorange observable?
Course Topics

- Relative positioning using carrier-beat phase
  - Relative positioning
  - What are the phase observables used in relative positioning?
  - Data processing considerations
  - Orbit estimation
Course Topics

• Terrestrial reference frames
  – Definition
  – Relevance to GPS
  – How do we establish reference frames?
Course Topics

• Stochastic filtering
  – Dynamic systems
  – Stochastic systems
  – Prior constraints and sequential least squares
  – The Kalman filter
Course Topics

• GPS errors
  – What are the main sources of error impacting GPS?
The Global Positioning System (GPS)

- The Global Positioning System (GPS) is a Global Navigational Satellite System (GNSS)
- US Department of Defense
- Operational 1993

Some web sites:
- [http://www.gps.gov](http://www.gps.gov) (Main government GPS portal; includes Civil GPS Service Interface Committee)

Other Global Satellite Navigational Systems:
- GLONASS (Russia, 1995)
- Galileo (Europe, 2019)
- Beidou/Compass (China, 2020)
The Global Positioning System

Three Segments:

1. Space segment

- Official constellation name: NAVSTAR (Navigation Signal Timing and Ranging)
- 31 usable satellites (1/15/2013)
- 20,000 km altitude
- Arranged in six orbital planes (4–6 SV each)
- 24-hr geosynchronous orbits
- Multiple satellite designs (blocks) in use at one time
- Constellation monitored by USNO (tycho.usno.navy.mil/gps.html)
The Global Positioning System

Three Segments:
1. Space segment
2. Control segment

- Monitor health of satellites
- Uplink to satellites (commands, data)
- Performs solutions and uploads ephemerides and clocks to SVs
The Global Positioning System

Three Segments:
1. Space segment
2. Control segment
3. **User segment**
   1. Science
   2. Agriculture
   3. Aviation
   4. Civilian navigation
   5. Surveying
   6. Recreation
   7. Space
   8. Timing
GPS is a Timing System

At 10:00:00, your GPS receiver hears
“The current time at satellite #24 is 10:00:00.00”

Time for the signal to reach us: **0.080 000 000** seconds
Speed of light: **299 792 458** m / s
Distance to #24: **299 792 458 \times 0.08 = 23,983.39664** km

“The current time at satellite #24 is 10:00:00.00”
Time Systems

- **TAI (International Atomic Time)**
  - Time kept by atomic frequency standards
  - *Second* is $9,192,631,770$ cycles of radiation from transition between two hyperfine levels of Cesium-133 ground state

- **UTC (Universal Coordinated Time)**
  - Atomic time scale that gives mean solar time
  - Sometime called “Greenwich Time” since predecessor was GMT (Greenwich Mean Time)
  - Offset from TAI by leap seconds

- **GPS time**
  - Runs with TAI, except offset: $\text{TAI} = \text{GPS} + 19\ s$
  - GPS time and UTC agreed on 1980 January 6.0 (GPS standard epoch)
  - Therefore, GPS time and UTC time disagree by the number of leap seconds since 1980 January 6.0
  - GPS time is often measured in weeks and seconds of week since GPS standard epoch
Time and Frequency

• Time is now measured using a frequency standard
• The phase of a uniformly running frequency standard is \( \phi(t) = 2\pi f_\circ t + \phi_\circ \)
• An error in the frequency therefore causes a time-dependent error in the phase of \( \delta \phi(t) = 2\pi \delta f(t) t \)
• This is known as a clock error
GPS Clock Errors

- The GPS satellites carry on-board atomic clocks (Cs at first, Rb for latest SVs)
- The clocks are maintained (clock corrections uploaded) by monitoring by the GPS ground segment
- GPS receivers also have accurate clocks and can also be connected to external 1 pps source
- The differences between the clock readings and the “true” time measured in a common system are the clock errors:
  \[ t(\text{rec}) = t + \delta_r \quad t(\text{sat}) = t + \delta^s \]
GPS Clock Errors

- The *observed* propagation time of a GPS signal from SV to Rxr is the receive time *measured by the receiver* minus the transmit time measured by the satellite:
  \[
  \tau_{\text{obs}} = t_r(\text{rec}) - t^s(\text{sat}) = t_r + \delta_r - t^s - \delta^s = \tau_{\text{true}} + \delta_r - \delta^s
  \]
- Thus, clock errors bias the estimates of the propagation time
- The electrical path length between transmitter and receiver is related to the propagation time by \( \rho = c\tau \) where \( c \) is the speed of light in vacuum
Pseudorange

- Assuming travel through vacuum, the pseudorange is the observed geometric range, i.e., the true range biased by clock errors:

\[ \rho(t) = cT_{\text{true}} + c(\delta_r - \delta^s) = |x^s(t - \tau) - \vec{r}_r(t)| + c(\delta_r - \delta^s) \]

- The pseudorange is sometimes called the code observable for reasons we will discuss several classes from now
Suggested Background Resources

